Developing coupled data assimilation methods in the presence of model error

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Aim of my PhD

- **What?** Develop data assimilation methods that will improve the estimation of atmospheric and oceanic initial conditions.
- **Why?** Contribute towards improved accuracy of atmospheric and oceanic circulation forecasts at seasonal to decadal time-scales.
- **How?**
  - Improved use of data and information during the data assimilation process e.g. atmospheric observational data influencing the estimation of the oceanic initial conditions and vice versa,
  - allowing for the fact that models contain error.

Seasonal to decadal forecasts

coupled atmospheric-oceanic model

$$\mathbf{x}_{i+1} = m_i(\mathbf{x}_i)$$

taking the model state vector **x** from time i to i+1

- Problem: need to **estimate** the **initial conditions $\mathbf{x}_0$**.
- Solution: **4DVAR** (four-dimensional variational data assimilation) uses observations over a time window, a background estimate (at the initial time) and a model to produce the analysis which is the **best estimate of the initial conditions of the states**.

4DVAR assimilation window

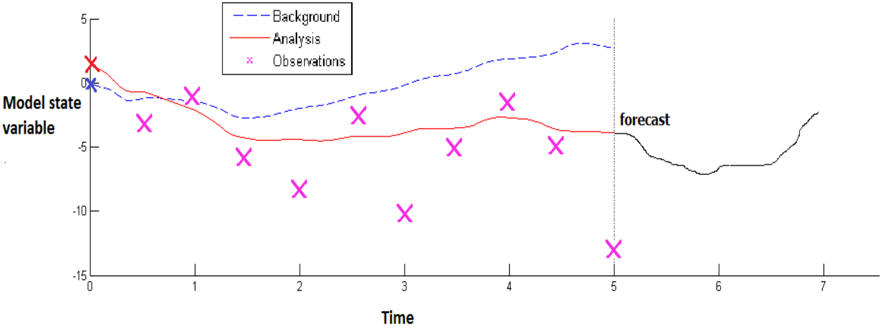


Figure: 4DVAR assimilation window

4DVAR cost function

Obtain solution by minimising the following cost function with respect to the **initial conditions of the model state**, subject to the model,

$$
\begin{aligned}
J(\mathbf{x}_0) &= \frac{1}{2}(\mathbf{x}_0 - \mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_b) \\
&+ \frac{1}{2}\sum_{i=0}^{n}((\mathbf{y}_i - \mathbf{h}_i(\mathbf{x}_i))^T \mathbf{R}_i^{-1}(\mathbf{y}_i - \mathbf{h}_i(\mathbf{x}_i)),
\end{aligned} \tag{1}
$$

- $\mathbf{x}_0$ initial conditions of model state variables at time t_0,
- $\mathbf{x}_b$ background estimates of model state variables at time t_0,
- $\mathbf{x}_i$ model state variables at time t_i,
- $\mathbf{y}_i$ observations at time t_i,
- $\mathbf{h}_i$ observation operator at time t_i,
- $\mathbf{B}$ background error covariance matrix at time t_0,
- $\mathbf{R}_i$ observation error covariance matrix at time t_i.

Incremental 4DVAR

- **Outer loop**: uses non-linear model at high resolution,
  - initial guess $\mathbf{x}_0{}^{(k)}$,
  - produce model trajectories $\mathbf{x}_i{}^{(k)}$,
  - calculate departures $\mathbf{d}_i{}^{(k)} = \mathbf{y}_i - \mathbf{h}_i(\mathbf{x}_i{}^{(k)})$.

- **Inner loop**: uses linear observation operator and tangent linear model at lower resolution,
  - $\delta\mathbf{x}_0{}^{(k)} = \mathbf{x}_0{}^{(k+1)} - \mathbf{x}_0{}^{(k)}$,
  - apply minimisation algorithm (e.g. conjugate gradient) to **minimise the incremental cost function** to obtain model state increment $\delta\mathbf{x}_0{}^{(k)}$,
  - $\mathbf{x}_0{}^{(k+1)}$ is subsequently used in the outer loop.

Treats the minimisation process as a sequence of quadratic problems. The solution of the incremental problem will converge to the solution of the original problem.

Incremental 4DVAR pictorially

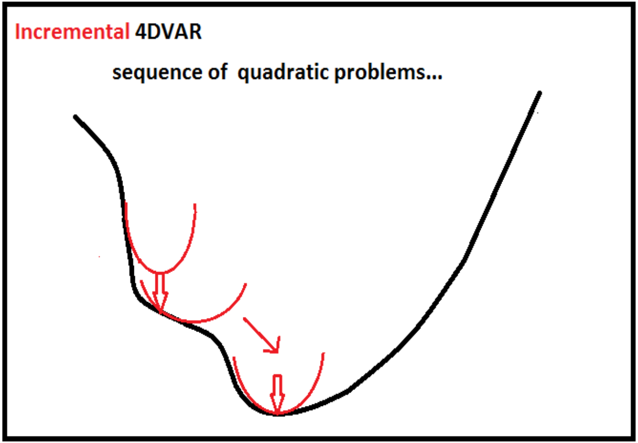


Figure: Incremental 4DVAR

Operational forecasting centres...currently

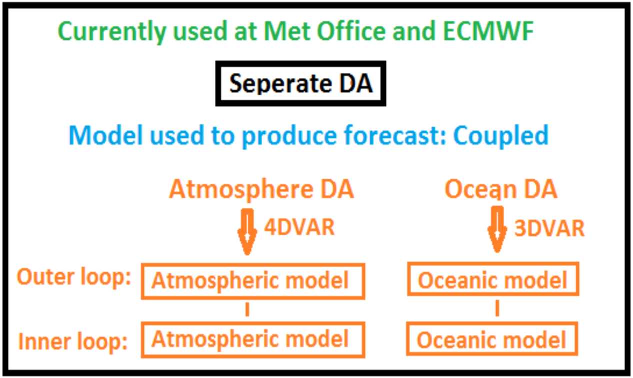


Figure: Current operational DA method

Separate DA...problems

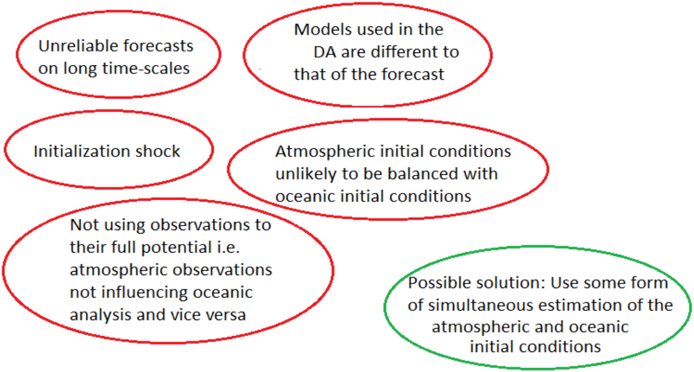


Figure: Problems with current operational DA method

Operational forecasting centres...working on

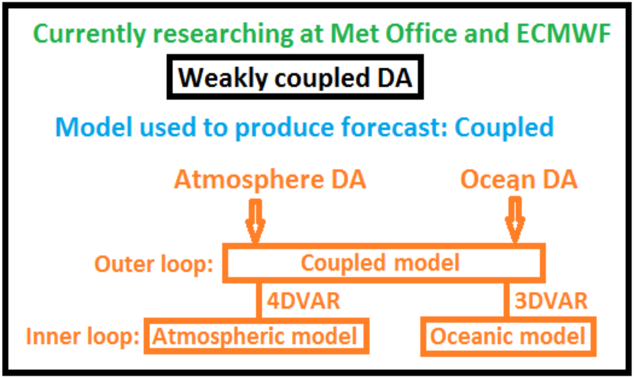


Figure: Operational centres investigating DA method

Weakly coupled DA...positive outcomes

- Both the Met Office and ECMWF have developed weakly coupled DA systems and ran some initial tests with positive results.

- Atmospheric assimilation window length used in weakly coupled DA with negligible impact on ocean
  - e.g. Met Office...
  - 6hr hour window for atmosphere only DA
  - 24hr window for ocean only DA
  - 6hr window for weakly coupled DA

Weakly coupled DA...problems

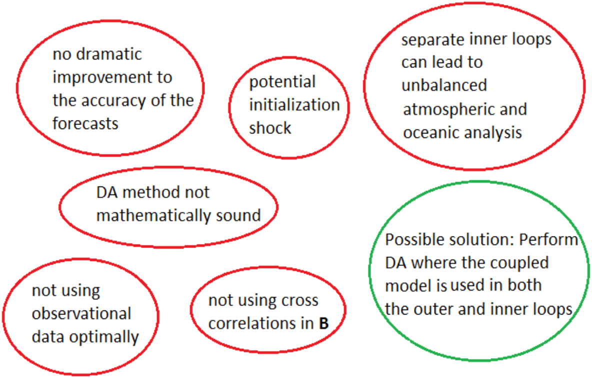


Figure: Problems with weakly coupled DA method

Strongly coupled DA

A possible future direction for operational forecasting centres such as the Met Office is...

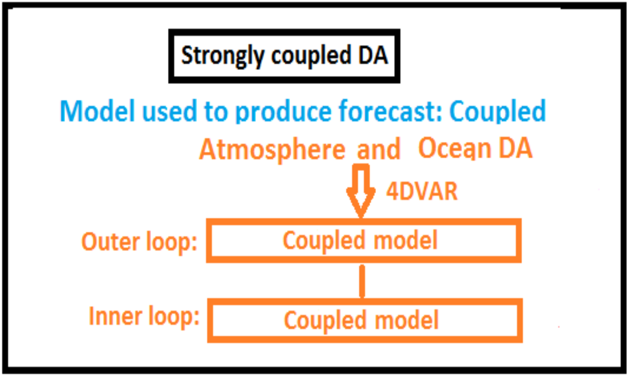


Figure: Strongly coupled DA method

Strongly coupled DA...why?

Why?

- Mathematically sound method,
- can explicitly specify cross correlations of atmospheric-oceanic background errors in $\mathbf{B}$,
- implicit cross correlation information in $\mathbf{B}$ throughout DA process,
- optimal use of observations i.e. atmospheric observations directly affect ocean analysis and vice versa.
- Should lead to
  - improved balance in analysis,
  - reduction in initialization shock.

All the above should contribute towards **improved forecast skill** of **atmospheric** and **oceanic circulation** at **seasonal** to **decadal time-scales**.

Strongly coupled DA...consideration of model error

- The accuracy of atmospheric and oceanic forecasts are bounded by the fact that operational models are a representation of the true dynamical system and contain model error.

- Therefore, methods of **allowing for this model error** while **using strongly coupled data assimilation** will be now be investigated.

Model error

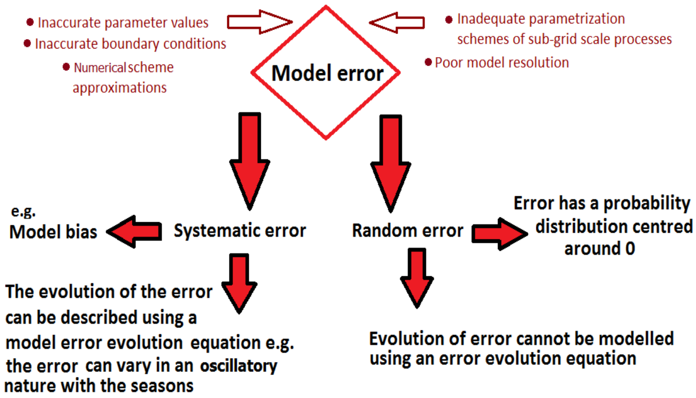


Figure: Model error

(Griffith and Nichols, 2000)

Operational centres...model error

Met Office and ECMWF use **bias correction scheme** in Ocean DA,

- model state augmented with systematic model variables,
- need information about the evolution of the errors and balance constraints.

(Martin et al. 2009)

Possible future aims,

- estimate biases in coupled model (Met Office)
- use weak constraint 4DVAR (ECMWF).

Bias in a simple linear model

Consider

- perfect model $\mathbf{M}$,
- biased model containing vector of model bias $\boldsymbol{\eta}$,
- observations $\mathbf{y}$ only at one time t_1,
- model state analysis statistics...

Model used in DA	Expected error	Covariance
Unbiased	0	$\mathbf{A}$
Biased	$-\mathbf{K}_1\mathbf{H}\eta$	$\mathbf{A}$

$$\text{where } \mathbf{A} = (\mathbf{B}^{-1} + \mathbf{M}^T\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\mathbf{M})^{-1}$$
$$\text{and } \mathbf{K}_1 = \mathbf{A}\mathbf{M}^T\mathbf{H}^T\mathbf{R}^{-1}.$$

- biased model equations lead to biased model state analysis unless the model bias is in the nullspace of the observation operator

Can we reduce the effect the model bias has on the model state analysis?

Coupling parameter estimation

A scalar coupling parameter can be estimated along with the model state by minimising the cost function,

$$\begin{aligned} J(\mathbf{x}_0, \alpha) &= \frac{1}{2}(\mathbf{x}_0 - \mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_b) \\ &+ \frac{1}{2}\sum_{i=0}^{n}((\mathbf{y}_i - \mathbf{h}_i(\mathbf{x}_i))^T \mathbf{R}_i^{-1}(\mathbf{y}_i - \mathbf{h}_i(\mathbf{x}_i)) + \frac{1}{2}\frac{(\alpha - \alpha_b)^2}{\sigma_A^2}, \quad (2) \end{aligned}$$

- minimise with respect to both $\mathbf{x}_0$ and α (coupling parameter),
- α_b background coupling parameter,
- σ_A^2 variance of the error in the background coupling parameter.

Bias in a simple linear model

Consider the simple biased linear model,

$$\mathbf{x}_i = \mathbf{M}(\alpha)\mathbf{x}_{i-1} + \boldsymbol{\eta}, \qquad 1 \leq i \leq n, \tag{3}$$

$$\text{where } \mathbf{M} = \begin{pmatrix} 1 & \alpha \\ 0 & 1 \end{pmatrix} \text{ and}$$

$$\boldsymbol{\eta} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

in a simple situation where there are only observations at one time.
Analysis results from a sample of 1000 DA runs...

Coupling parameter estimation...helps compensate for bias in a simple linear model

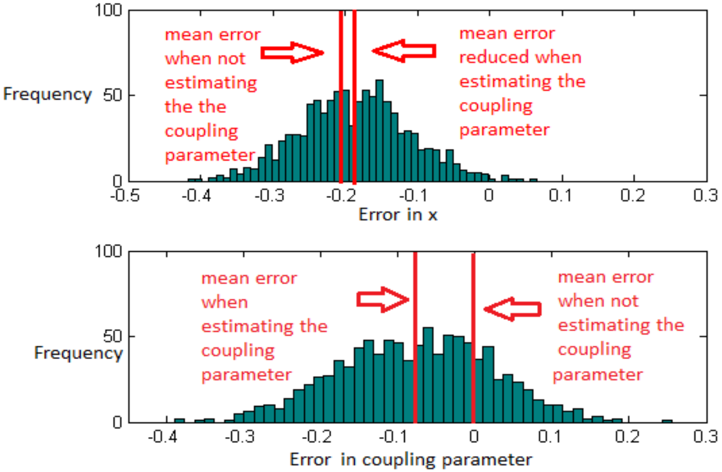


Figure: Error spread in analysis

Simple non-linear model

Couples the Lorenz 63 system and 2 linear equations (Molteni et al. 1993),

$$\begin{aligned}
\dot{x} &= -\sigma x + \sigma y + \alpha, \\
\dot{y} &= -xz + rx - y + \alpha w, \\
\dot{z} &= xy - bz, \\
\dot{w} &= -\Omega v - k(w - w^*) - \alpha y, \\
\dot{v} &= \Omega(w - w^*) - kv - \alpha x,
\end{aligned} \tag{4}$$

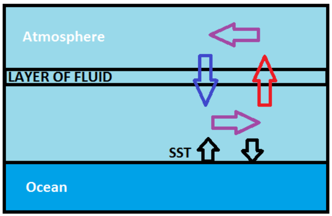
where $\sigma = 10$, $r = 30$, $b = \frac{8}{3}$, $k = 0.1$, $\Omega = \frac{\pi}{10}$ and $w^* = 2$.

- x, y and z are the atmosphere state variables, w and v are the ocean state variables and α is the coupling parameter.

Simple non-linear model...why this model?

Model description

Temperature between the top and bottom of a uniform depth layer of fluid in the atmosphere is kept constant. Convection will develop, movement of fluid and a change in the temperature profile at unstable solutions.



Model state variables

- $x \propto$ intensity of the convective motion,
- $y \propto$ temperature difference between ascending and descending currents,
- $z \propto$ distortion of the vertical temperature profile from linearity,
- α heat flux (rate of heat energy transfer between the atmosphere and the ocean),
- w and v represent equatorial SST (sea surface temperature) anomalies influence on the global system.

Coupling parameter estimation...reduction in error norms

- Runge-Kutta 2nd order method applied to provide a time-stepping method for approximating the solution of the coupled system ODE's,
- a model bias term 0.01 added to the equation for atmospheric variable x.

State	State and coupling parameter estimation	State estimation
x	0.397071197	2.67397453
y	0.574463196	4.20221004
z	0.68507461	4.57753998
w	0.299153876	0.65327516
v	0.110780612	1.03672316

Table: Error norms over forecast window t=0 to t=2.5, biased model used in data assimilation, assimilation window length 5

Error in coupling parameter analysis value 0.4%.

Coupling parameter estimation...considerations

The variance of the error in the background coupling parameter has to be

- large enough to enable the coupling parameter to compensate for model bias,
- small enough so that the estimation of the coupling parameter could be in a realistic range,
- not too small.....else the minimisation algorithm can find a local minima of the cost function.

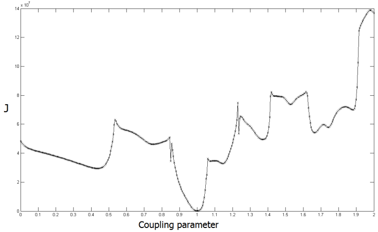


Figure: Cost function respect to the coupling parameter.

Weak constraint 4DVAR...introduction

Consider an imperfect forecasting model of the form,

$$\mathbf{x}_i = \mathbf{m}_i(\mathbf{x}_{i-1}) + \boldsymbol{\eta}_i, \qquad 1 \leq i \leq n, \tag{5}$$

where $\boldsymbol{\eta}_i$ is the vector of **random, unbiased, Gaussian model errors** with model error covariance matrix $\mathbf{Q}_i$ at time t_i.

As the model equations are not perfect, it makes sense for them to be only satisfied approximately.

Weak constraint 4DVAR...cost function

The model equations are treated as weak constraints in the data assimilation formulation,

$$
\begin{aligned}
J(\mathbf{x_0}, \boldsymbol{\eta}_1, ..., \boldsymbol{\eta}_n) &= \frac{1}{2}(\mathbf{x}_0 - \mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_b) \\
&+ \frac{1}{2}\sum_{i=0}^{n}((\mathbf{y}_i - \mathbf{h}_i(\mathbf{x}_i))^T \mathbf{R}_i{}^{-1}(\mathbf{y}_i - \mathbf{h_i}(\mathbf{x}_i)) \\
&+ \frac{1}{2}\sum_{i=1}^{n}\boldsymbol{\eta}_i{}^T \mathbf{Q_i}{}^{-1}\boldsymbol{\eta}_i,
\end{aligned} \tag{6}
$$

is to be minimised with respect to

- $\mathbf{x}_0$ initial conditions of the model state,
- $\boldsymbol{\eta}_i$ vectors at each time t_i.

Weak constraint 4DVAR...the model error covariance matrix

What are the entries in $\mathbf{Q}_i$?...For each time t_i,

- diagonal elements: model error variances for each model state variable,
- off diagonal elements: model error covariances between each of the model state variables.

Problems:

- model errors, model error variance and model error covariance relationships are unknown

Therefore this information needs to be obtained using mathematical and statistical methods.

Weak constraint 4DVAR...work previously done on estimating a static Q

- Forcing $\mathbf{Q} \propto \mathbf{B}$ (Y. Trémolet 2007)
  - Disadvantage: Initial condition analysis increment and model error analysis increment in the same solution direction and can be incorrectly transferred.
- Comparing model drift tenancies (Fisher et al. 2011)
  - Disadvantage: Only captures large scale features (systematic error including model bias).
  - It has been shown that a slightly degraded forecast is produced if weak constraint 4DVAR is used to capture constant model error.
- Comparing an ensemble of model tendencies using 6hr windows (Fisher et al. 2011)
  - Disadvantage: This captures small scale structures, but how can model error statistics be distinguished from background error statistics?

Future work... Key aim of my PhD

- Develop a method to estimate $\mathbf{Q}$ for a coupled atmosphere-ocean system.
- This will be a static matrix $\mathbf{Q}$ and will contain statistical information about random model error.

- How to tackle this problem?
  1. Using prior knowledge about possible origins of errors in the model,
     - perturb model and use model tendency information.
  2. Using the assimilation process,
     - run weak constraint 4DVAR over a number of different windows to supply model error estimates.

F. Molteni, L. Ferranti, T.N. Palmer, P. Viterbo: A dynamical interpretation of the global response to equatorial Pacific SST anomalies *Journal of climate*, **vol.6**, 1993, pp. 777-795.

M. Fisher, Y. Tremolet, H. Auvinen, D. Tan, P. Poli: Weak-Constraint and Long-Window 4D-Var *ECMWF Technical Report*, **report.655**, 2011.

D. P. Dee: Bias and data assimilation *Quarterly Journal of the Royal Meteorological Society*, **vol.131**, 2005, pp. 3323-3343.

A. K. Griffith, N. K. Nichols: Adjoint methods in data assimilation for estimating model error *Flow, turbulence and combustion*, **vol.65**, 2000, pp. 469-488.

M.J. Martin, M.J. Bell, N.K. Nichols: Treatment of systematic errors in sequential data assimilation *Ocean Applications, Meteorological Office*, **tech note.21** 1999.

Y. Trémolet: Model-error estimation in 4D-Var *Quarterly Journal of the Royal Meteorological Society*, **vol.133**, 2007, pp. 1267-1280.

Questions

Thank you for listening

Any questions?