Model error in coupled atmosphere-ocean data assimilation

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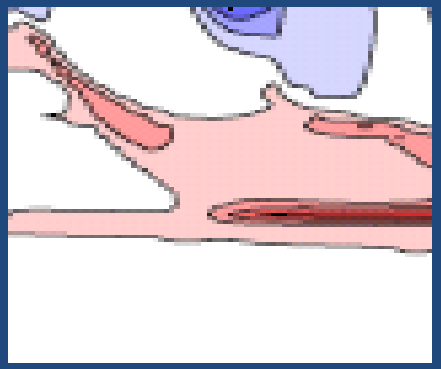

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Introduction

- Coupled atmosphere-ocean DA schemes have been developed for the initialisation of coupled models, typically used to produce long-term forecasts.
- An outstanding problem is the choice of assimilation window length in coupled 4DVar algorithms.
  - Due to the chaotic nature of the atmosphere a short window length of approximately 12hrs is used.
  - However the analysis of the ocean would benefit from a longer window length of a few days due to the slower nature of the fluid and the sparse number of observations.
- A solution to this could be the development of weak constraint coupled 4DVar.
- As a first step we look at the effect of model error in strong constraint coupled 4DVar.

Model error in 4D-Var

4D-Var problem formulated as minimising the following cost function w.r.t. x_0

$$J = \frac{1}{2}(\mathbf{x}^b - \mathbf{x}_0)^{\mathrm{T}}\mathbf{B}^{-1}(\mathbf{x}^b - \mathbf{x}_0) + \frac{1}{2}(\hat{\mathbf{y}} - \hat{\mathcal{H}}(\mathbf{x}_0)))^{\mathrm{T}}\hat{\mathbf{R}}^{-1}(\hat{\mathbf{y}} - \hat{\mathcal{H}}(\mathbf{x}_0)))$$

The state found is the maximum a-posteriori state assuming that

$$\mathbf{x}^b \sim N(\mathbf{x}_0^t, \mathbf{B})$$ and $$\hat{\mathbf{y}} \sim N(\hat{\mathbf{y}}^t, \hat{\mathbf{R}}),$$

and $$\hat{\mathbf{y}}^t = \hat{\mathcal{H}}(\mathbf{x}_0^t)$$

Model error in 4D-Var

In the presence of model error this no longer holds as

$$\hat{\mathbf{y}}^t = \hat{\mathcal{H}}(\mathbf{x}_0^t) + \epsilon^{\hat{\mathcal{H}}} = \hat{\mathcal{H}}^t(\mathbf{x}_0^t)$$

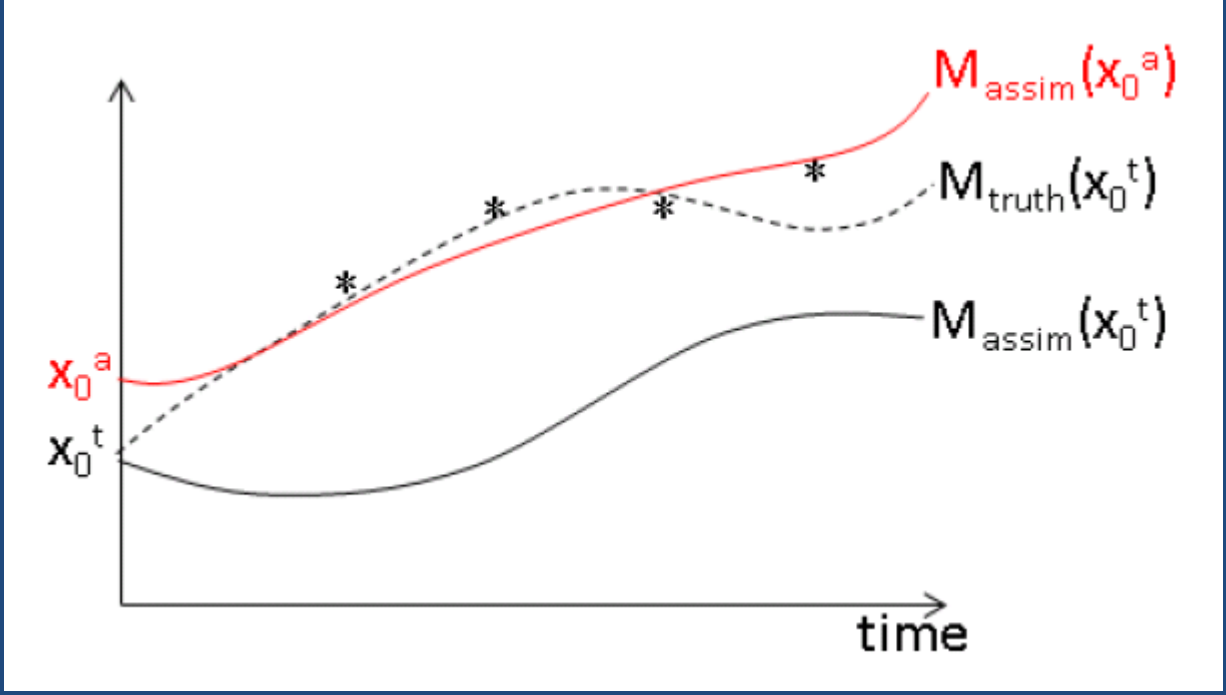
If the model error is unaccounted for it has the effect of increasing the analysis error covariances

$$E[\boldsymbol{\epsilon}^{\mathrm{a}}(\boldsymbol{\epsilon}^{\mathrm{a}})^{\mathrm{T}}] = \mathbf{P}^{\mathrm{a}}_{\mathrm{nm}} + \mathbf{K}E[\epsilon^{\hat{\mathcal{H}}}(\epsilon^{\hat{\mathcal{H}}})^{\mathrm{T}}]\mathbf{K}^{\mathrm{T}}$$

And if the model error is biased then the analysis will also be biased

$$E[\boldsymbol{\epsilon}^{\mathrm{a}}] = \mathbf{K}E[\epsilon^{\hat{\mathcal{H}}}]$$

Model error in 4D-Var



Model error in incremental 4D-Var

$$J = \frac{1}{2}(\mathbf{x}^b - \mathbf{x}_0)^{\mathrm{T}}\mathbf{B}^{-1}(\mathbf{x}^b - \mathbf{x}_0) + \frac{1}{2}(\hat{\mathbf{y}} - \hat{\mathcal{H}}(\mathbf{x}_0)))^{\mathrm{T}}\hat{\mathbf{R}}^{-1}(\hat{\mathbf{y}} - \hat{\mathcal{H}}(\mathbf{x}_0)))$$

In incremental 4D-Var use the TL approximation to linearise the problem

$$\hat{\mathcal{H}}(\mathbf{x}_0^t) \approx \hat{\mathcal{H}}(\mathbf{x}^b) + \hat{\mathbf{H}}|_{\overline{\mathbf{x}}^b}(\mathbf{x}_0^t - \mathbf{x}^b)$$

The error in the TL approximation is given by

$$\epsilon^{\mathrm{TL}} = \hat{\mathcal{H}}(\mathbf{x}_0^t) - \hat{\mathcal{H}}(\mathbf{x}^b) - \hat{\mathbf{H}}|_{\overline{\mathbf{x}}^b}(\mathbf{x}_0^t - \mathbf{x}^b)$$

Therefore in incremental 4D-Var the model error becomes

$$\epsilon^{\hat{\mathbf{H}}} = \epsilon^{\hat{\mathcal{H}}} + \epsilon^{\mathrm{TL}}$$

Model error in coupled DA

Different coupling strategies are available which make different approximations when minimising the full non-linear coupled cost function and this leads to different model error

- Strongly coupled DA

$$\epsilon^{\hat{\mathbf{H}}} = \hat{\mathcal{H}}^t(\mathbf{x}_0^t) - \hat{\mathcal{H}}^c(\mathbf{x}^b) - \hat{\mathbf{H}}^c|_{\overline{\mathbf{x}}^b}(\mathbf{x}_0^t - \mathbf{x}^b)$$

- Weakly coupled DA

$$\epsilon^{\hat{\mathbf{H}}} = \hat{\mathcal{H}}^t(\mathbf{x}_0^t) - \hat{\mathcal{H}}^c(\mathbf{x}^b) - \hat{\mathbf{H}}^{uc}|_{\overline{\mathbf{x}}^b}(\mathbf{x}_0^t - \mathbf{x}^b)$$

- Uncoupled DA

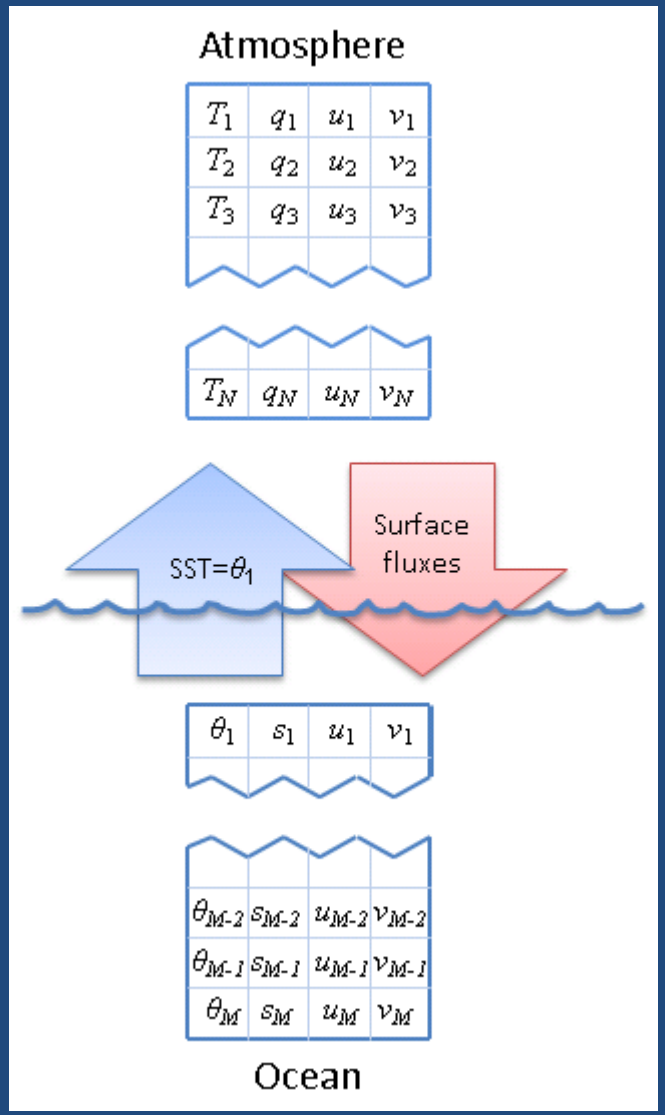
$$\epsilon^{\hat{\mathbf{H}}} = \hat{\mathcal{H}}^t(\mathbf{x}_0^t) - \hat{\mathcal{H}}^{uc}(\mathbf{x}^b) - \hat{\mathbf{H}}^{uc}|_{\overline{\mathbf{x}}^b}(\mathbf{x}_0^t - \mathbf{x}^b)$$

Treats atmosphere and ocean as a coherent system

Approximation to strongly coupled DA but without the need for a coupled TL and AD

Finds a separate analysis for the atmosphere and ocean and then stitches them back together

Generating model error



Atmosphere

Assimilation model has missing physics and a bias in the large scale forcing.

Ocean

Assimilation model used perturbed diffusion parameters.

Model error in coupled DA

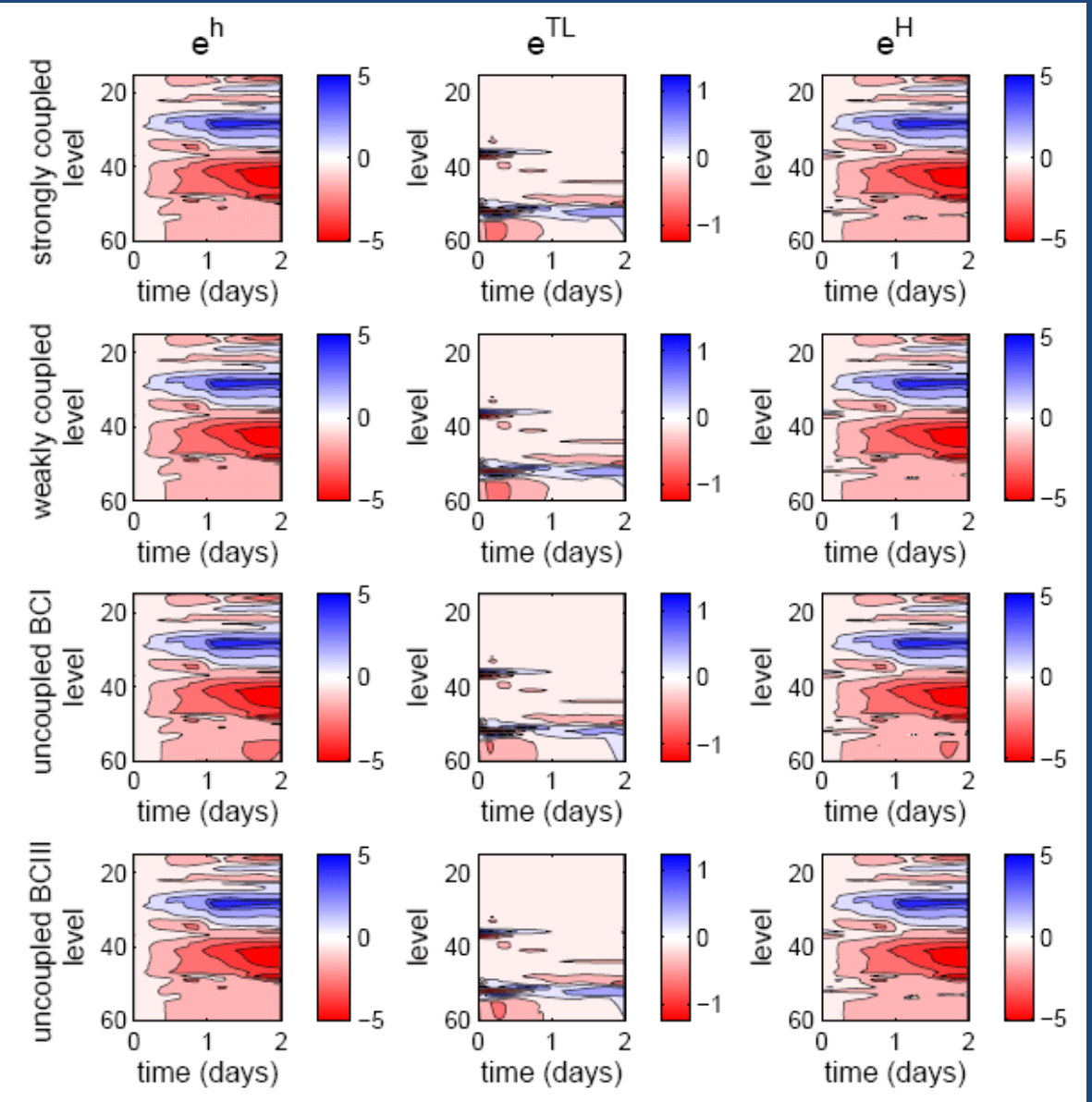


Figure: Model error over 2 days for atmospheric temperature

Observation error standard deviation is 1K.

Uncoupled DA with a poor estimate of the BCs

Uncoupled DA with a good estimate of the BCs

Model error in coupled DA

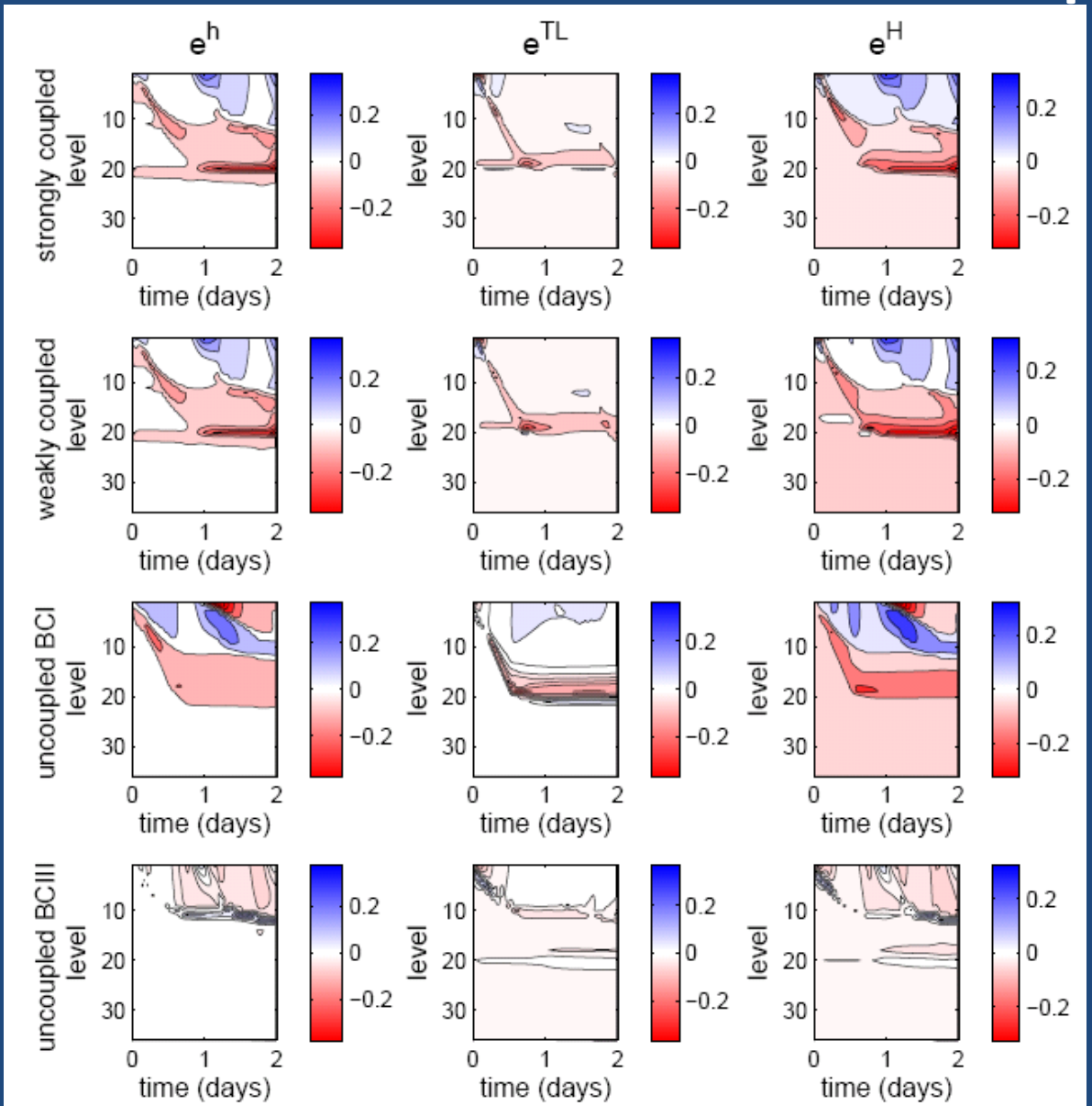


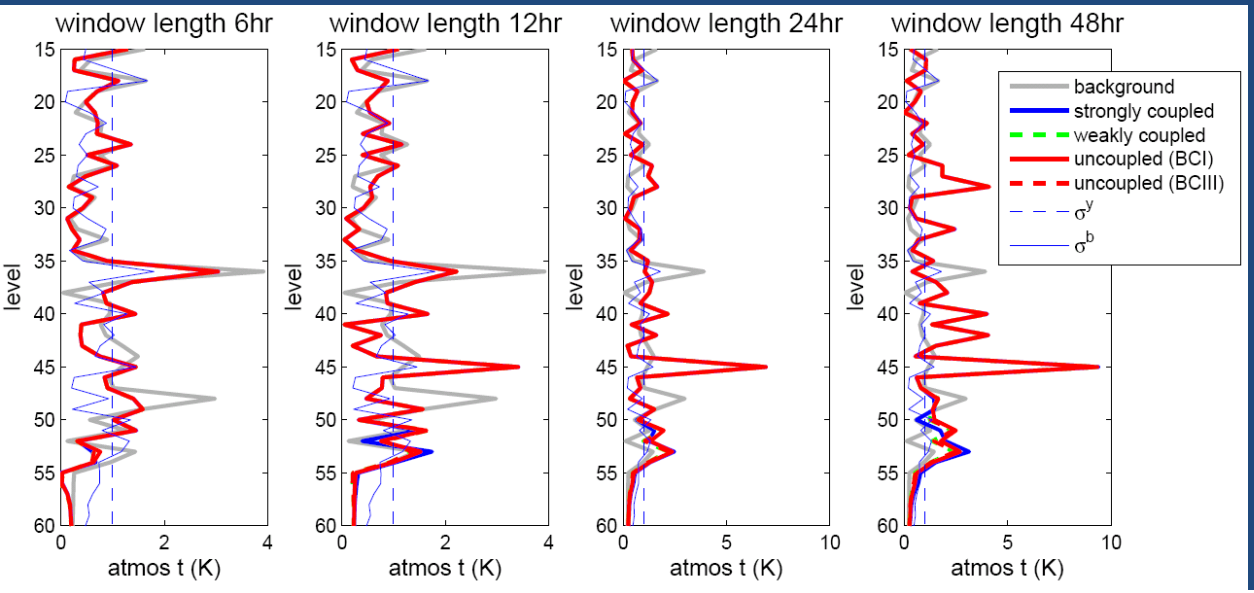
Figure: Model error over 2 days for oceanic temperature

Observation error standard deviation is 0.1K.

Uncoupled DA with a poor estimate of the BCs

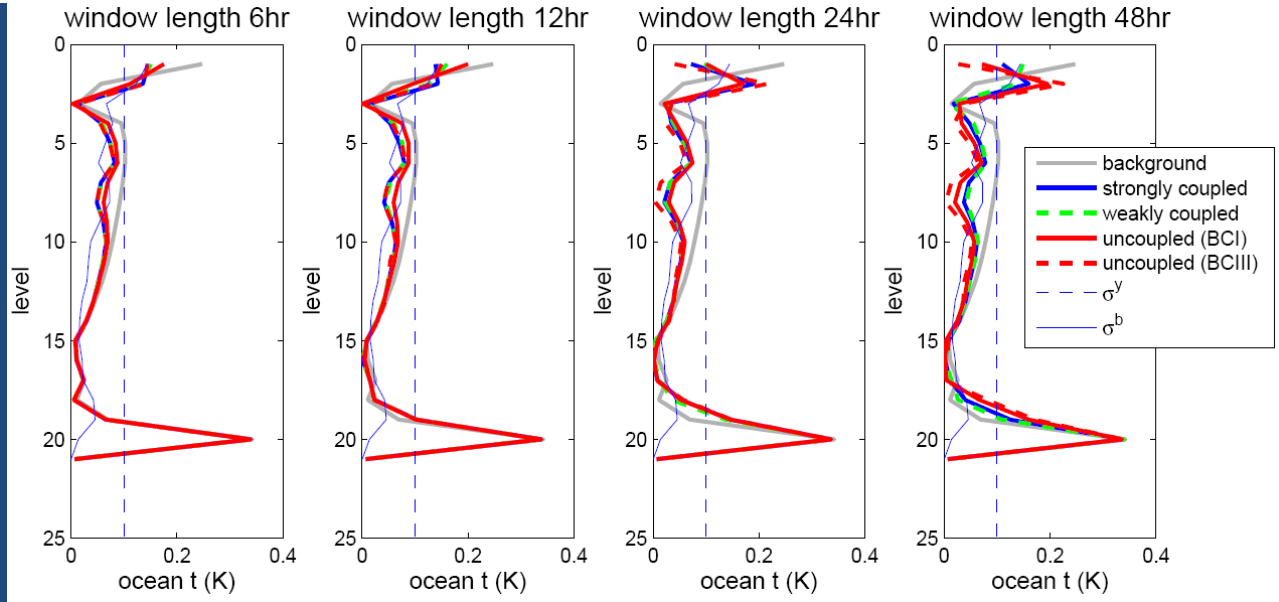
Uncoupled DA with a good estimate of the BCs

Effect of model error on the analysis



Figures: Absolute error in temperature at the initial time, in the atmosphere (left) and ocean (below).

Atmospheric observations every 3 hours, ocean observations every 6 hours



Effect of model error on the initialised forecast

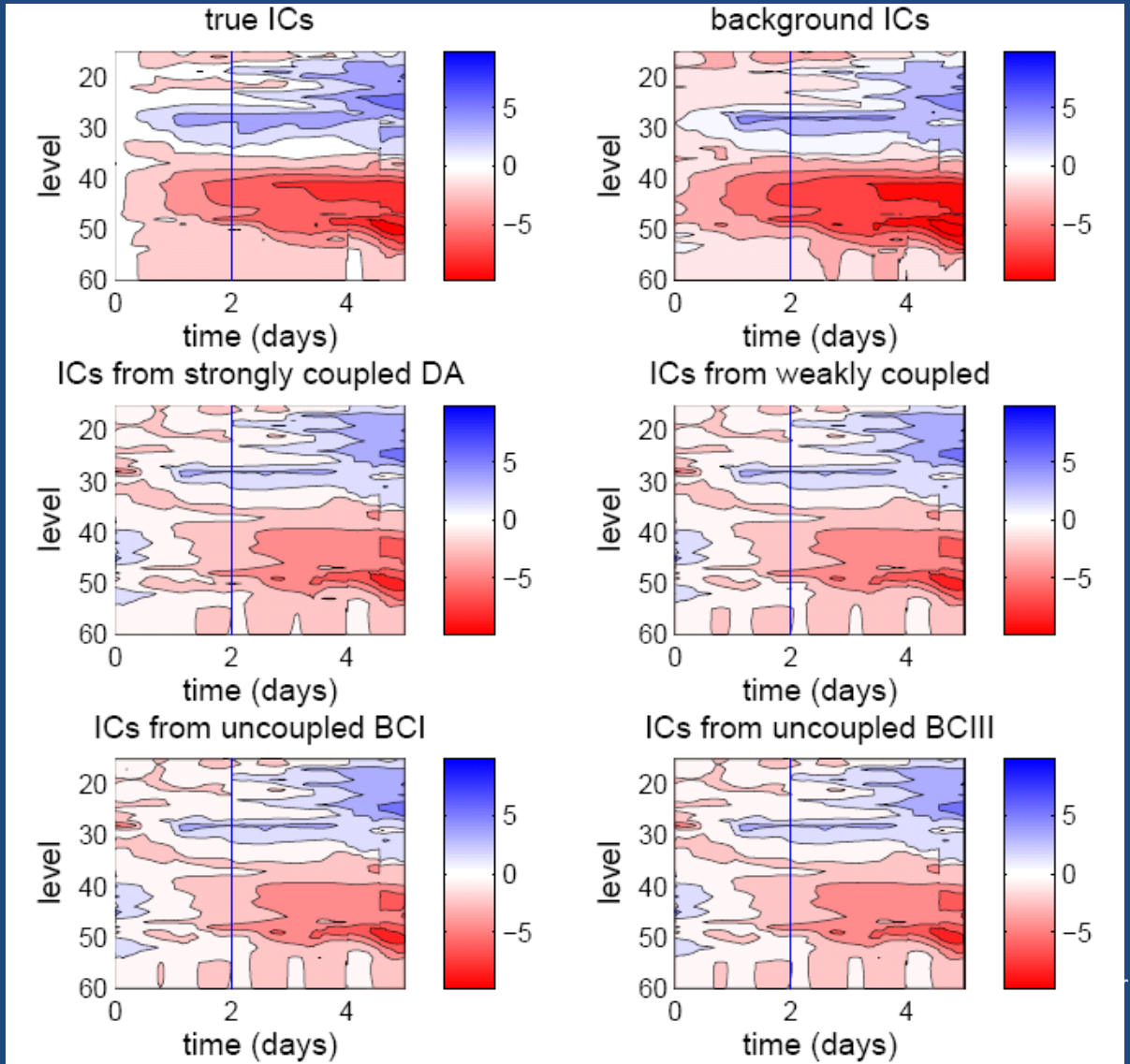


Figure: Forecasts of atmospheric temperature initialised using different analyses computed using a 2 day assimilation window

Effect of model error on the initialised forecast

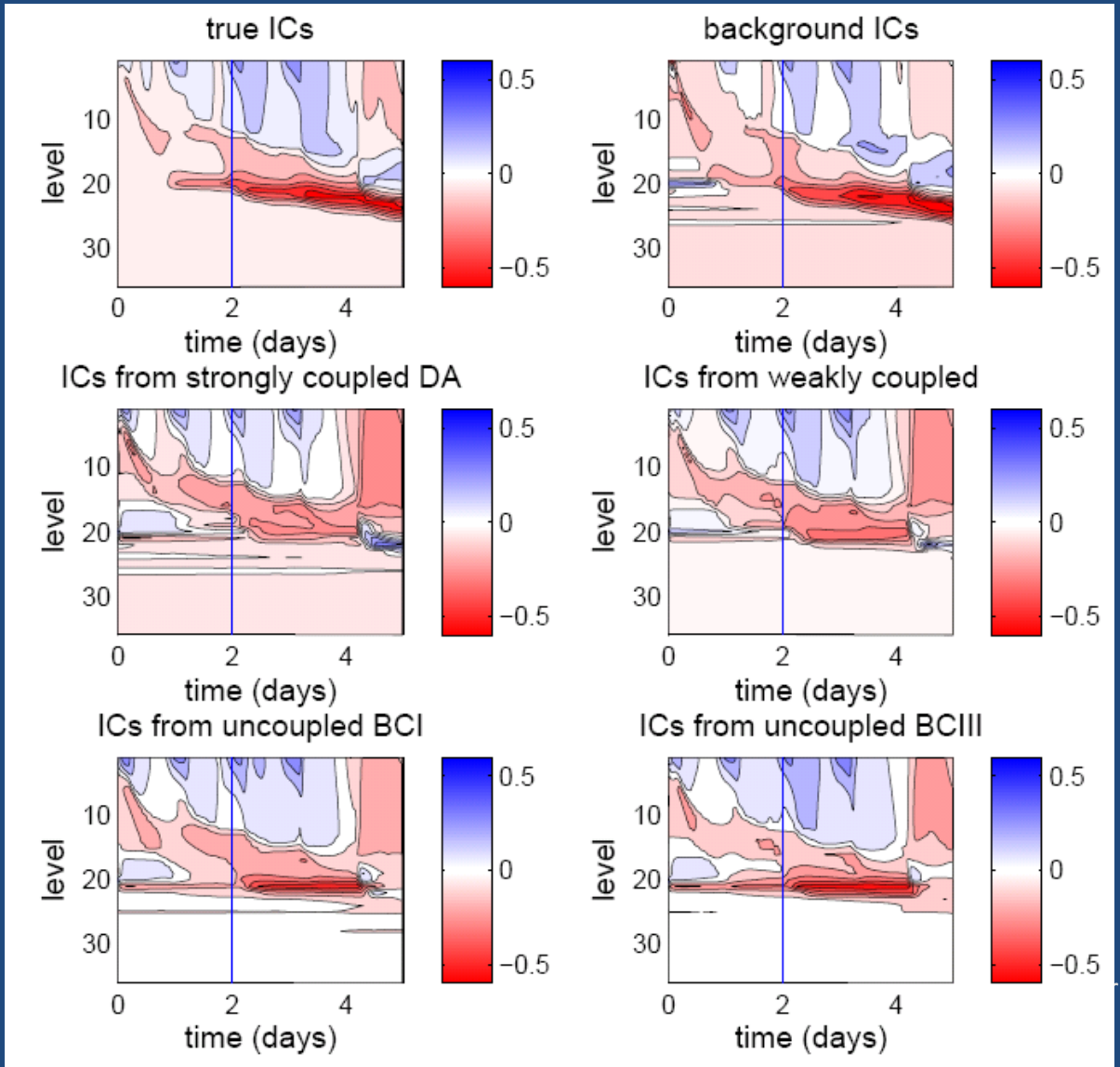
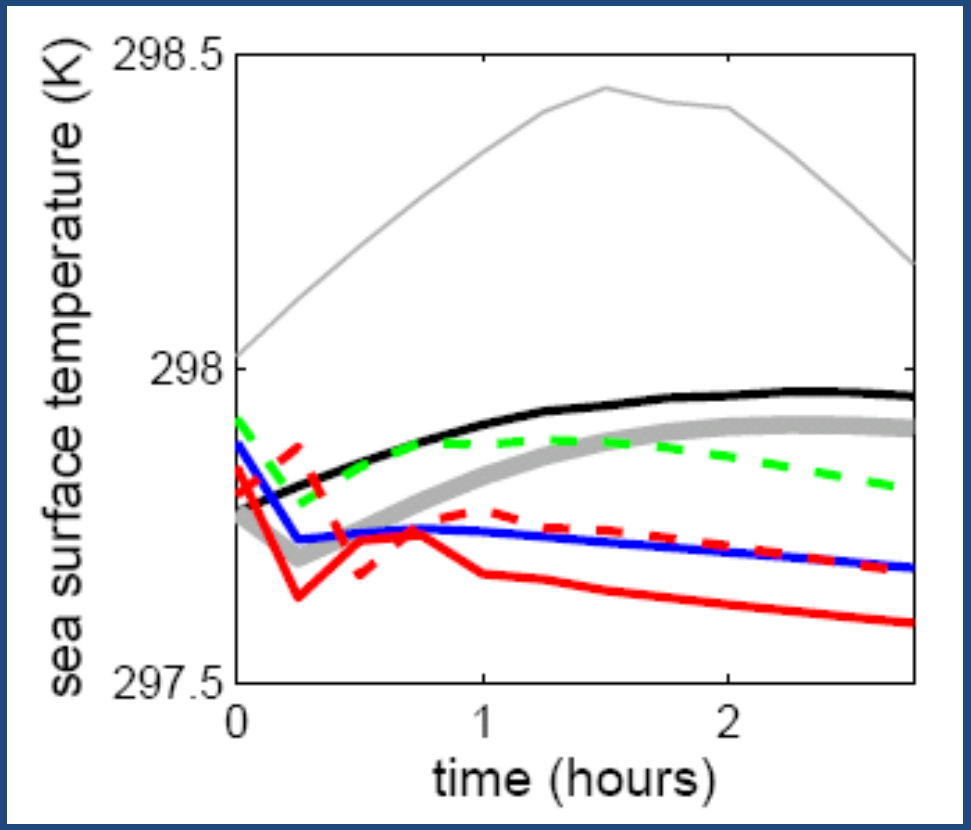


Figure: Forecasts of oceanic temperature initialised using different analyses computed using a 2 day assimilation window

Linearisation shock



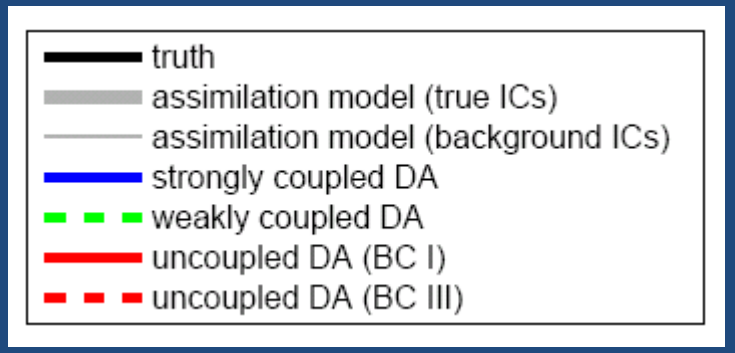


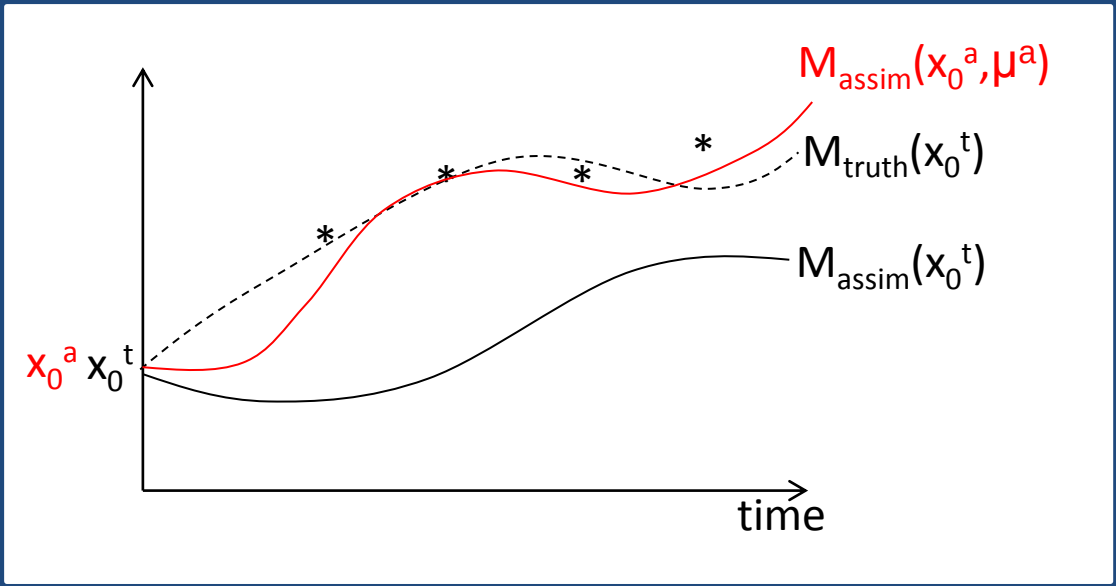
Figure: Forecasts of SST initialised using different analyses computed using a 2day assimilation window

Summary: What's unique about coupled DA?

1. The two fluids have different time scales- clear physical separation.
2. The coupling of the ocean to the atmosphere introduces a new source of error with a different timescale present to that in the rest of the ocean.
3. However, there are still benefits to coupled DA as it imposes balanced in the atmosphere-ocean analysis.

Conclusions: What does this mean for the design of weak constraint coupled DA?

Weak constraint 4DVar allows you to estimate the model error along with the initial state. This could be in the form of a **constant bias** over the assimilation window or an additional **forcing term** at each time step.



Conclusions: What does this mean for the design of weak constraint coupled DA?

- Assume that over 2 days the error in the ocean is still negligible if the model equations in the atmosphere are corrected.
- What will estimating model error do to the balance of the atmosphere and ocean?
- Can we use the estimate of model error in the resulting forecast to help reduce initialisation shock?