Sequential data assimilation for subsurface flow modeling in civil engineering

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Today's Work, Tomorrow's Heritage

Roles of subsurface flow analysis in civil engineering

- Impact assessment of the construction in design phase
- Regular monitoring in operation and maintenance
- Safety assessment with long-term forecast for geological nuclear waste disposal

Geological disposal of radioactive waste

Groundwater contamination
at Fukushima nuclear power plant

Difficulties in the subsurface flow simulation

We have carried out deterministic simulation under huge uncertainties.

- Sparse observations
- Geological heterogeneity and discontinuity
- Incomplete knowledge about hydraulic parameters and boundary conditions

Objectives

- ➢ Estimate the hydraulic parameters that can represent the current state of subsurface flow.
- ➢ Construct the reliable subsurface flow model for forecasting

Our solution : sequential data assimilation

- Efficient use of observations
- Easy implementation
- Online method

Sequential-based data assimilation methods

- Particle Filter
  - No assumption
  - Very easy implementation
  - Expensive computation cost
- Ensemble Kalman Filter (EnKF)
  - Relatively small ensemble size
  - Gaussian assumption

Versatile PF or Efficient EnKF ?

Numerical experiments

Application to an actual full-scale model

Step1: Twin experiments with synthetic data

- Evaluate the performance of PF and EnKF
- Identify the issues of the data assimilation method

Step2: Assimilation test using real observation (in progress)

- Identify the issues of the model and observations

Kikuma underground oil storage

Saturated-unsaturated subsurface flow model equation

$$\frac{\partial}{\partial x}\left(K\frac{\partial h}{\partial x}\right)=(C+\beta S_S)\frac{\partial h}{\partial t}$$

Discretize in time using $\frac{\partial h}{\partial t}\approx\frac{h_t-h_{t-1}}{\Delta t}$

$$h_t=\left\{1+\frac{\Delta t}{C+\beta S_S}\frac{\partial}{\partial x}\left(K\frac{\partial}{\partial x}\right)\right\}h_{t-1}$$

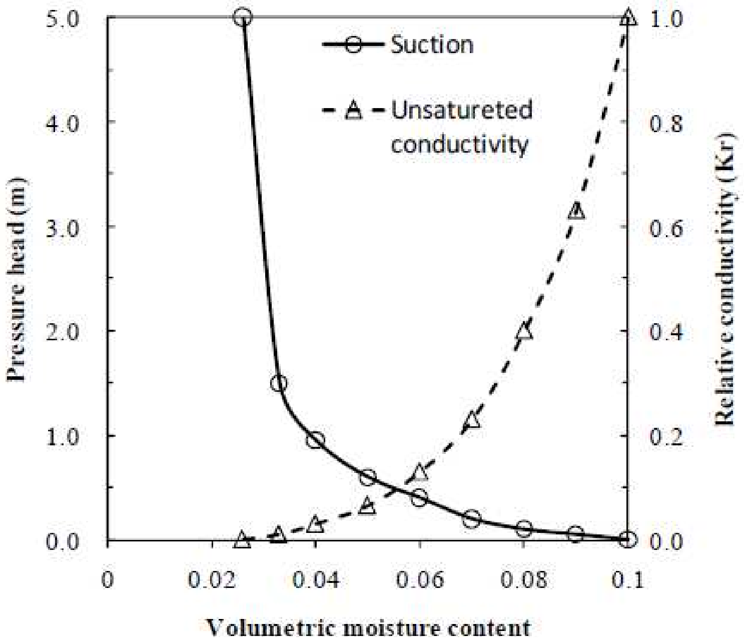
h : Pressure head

K : Permeability

C : Specific water capacity (if saturated, C=0)

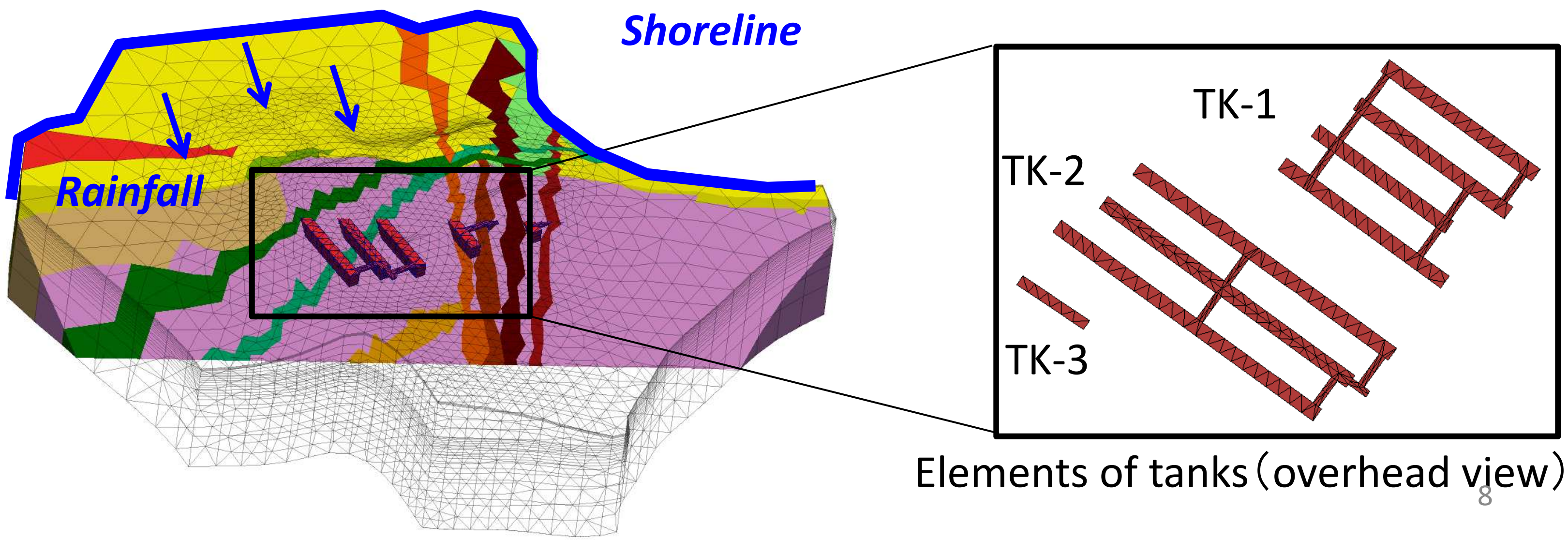
S_S : Specific storage

β = 1 saturated / = 0 not saturated



Summary of subsurface flow simulation

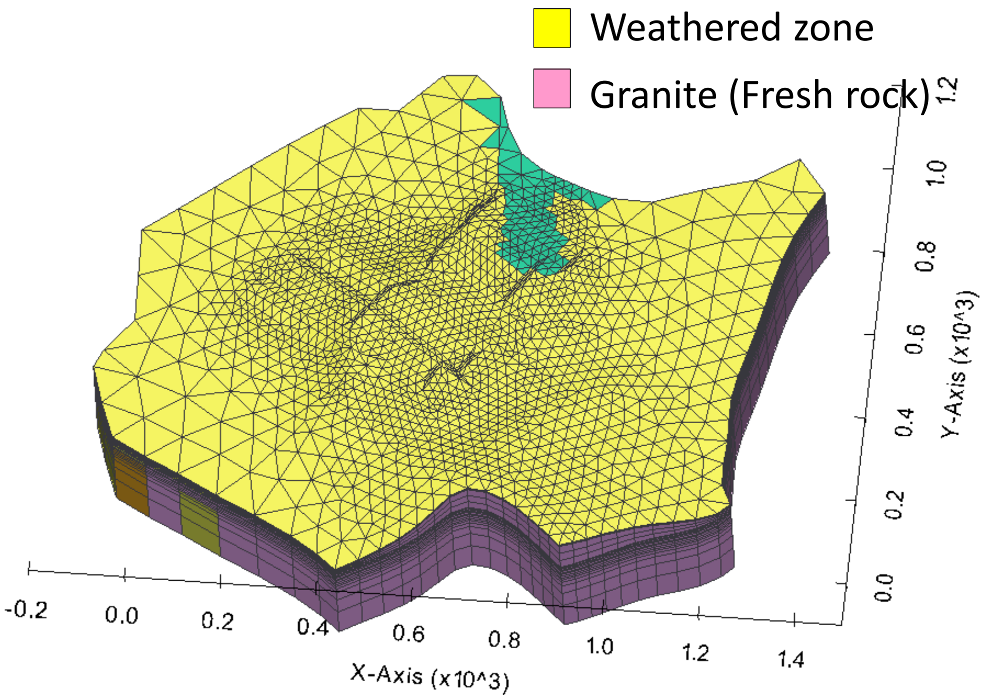
Solver	Finite Element Method
Dimension	34,000 nodes 50,000 elements
Boundary conditions	• Water seal pressure • Internal tank pressure (Oil level) • Groundwater recharge • Shoreline
Initial condition	Use the result of static analysis



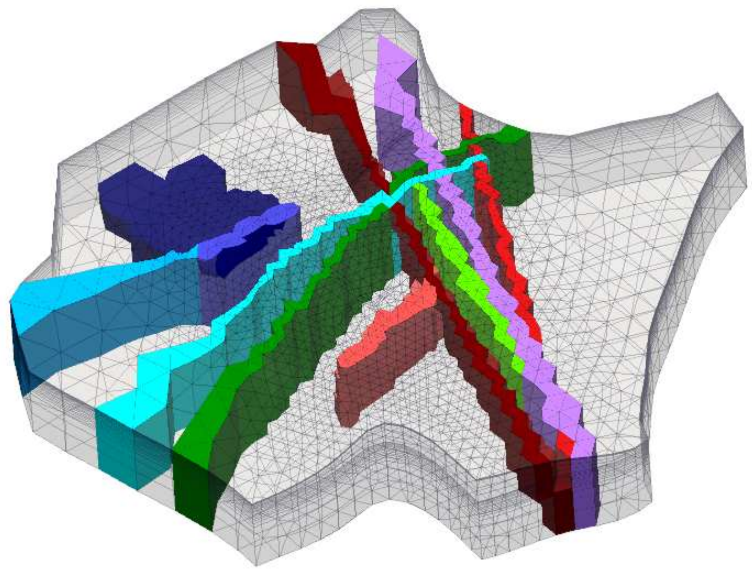
Elements of tanks（overhead view）

Geology

5 Geological units



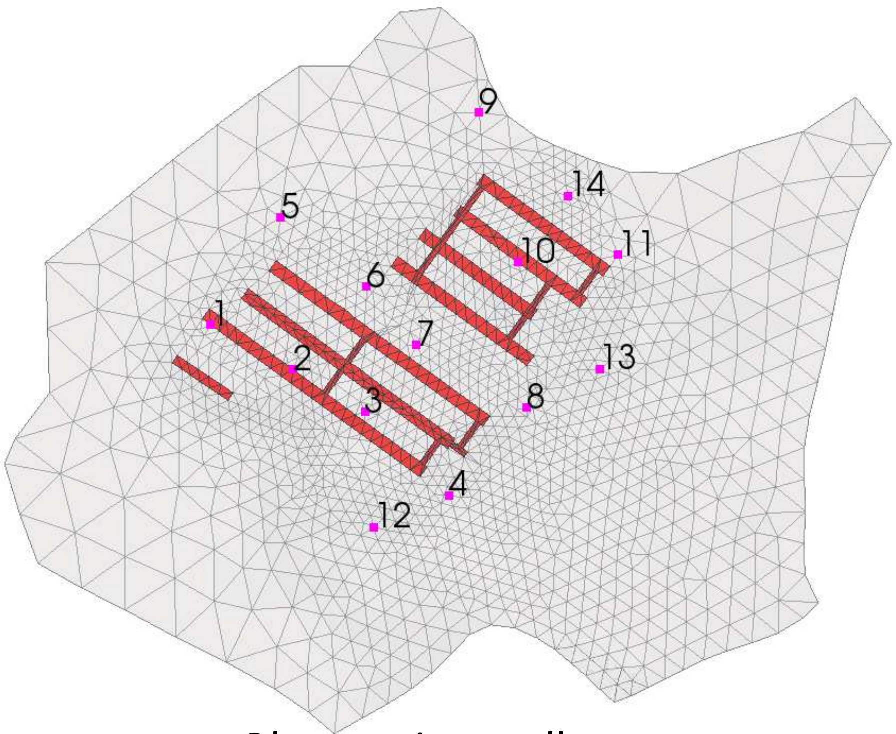
10 Fracture zones



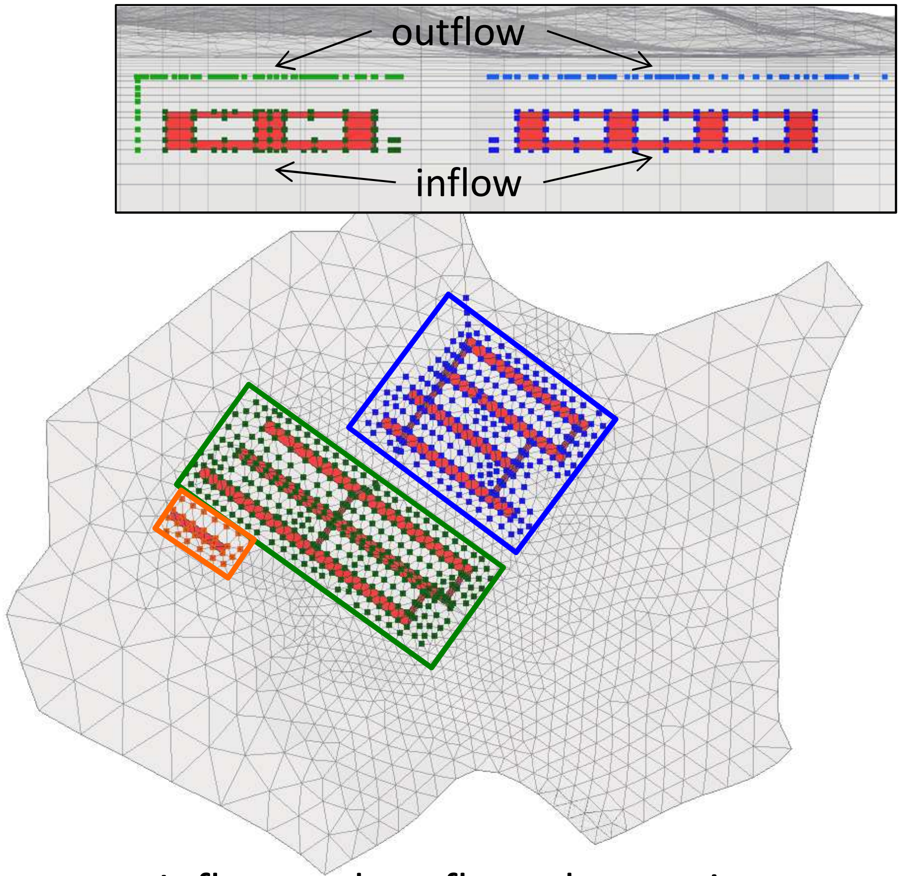
We estimate 15 hydraulic parameters

Observations

- Groundwater level at 14 boreholes
- Inflow and outflow volumes around 3 oil tanks (total 6 observations)



Observation wells



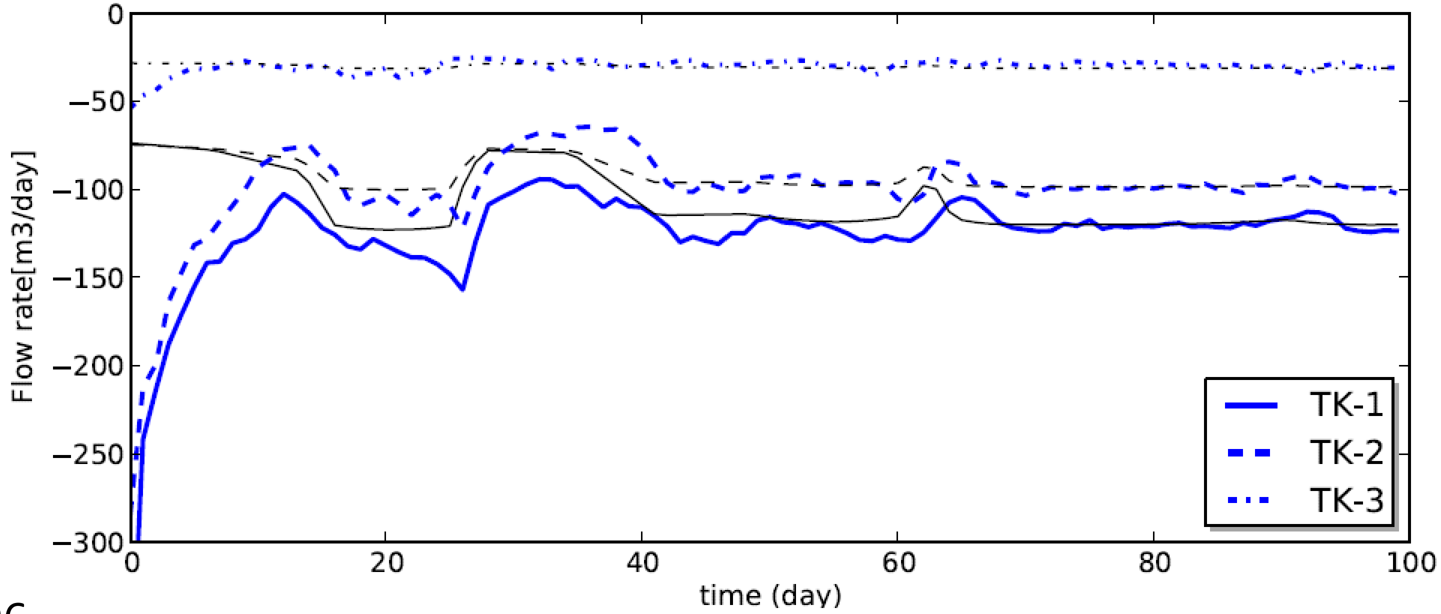
Inflow and outflow observations

Summary of Condition of data assimilation

	PF (SIR)	**EnKF** (PO)
Estimated parameter	15 permeability parameters $\log(K)$	
Observations	20 observations (Pressure head and flow volume)	
State vector	$\mathbf{x}=(h_1, \dots, h_{Nn}, Q_1, \dots, Q_{Nn}, \mathrm{Log}(K)_1, \dots, \mathrm{Log}(K)_{15})^{\mathrm{T}}$ h : Pressure head, Q : Flow rate, Nn : number of nodes	
System noise	$\log(K_{t,j}) = \log(K_{t-1,j}) + v_{t,j}$ $v_{t,j} \sim N(0,\sigma)$	None
Observation noise	$N(0,\sigma_j^{o})$	
Initial distribution of log(K)	Geological unit : $N(-4,\sigma_j^{\mathrm{ini}})$ Fracture zone : $N(-2,\sigma_j^{\mathrm{ini}})$	

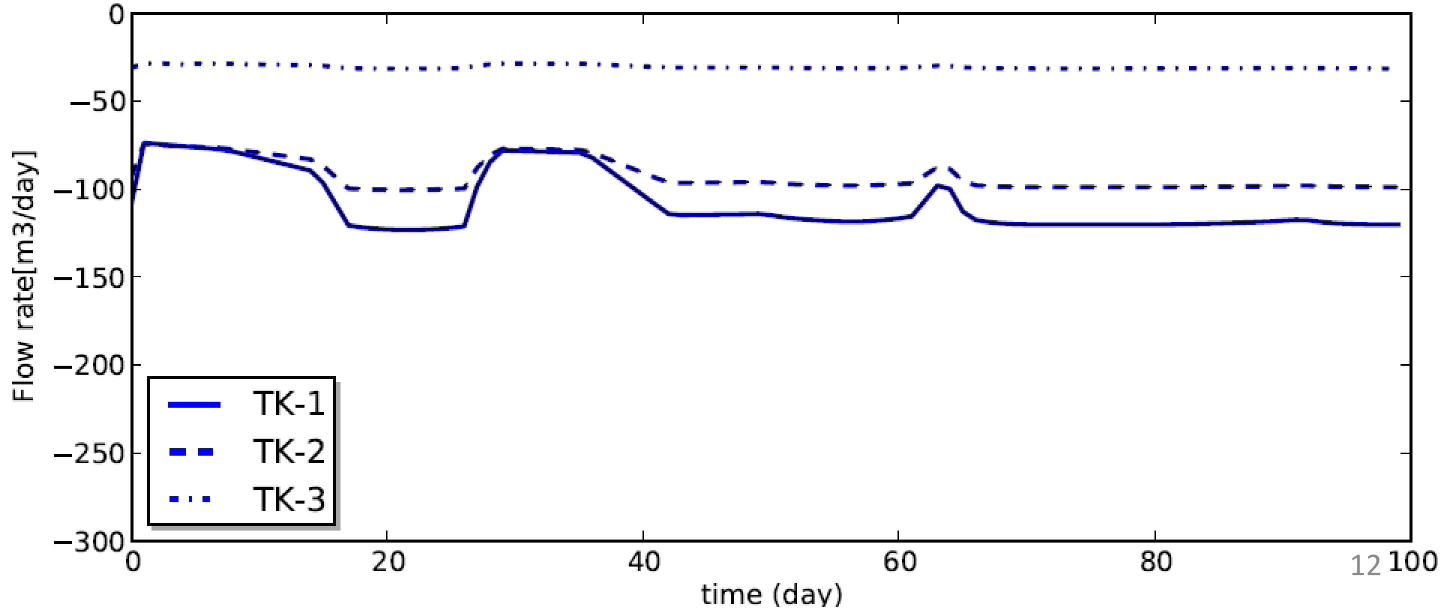
Assimilation results of water inflows in tanks

PF



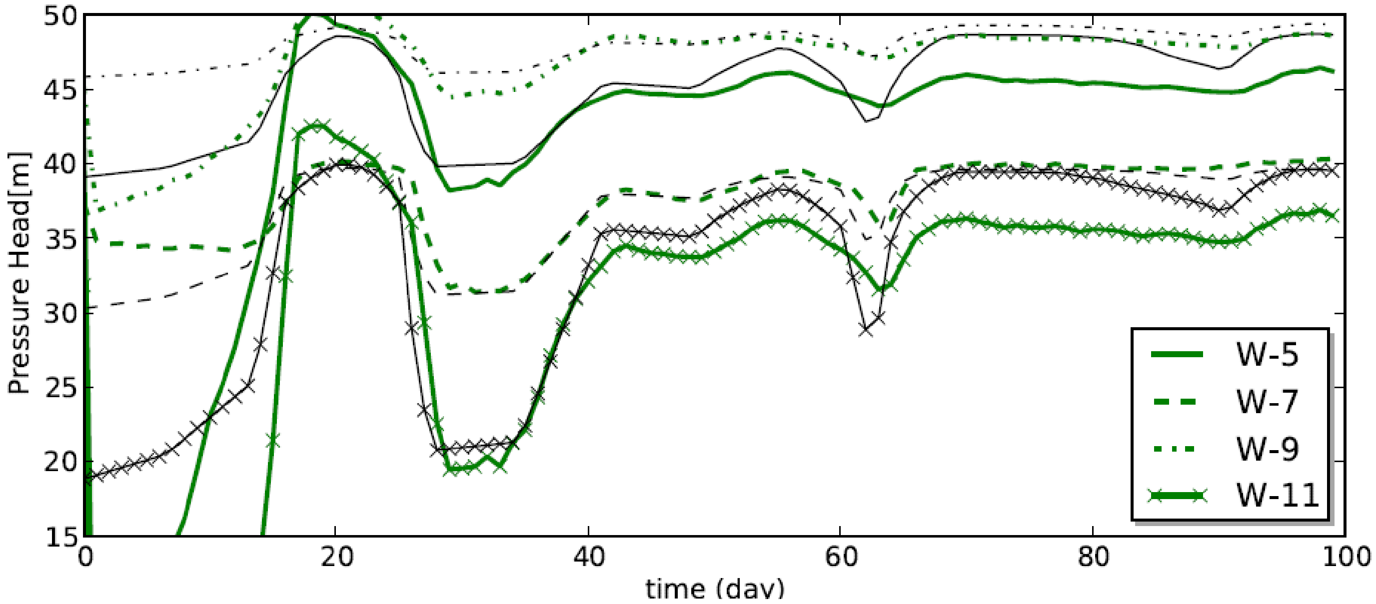
Ensemble size = 96

EnKF



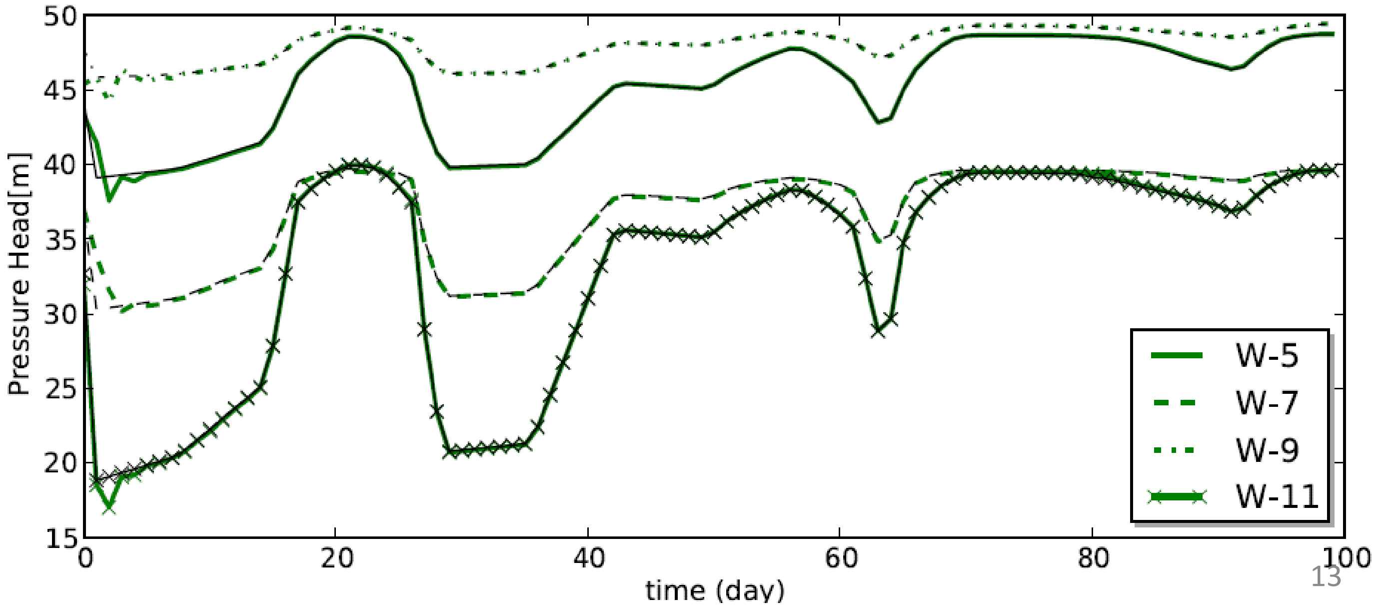
Assimilation results of pressure head

PF



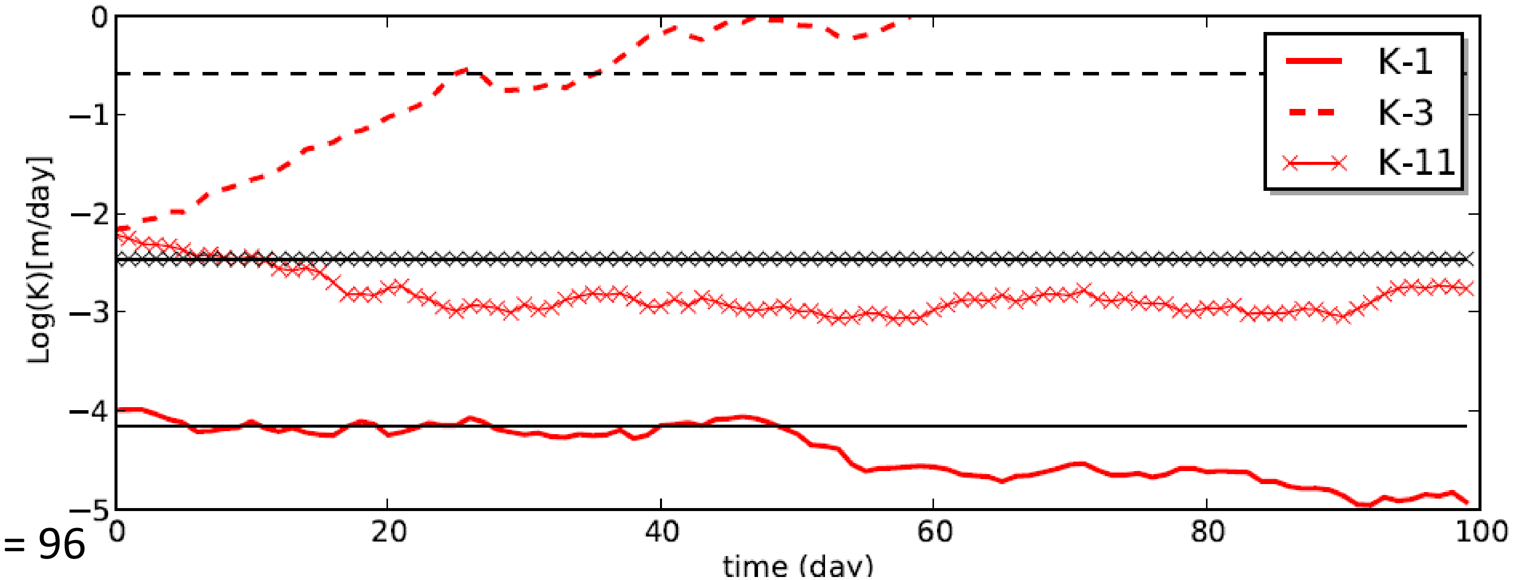
Ensemble size = 96

EnKF



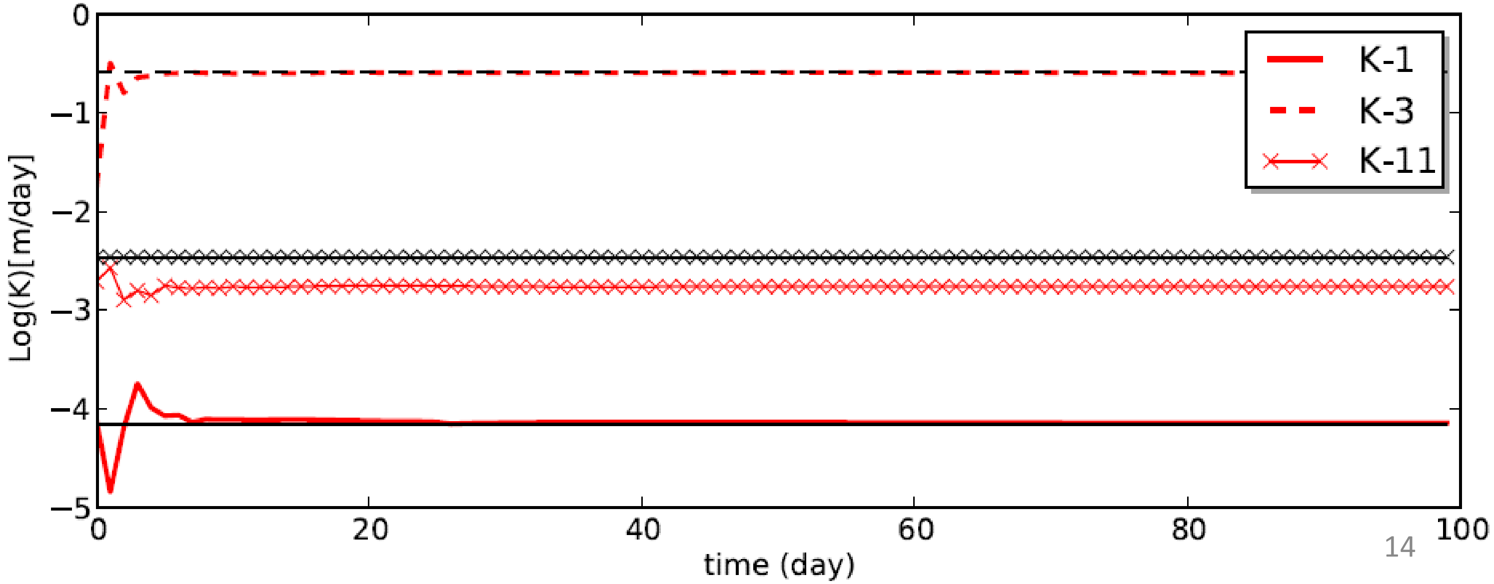
Estimation results of log(K)

PF

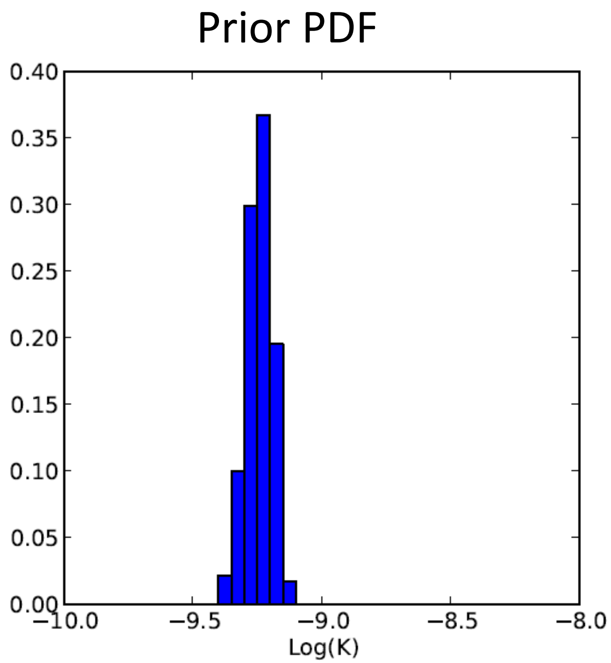


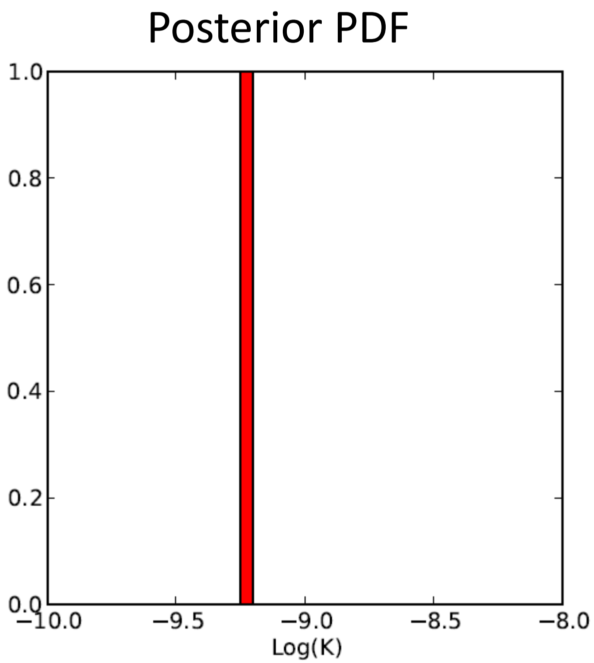
Ensemble size = 96

EnKF



Degeneration of particle filter



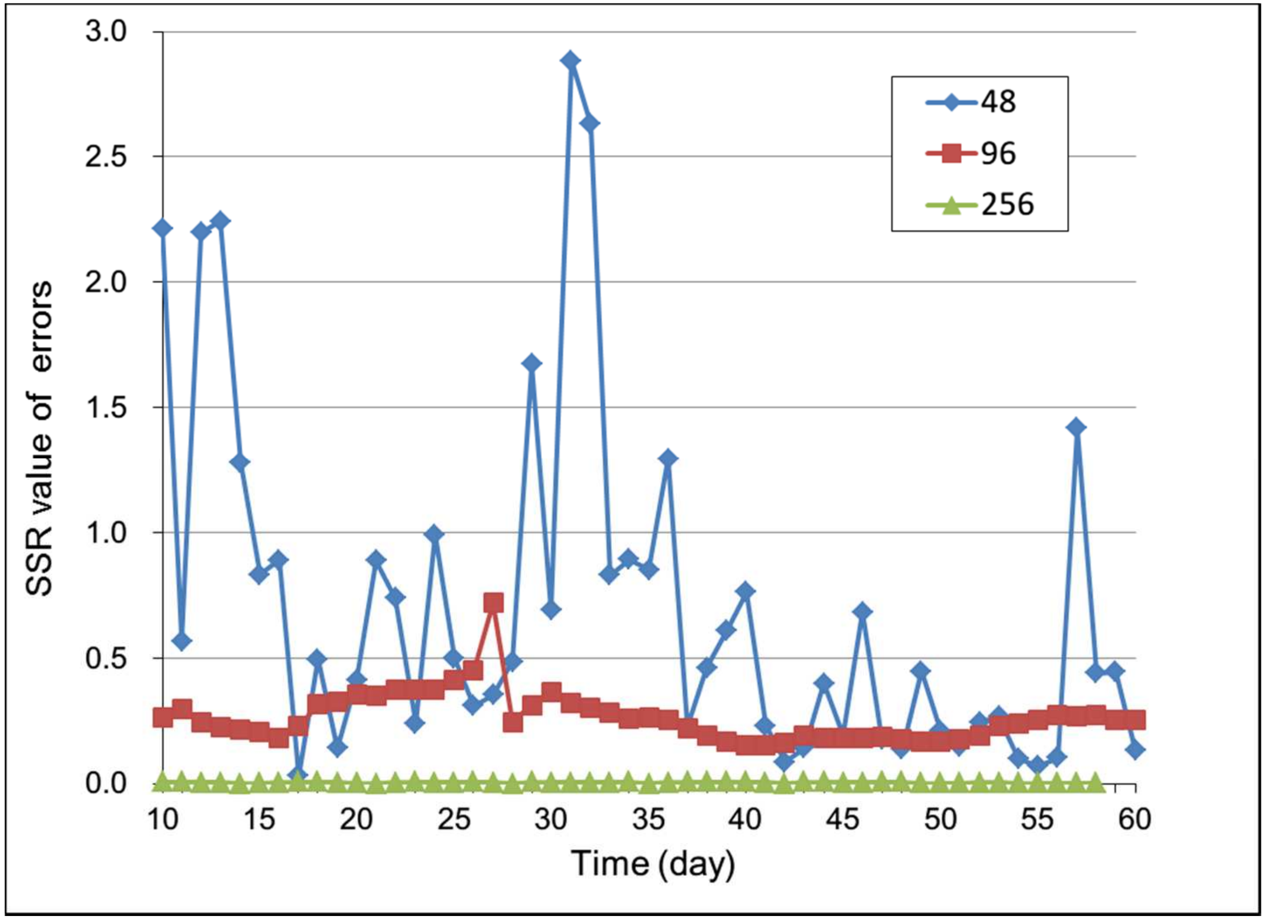


Probability distribution of log(K) at 60th day (ensemble size=512)

- PF collapses completely in every time step.

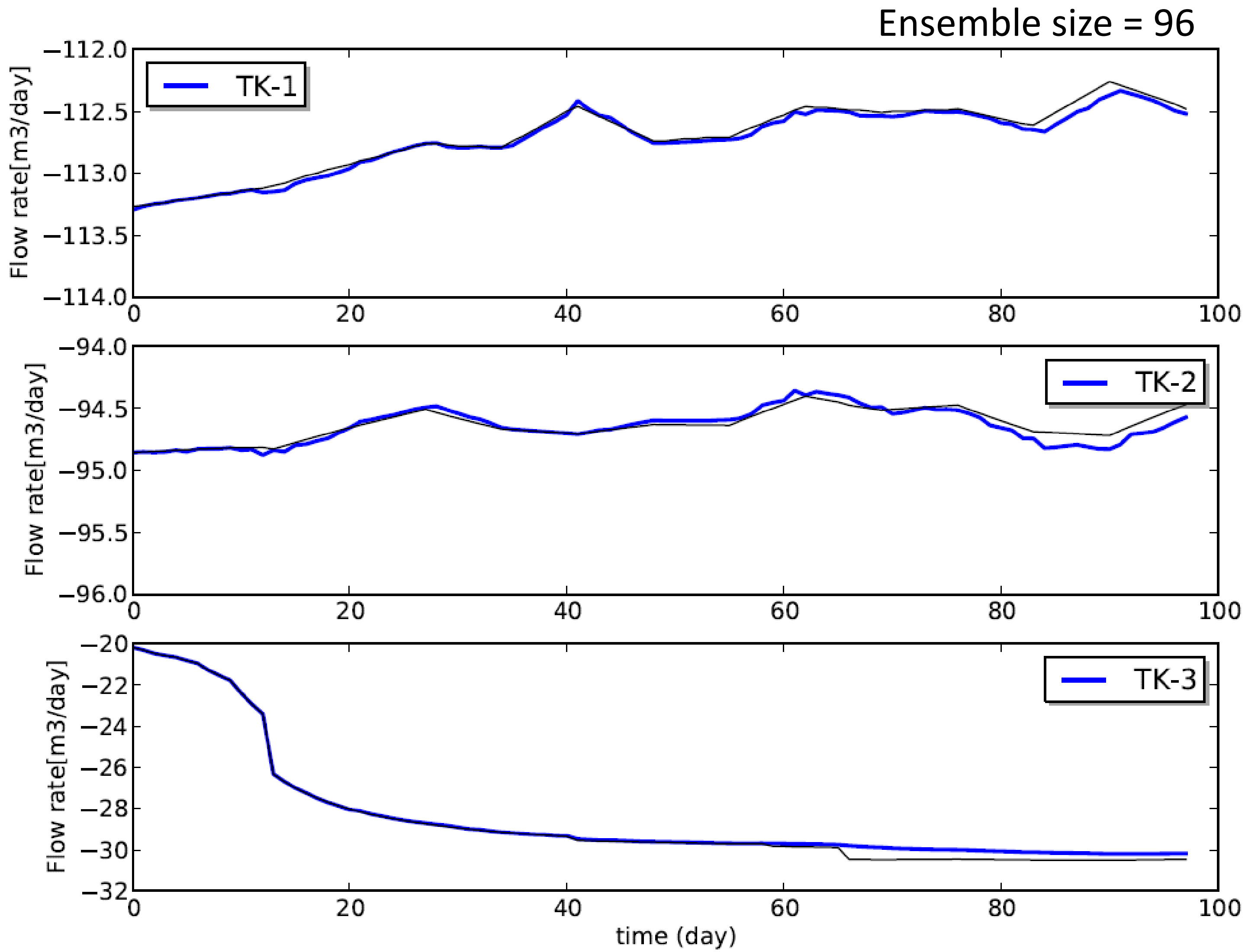
Using more than thousands ensemble member is impractical!

Number of ensemble members in EnKF



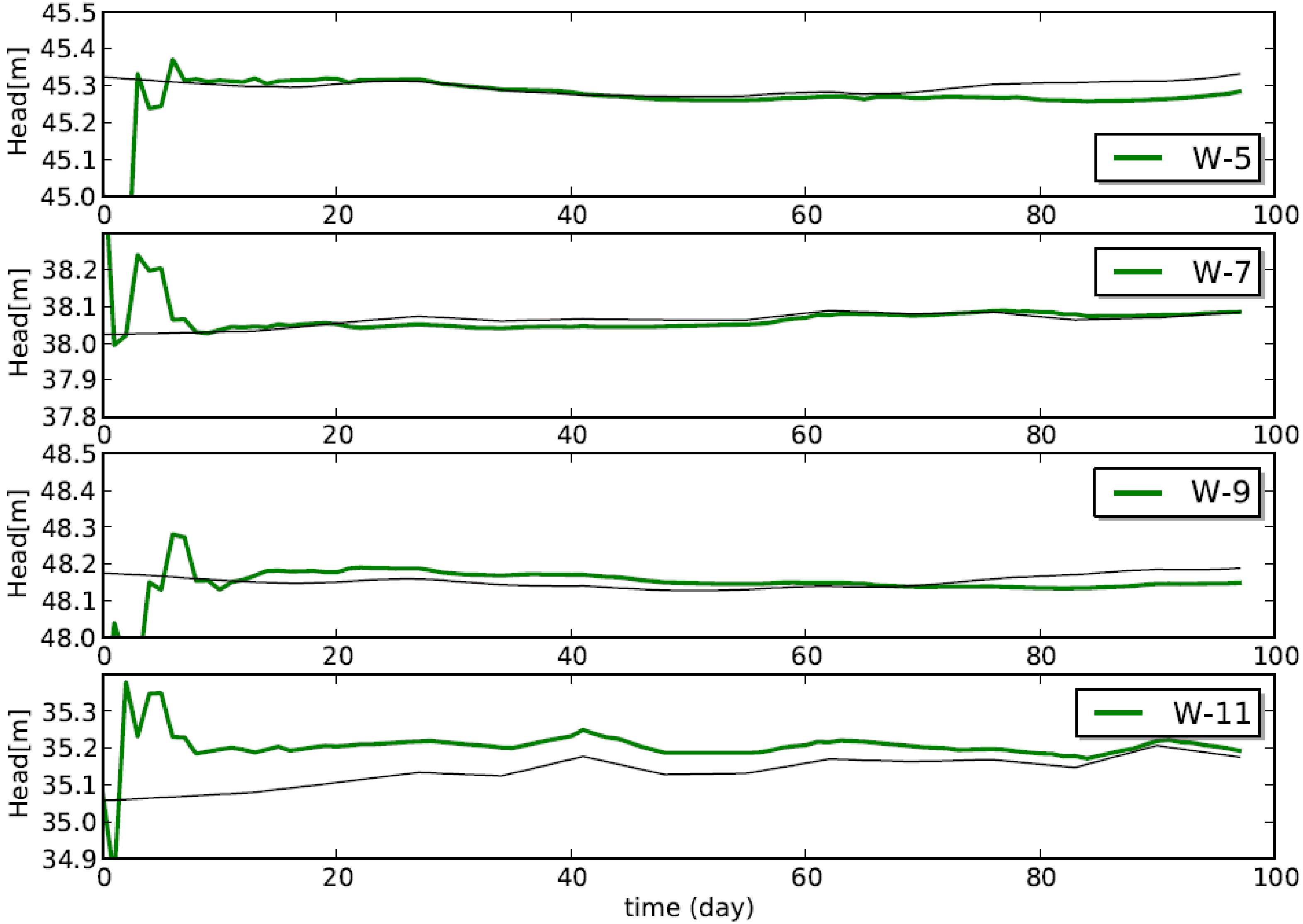
Comparison of the errors of pressure head observations

Assimilation results of water inflows with EnKF (unsaturated model)



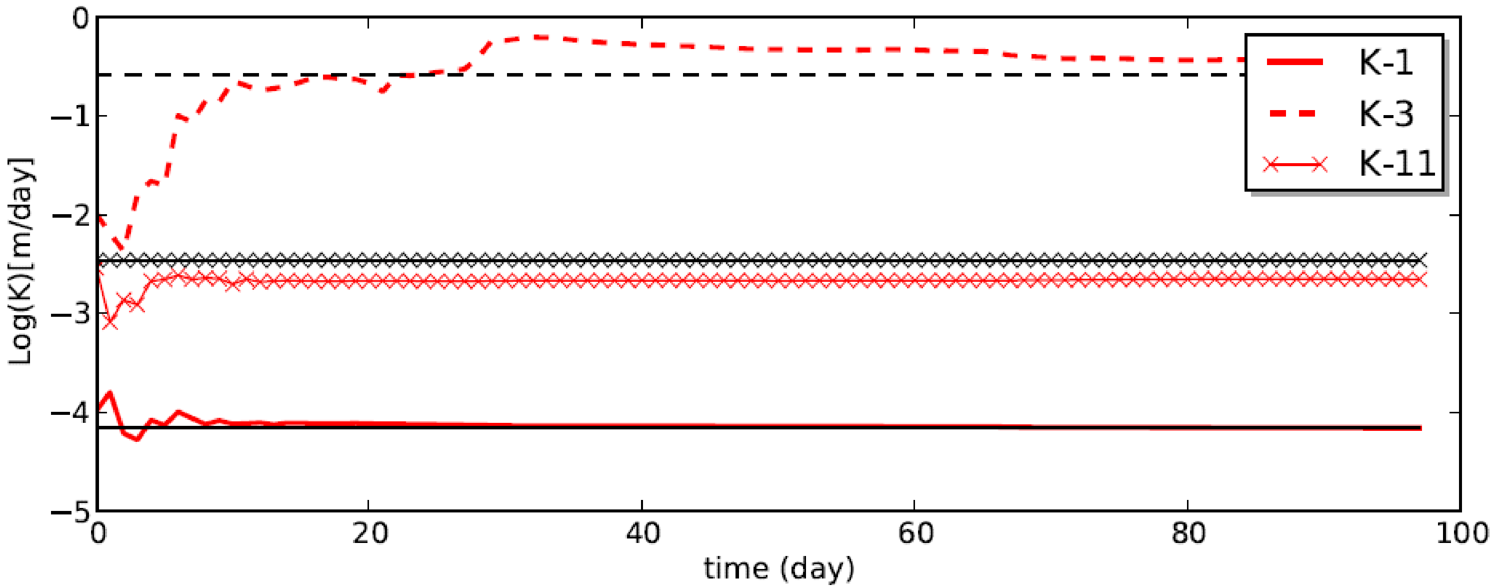
Assimilation results of pressure head with EnKF (unsaturated model)

Ensemble size = 96



Estimation results of log(K) (Unsaturated model)

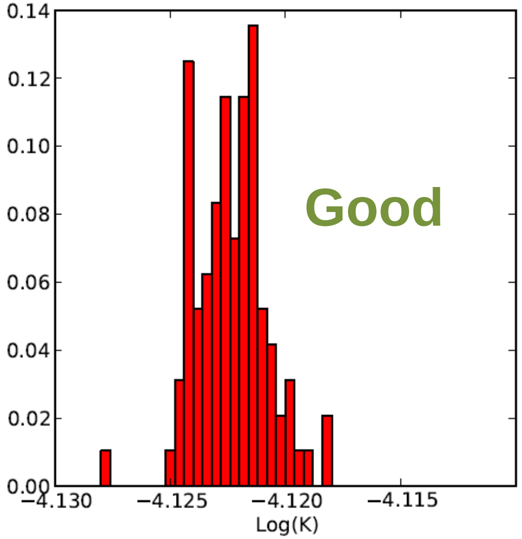
Ensemble size = 96



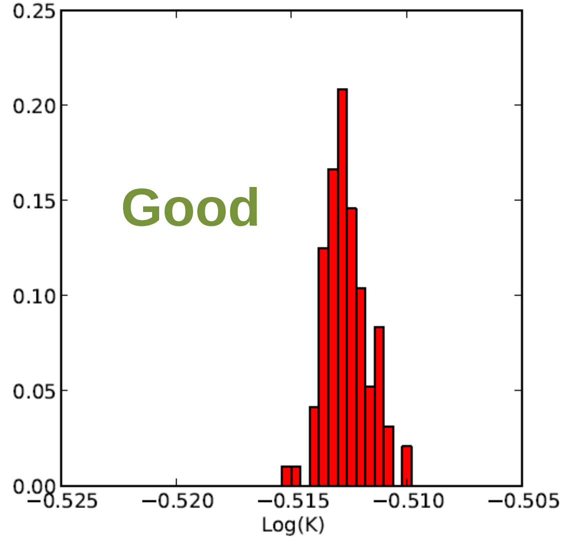
Non-linearity of unsaturated model

Ensemble size = 96

Prior PDF of Permeability ($\mathrm{Log}(K)$)

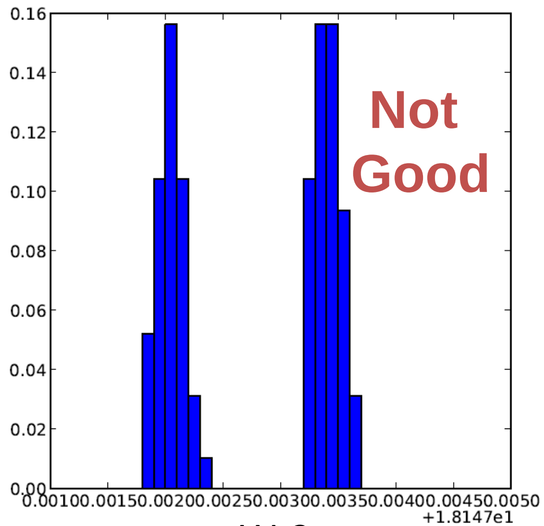


K-1

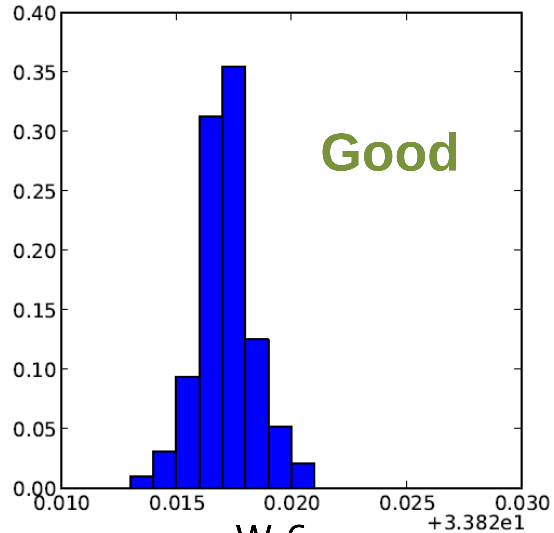


K-11

Prior PDF of Pressure head



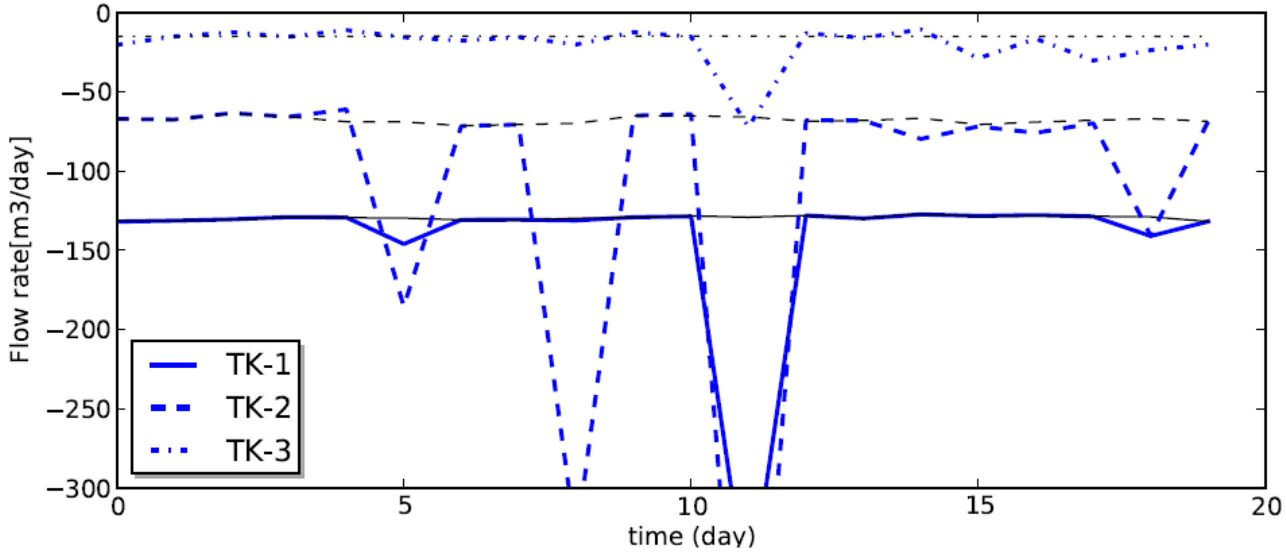
W-3



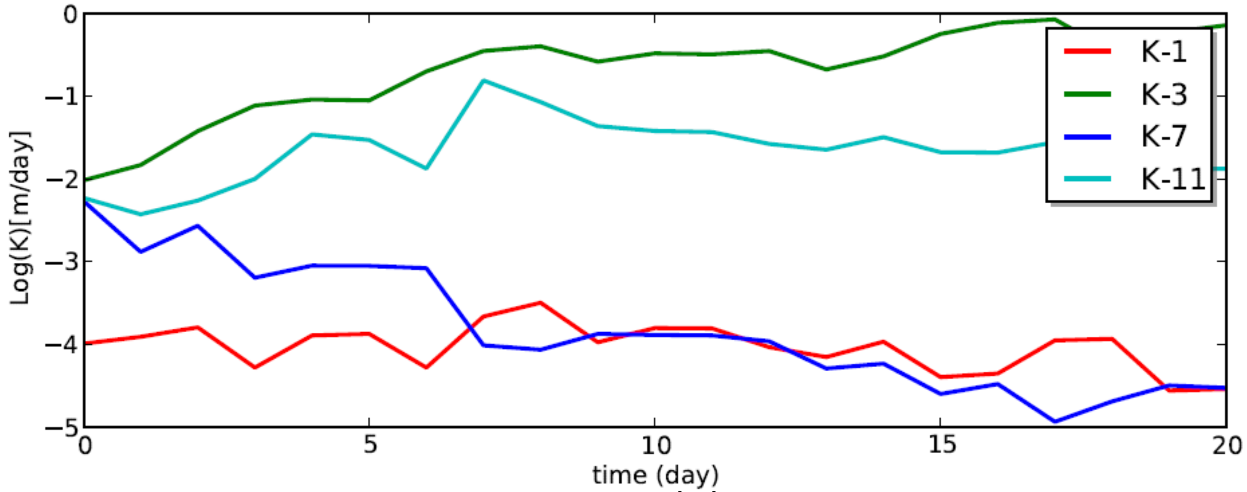
W-6

Data assimilation using real observations with EnKF (in progress)

Ensemble size = 96



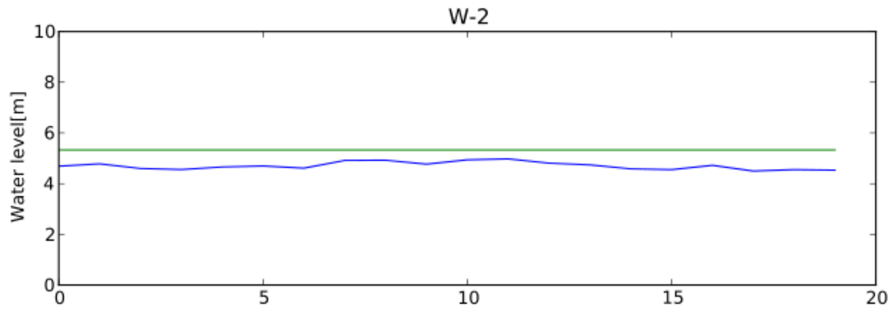
Inflow volume

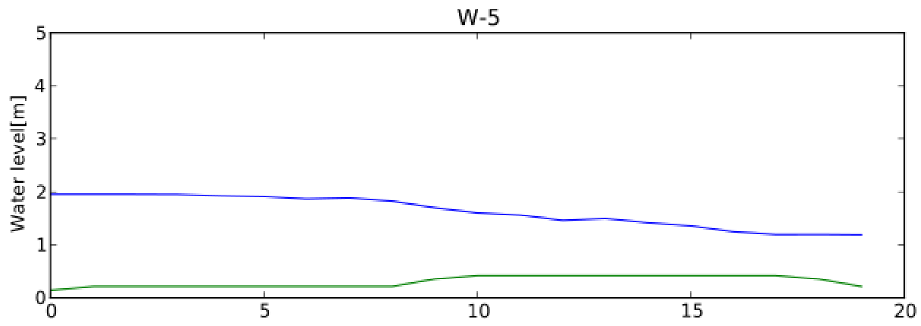


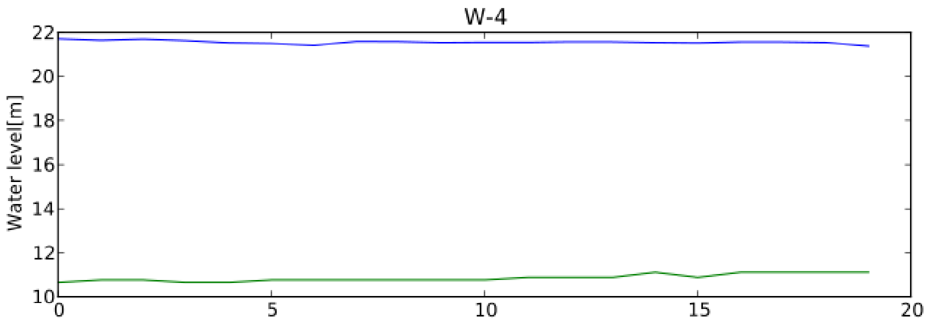
Log(K)

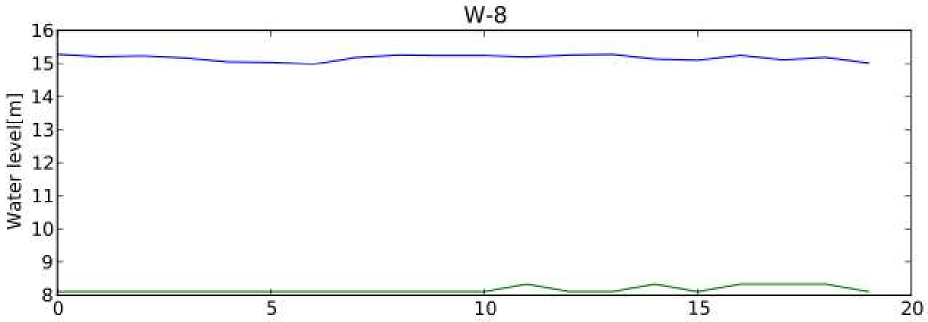
Data assimilation using real observations (in progress)

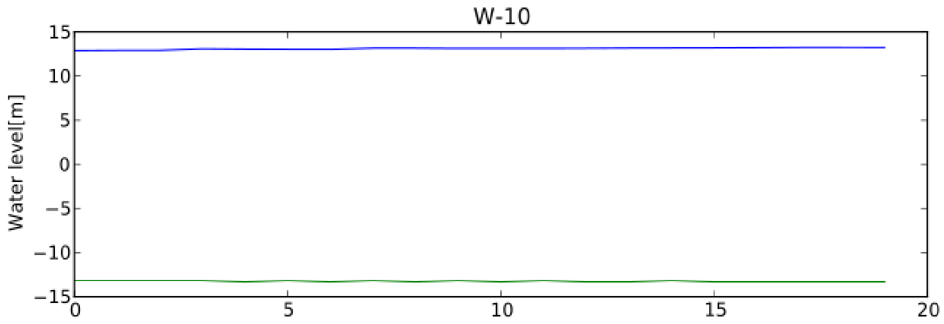
Ensemble size = 96

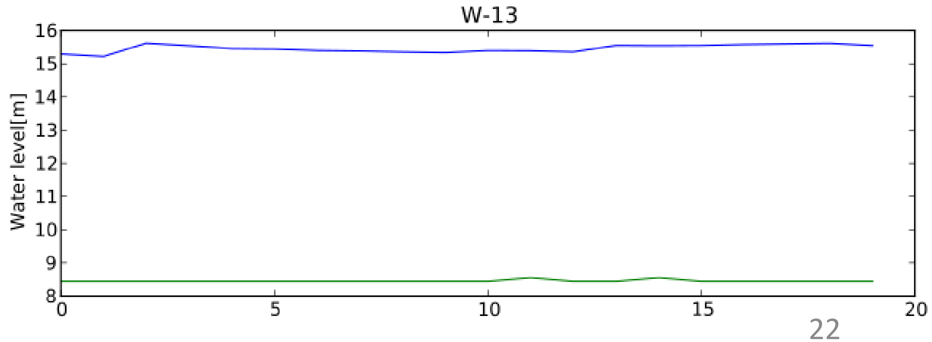












Summary of the results

- EnKF produces good performance under the twin experiments with perfect model and observations.
- Application of particle filter is difficult at acceptable computation cost.
- The non-linearity of the unsaturated model collapses Gaussian distributions of state variables and reduces the accuracy.
- The assimilation result using real observations has biased errors. This implies that the current model has a limitation.

Future tasks

- Model evaluation and refinement
  - More accurate geological model
  - Considering the uncertainty of the groundwater recharge

- Find a solution to deal with non-gaussian distributions in EnKF
  - Gaussian mixture density

- Build the appropriate scheme for successful data assimilation in an actual operation.
  - Assimilation cycle
  - Ensemble size