NOTES AND CORRESPONDENCE

A Simple Method to Determine Obukhov Lengths for Suburban Areas

C. S. B. GRIMMOND
Climate and Meteorology Program, Geography Department, Indiana University, Bloomington, Indiana

H. A. CLEUGH
School of Earth Sciences, Macquarie University, North Ryde, New South Wales, Australia

1 June 1993 and 9 September 1993

ABSTRACT

A simple scheme is presented to calculate Obukhov stability lengths L for suburban areas when measurements of sensible heat flux Q_H or temperature gradients are not available. The scheme replaces Q_H with χQ^*, where Q^* is net all-wave radiation, and $\chi = Q_H/Q^*$, which is based on data collected in North American cities. The results suggest that the simple two-part model with either a variable or fixed value of χ for daytime works satisfactorily.

1. Introduction

Here we present a simplified method to compute the Obukhov length L and thus atmospheric stability functions ψ for the surface layer above suburban land use. Monin–Obukhov similarity theory (MOST) is the accepted scaling theory for the turbulent surface layer (Stull 1988). Numerous studies have shown that a scaling length for surface-layer turbulence is

$$L = -\frac{u_*^3 \rho c_P \theta}{k g Q_H}, \qquad (1)$$

where u_* is the friction velocity, ρc_P is the heat capacity for air at constant pressure, θ is the dry-bulb potential temperature, k is the von Kármán constant, g is the acceleration due to gravity, and Q_H is the turbulent sensible heat flux.

From MOST, similarity functions are derived that relate mean scalar concentration gradients to their fluxes in the surface layer where vertical variations of fluxes can be ignored. So, the relationship between wind speed U measured at height z and u_* is

$$U(z-d) = \frac{u_*}{k\{\ln[(z-d)/z_0] - \psi_M[(z-d)/L] + \psi_M(z_0/L)\}}, \qquad (2)$$

where d is the zero-plane displacement length, z_0 is the roughness length, and ψ_M is the integrated form of the similarity function for momentum. For unstable conditions ($L < 0$) the set of expressions used here for ψ_M is determined from (van Ulden and Holtslag 1985)

$$\psi_M = 2\ln\left(\frac{1+X}{2}\right) + \ln\left(\frac{1+X^2}{2}\right) - 2\tan^{-1}(X) + \frac{\pi}{2}, \qquad (3)$$

where X determined by Dyer (1974) is

$$X = \left[1 - \frac{16(z-d)}{L}\right]^{0.25}, \qquad (4)$$

and alternatively by Dyer and Bradley (1982) is

$$X = \left[1 - \frac{28(z-d)}{L}\right]^{0.25}. \qquad (5)$$

For the stable ($L > 0$) case the following from van Ulden and Holtslag (1985) is used:

$$\psi_M = -17\left\{1 - \exp\left[-\frac{0.29(z-d)}{L}\right]\right\}. \qquad (6)$$

The dilemma often faced by researchers is the need for the knowledge of surface turbulent fluxes (e.g., for predicting evaporative water losses or mixed-layer rise) when scalar concentration measurements are made only at a single point or as a gradient. Incorporating the similarity functions requires a knowledge of u_* and

---

Corresponding author address: C. S. B. Grimmond, Climate and Meteorology Program, Geography Department, Student Building 120, Indiana University, Bloomington, IN 47405.

TABLE 1. Comparison between measured and modeled values of (a) u_* and (b) $(z - d)/L$ with different stability functions ($n = 28$): Dyer (1974) and Dyer and Bradley (1982), for Vancouver, British Columbia, summer 1986.

	Measured	Dyer (1974)	Dyer and Bradley (1982)
(a) u_* (m s^{-1})			
Mean	0.344	0.362	0.383
std dev	0.082	0.098	0.103
rmse	—	0.071	0.081
IA (Willmott 1981)	—	0.825	0.796
(b) $(z - d)/L$			
Mean	−0.80839	−0.709	−0.592
std dev	0.57445	0.492	0.397
rmse	—	0.552	0.556
IA (Willmott 1981)	—	0.684	0.645

Q_H a priori to compute L. While Eqs. (1) and (2) can be used to compute u_* iteratively if u is measured at one height and z_0 is known, it is much harder to obtain Q_H. Even if direct measurements were available, there are modeling situations where it is inappropriate to use measured values. For example, if the objective of the model is to determine Q_H, it is obviously inappropriate to require Q_H as a measured input. Clearly, a simple estimation technique is required, especially in urban areas where Q_H is likely to be a dominant flux and is often required for air pollution dispersion modeling.

2. Calculation of stability functions

Measured values of u_* and $(z - d)/L$ are compared with those calculated from (1) and (2) (with Q_H directly measured) using both the Dyer (1974) and Dyer and Bradley (1982) stability function [Eqs. (4) and (5), Table 1, and Fig. 1]. The measured data were collected by Roth et al. (1989) in suburban Vancouver, British Columbia, Canada, in late summer and comprise 28 h of turbulence data representing unstable conditions. The measurements were conducted using a two-propeller Gill anemometer at an effective height of 22.5 m and a Campbell Scientific sonic anemometer and fine-wire thermocouple system at 19 m. The z_0 and d heights were determined by Steyn (1980) based on land-use roughness element analysis, with the method of Lettau (1969) used for z_0.

Comparisons between the measured and modeled u_* and $(z - d)/L$ with the two stability functions are shown in Fig. 1. Table 1 summarizes the statistical results. The index of agreement (IA) ranges between 0 and 1, where 1 indicates perfect agreement between the observed and predicted values (Willmott 1981):

$$\mathrm{IA} = 1 - \frac{\sum_{i=1}^{n} (P_i - O_i)^2}{\sum_{i=1}^{n} (|P_i - \bar{O}| + |O_i - \bar{O}|)^2},$$

where P is predicted and O is observed, and the overbar is the mean. The highest IA value and lowest rmse (root-mean-square error) were obtained using the Dyer (1974) stability function for both u_* and $(z - d)/L$. It should be noted that the ability to predict $(z - d)/L$ is not as good as for u_*, because L is proportional to u_*^3, which thus creates more scatter. No data were available to test stability formulations for stable conditions.

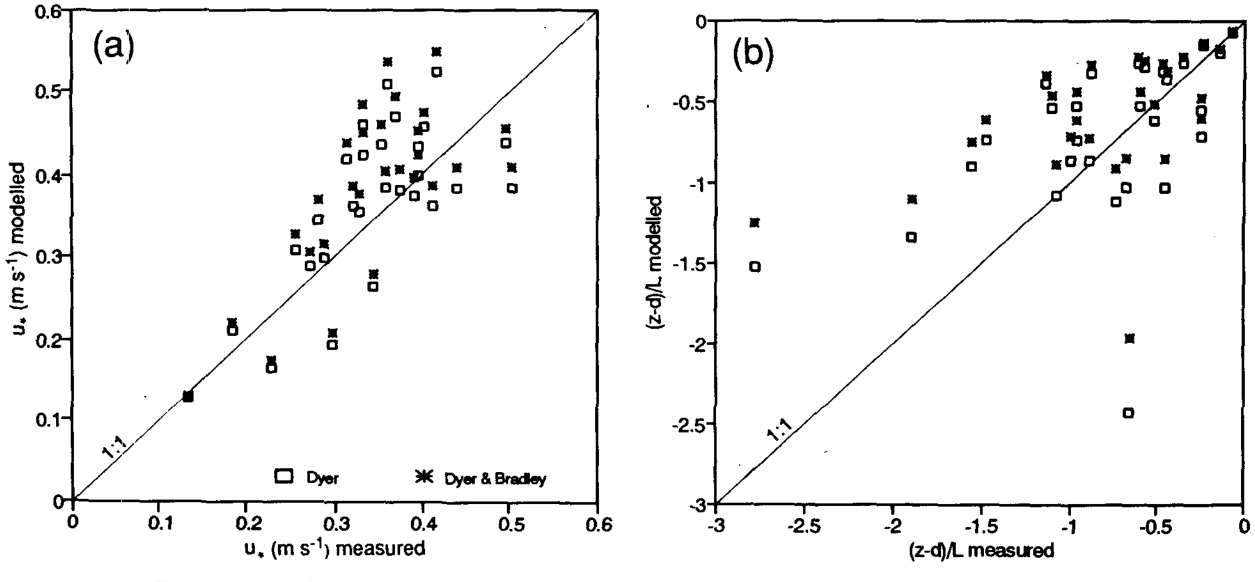

FIG. 1. Measured versus modeled (a) u_* and (b) $(z - d)/L$ for suburban Vancouver, British Columbia, for August 1986 using Dyer (1974) and Dyer and Bradley (1982) stability functions.

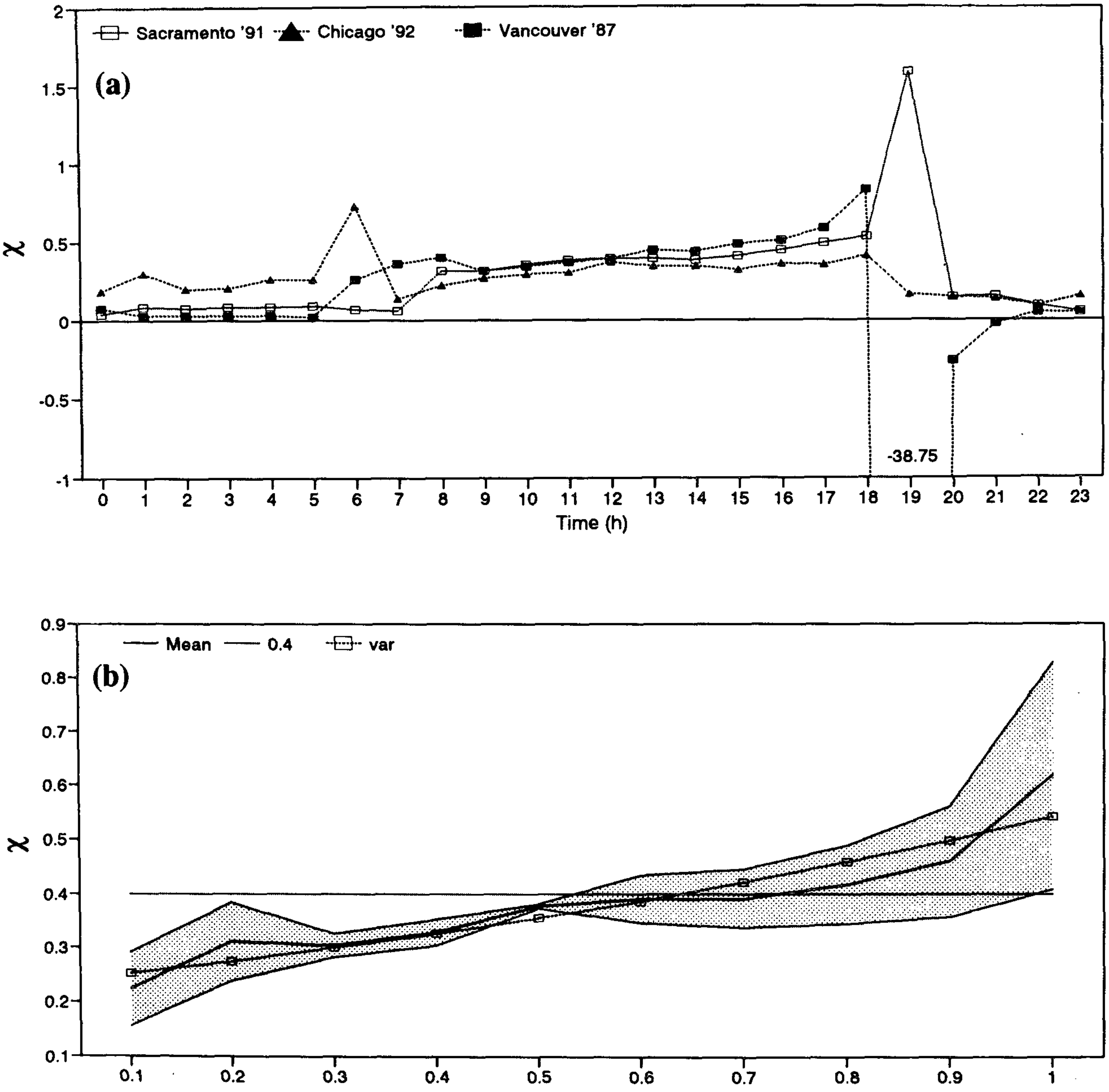

FIG. 2. (a) The diurnal path of χ based on ensemble hourly values for Chicago 1992, Sacramento 1991, and Vancouver 1987. (b) The daytime ($Q^* > 20$ W m^{-2}) function of χ. Plotted is the mean of the Chicago and Vancouver ensemble data, Eqs. (7a) and (8). The shaded areas represent the range between the ensemble Chicago and Vancouver data. Note "var": refers to a time-varying value of χ as defined in Eq. (7a), and 0.4 refers to a fixed daytime χ values of 0.4 [Eq. (8)].

3. Simple parameterization of Q_H for estimating L and stability functions

Briggs (1982) recognized that although L was most highly correlated with wind speed, improved values of L might result with the provision of more accurate estimates of Q_H. He also noted these estimates did not need to be "accurate to within a few percent" to provide improved estimates of L. On this basis he suggests substituting a fraction of solar radiation or of net all-wave radiation in place of Q_H in (1). More recently Hanna and Chang (1992) have developed a simple parameterization for Q_H in any environment that uses a moisture availability factor. This is somewhat difficult to assign a priori in urban areas (e.g., see Ross and Oke 1988). We follow these ideas by replacing the Q_H term in (1) with

TABLE 2. Comparison of u_* and $(z - d)/L$ calculated with measured and modeled Q_H for suburban Sacramento during August 1991. Note: meas—$Q_{H\text{meas}}$ values used; variable—Eqs. (7a) and (7b) used; 0.4—Eq. (8) and Eq. (7b).

Predicted (P)	Observed (O)	P mean	O mean	P std dev	O std dev	Slope	Intercept	r^2	rmse	rmse (systematic)	rmse (unsystematic)	IA
(a) u^* for all hours ($n = 211$)												
Variable	$Q_{H\text{meas}}$	0.337	0.337	0.104	0.105	0.989	0.004	0.999	0.004	0.001	0.004	1.000
0.4	$Q_{H\text{meas}}$	0.336	0.337	0.105	0.105	0.996	0.000	0.999	0.004	0.001	0.004	1.000
(b) $(z - d)/L$ for all hours ($n = 211$)												
Variable	$Q_{H\text{meas}}$	−0.084	−0.069	0.193	0.143	1.278	0.004	0.901	0.074	0.043	0.060	0.950
0.4	$Q_{H\text{meas}}$	−0.055	−0.069	0.138	0.143	0.895	0.007	0.865	0.055	0.020	0.051	0.961
(c) $(z - d)/L$ for the hours when $Q^* > 20$ W m^{-2} [i.e., those hours when Eq. (7a) or (8) is used] ($n = 100$)												
Variable	$Q_{H\text{meas}}$	−0.185	−0.150	0.242	0.174	1.301	0.010	0.878	0.105	0.063	0.085	0.934
0.4	$Q_{H\text{meas}}$	−0.124	−0.150	0.175	0.174	0.921	0.015	0.839	0.076	0.030	0.070	0.951
(d) $(z - d)/L$ for the hours when $Q^* < -20$ W m^{-2} [i.e., those hours when Eq. (7b) is used] ($n = 111$)												
0.1	$Q_{H\text{meas}}$	0.007	0.005	0.017	0.013	0.210	0.006	0.025	0.020	0.011	0.017	0.506

χQ^* where Q^* is the net all-wave radiation and $\chi = Q_H/Q^*$.

The parameterization is based on direct measurements of sensible heat flux and net all-wave radiation over suburban land use in three cities: Vancouver, British Columbia; Chicago, Illinois; and Sacramento, California. These measurements were obtained in the summer. A full description of the methods used and complete results can be found in the following papers: Vancouver, Grimmond (1992); Sacramento, Grimmond et al. (1993a); Chicago, Grimmond et al. (1993b).

Figure 2 illustrates the diurnal path of χ based on ensemble hourly values for each field period. It shows that the diurnal cycle of χ has a similar trend for all cities. The difference in χ between sites during the daytime is ±0.2. At night the variations between cities is greater. At the transition periods, where χ is the ratio of two small numbers, the mean is very unstable. This time period is not considered further here.

In order to allow independent evaluation, a model is developed here using the data from only two of the cities: Vancouver (June 1987) and Chicago (July 1992). The model is subsequently evaluated using the Sacramento August 1991 data. The data from Chicago and Vancouver represent a range of cloud conditions. The expression χQ^* is proposed to replace Q_H in Eq. (1) (based on summer data only):

daytime hours ($Q^* > 20$ W m^{-2})

$$\chi = 0.232 \exp\left[0.847\left(\frac{t}{T}\right)\right] \quad \text{(unstable)} \quad (7a)$$

nighttime hours ($Q^* < -20$ W m^{-2})

$$\chi = 0.1 \quad \text{(stable)}, \quad (7b)$$

where the hour preceding $Q^* > 20$ W m^{-2} in the morning is defined as $t = 0$ (i.e., $t = 0$ when $Q^* < 20$; $t = 1$ for the first hour when $Q^* > 20$; $t = 2$ is the second hour $Q^* > 20$, etc.). Thus, t is incremented by 1 for each hour beyond $t = 0$ and T is the number of hours in the day in which $Q^* > 20$ W m^{-2}. We emphasize that these simple parameterizations of Q_H are for use in computation of L and are not intended as general models for the turbulent sensible heat flux in urban areas.

A second simpler version of (7a), at the level of the variability of the data (see comments above), is also proposed:

daytime hours ($Q^* > 20$ W m^{-2}):

$$\chi = 0.4 \quad \text{(unstable)}. \quad (8)$$

This is based on the mean daytime summertime results for Vancouver (June 1987) and Chicago (July 1992), and the winter and springtime results from Vancouver for January through May 1987 of Grimmond (1992), the summertime work of Cleugh and Oke (1986), and the general observations for suburban areas by Oke (1982). It is important to note that this is proposed as a first-order approximation. Figure 2b shows the range of the ensemble Chicago and Vancouver data and their mean, and (7a) and (8) plotted through the day.

4. Evaluation

In order to evaluate this parameterization L, $(z - d)/L$, and u_* were obtained using (7) and (8) with the Dyer (1974) stability functions. The simple sensible heat parameterization is evaluated using data collected in Sacramento, California, in late August 1991 (Grimmond et al. 1993a) by comparing cal-

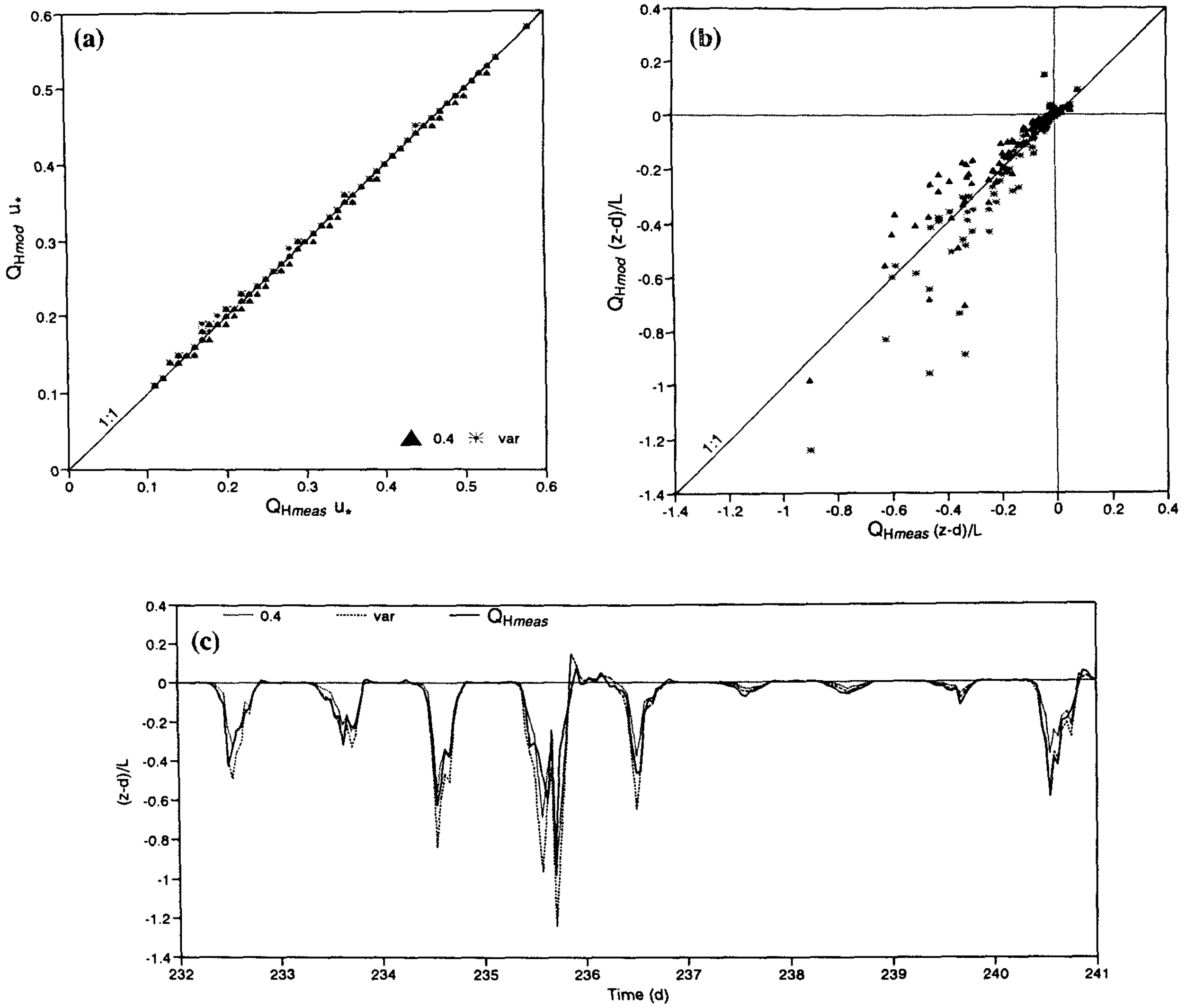

FIG. 3. Results from suburban Sacramento, August 1991, for Q_H measured and modeled: (a) u_*; (b) $(z - d)/L$ scatterplot; and (c) $(z - d)/L$ time series. "Note var:" refers to a time-varying value of χ as defined in Eq. (7a), and 0.4 refers to fixed daytime χ values of 0.4 [Eq. (8)]. Note Eq. (7b) used for nocturnal conditions in both cases.

culated u_* and $(z - d)/L$ based on "modeled" values of Q_H ($Q_{H\text{mod}}$) with those using directly measured Q_H ($Q_{H\text{meas}}$). In both cases, u_* is computed iteratively using (1) and (2).

The results for the u_* comparisons based on measured and modeled Q_H are extremely good (Table 2a), falling very close to a 1:1 line (Fig. 3a). There are only small differences between the variable [Eq. (7a)] and fixed χ model [Eq. (8)].

There is fairly good agreement for $(z - d)/L$ (Figs. 3b and 3c) in unstable conditions. There is increasing scatter with increasingly unstable atmospheric conditions $[(z - d)/L) \rightarrow -1]$. In stable conditions, which in urban areas are often close to neutrality, the model fit is not as good statistically (Table 2d). This is to be expected given the greater observed nocturnal intercity range of χ (Fig. 2). The statistics for the comparisons are presented in Tables 2b and 2c. It should be noted that the variable χ value [Eq. (7a)] influences only the daytime conditions (Table 2c). Again the statistics are only slightly better for the variable [Eq. (7a)] than the fixed χ value [Eq. (8)].

The time series of $(z - d)/L$ (Fig. 3c) show that the modeled values track the measured values well. The absolute modeled values are slightly larger than the measured values during the daytime hours on days 232–236 and then slightly smaller than those determined using measured values on the remaining days. Some of the explanation may relate to synoptic controls on the flux partitioning associated with the passage of cold fronts (Grimmond et al. 1993a).

5. Conclusions

The simple equations proposed to alleviate the need for directly measured Q_H or gradient data to calculate L and therefore also u_* appear to work well for unstable conditions. There is little difference in the performance of the model with a variable daytime value of χ [Eq. (7a)] and a fixed value of 0.4 [Eq. (8)]. Given this insignificant difference in performance and the more general basis for the fixed value of 0.4, until more data become available for the further development of equations of the form of (7a), it is suggested that (8) be used as a simple method to determine Obukhov lengths for suburban areas in the daytime.

Acknowledgments. The data that were made available by Dr. Matthias Roth are gratefully acknowledged. This research has been supported by USDA Forest Service Co-operative Agreement 23-546 (SG) and an Indiana University Summer Faculty Fellowship (SG).

REFERENCES

Briggs, G. A., 1982: Simple substitutes for the Obukhov length. *Proc., Third Joint Conf. on Applications of Air Pollution Meteorology,* San Antonio, TX, 68–71.

Cleugh, H. A., and T. R. Oke, 1986: Suburban–rural energy balance comparisons in summer for Vancouver, B.C. *Bound.-Layer Meteor.,* **36,** 351–369.

Dyer, A. J., 1974: A review of flux-profile relationships. *Bound-Layer Meteor.,* **7,** 363–372.

——, and C. F. Bradley, 1982: An alternative analysis of flux-gradient relationships at the 1976 ITCE. *Bound-Layer Meteor.,* **22,** 3–19.

Grimmond, C. S. B., 1992: The suburban energy balance: Methodological considerations and results from a mid-latitude west coast city under winter and spring conditions. *Int. J. Climatol.,* **12,** 481–497.

——, T. R. Oke, and H. A. Cleugh, 1993a: The role of "rural" in comparisons of observed suburban–rural flux differences. Exchange processes at the land surface for a range of space and time scales. *IAHS Publ.,* **212,** 165–174.

——, C. Souch, and G. Heisler, 1993b: The Chicago Urban Forest Climate Project: An overview and preliminary results. American Association of Geographers, Atlanta, GA.

Hanna, S. R., and J. C. Chang, 1992: Boundary-layer parameterizations for applied dispersion modelling over urban areas. *Bound.-Layer Meteor.,* **58,** 229–259.

Lettau, H. H., 1969: Note on aerodynamic roughness-element description. *J. Appl. Meteor.* **8,** 828–832.

Oke, T. R., 1982: The energetic basis of the urban heat island. *Quart. J. Roy. Meteor. Soc.,* **108,** 1–24.

Ross, S. L., and T. R. Oke, 1988: Tests of three urban energy balance models. *Bound.-Layer Meteor.,* **44,** 73–96.

Roth, M., T. R. Oke, and D. G. Steyn, 1989: Velocity and temperature spectra and cospectra in an unstable suburban atmosphere. *Bound.-Layer Meteor.,* **47,** 309–320.

Steyn, D. G., 1980: Turbulence, diffusion and daytime mixed layer depth over a coastal city. Ph.D. thesis, University of British Columbia, 161 pp.

Stull, R. B., 1988: *An Introduction to Boundary Layer Meteorology.* Kluwer Academic Publishers, 666 pp.

van Ulden, A. P., and A. A. M. Holtslag, 1985: Estimation of atmospheric boundary layer parameters for diffusion applications. *J. Climate Appl. Meteor.,* **24,** 1196–1207.

Willmott, C. J., 1981: On the validation of models. *Phys. Geogr.,* **2,** 184–194.