On admission to the examination room, you should acquaint yourself with the instructions below. You must listen carefully to all instructions given by the invigilators. You may read the question paper, but must not write anything until the invigilator informs you that you may start the examination.

You will be given five minutes at the end of the examination to complete the front of any answer books used.

DO NOT REMOVE THIS QUESTION PAPER FROM THE EXAM ROOM

April 2016 **MTMW15/ MT4YG 2015/16/ A001**

Answer Book
Data Sheet
Any bilingual English language dictionary permitted
Any non-programmable calculator permitted

UNIVERSITY OF READING

Extra-tropical weather systems (MTMW15)

Two hours

Answer **ANY TWO** questions

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100.

1. (a) Starting from the Boussinesq form of the vertical momentum equation

$$f_0 \frac{\partial \psi_g}{\partial z} = b',$$

where b' is buoyancy and ψ_g is geostrophic streamfunction, and given the vertical component of vorticity

$$\xi = \nabla^2 \psi_g,$$

derive an expression for thermal wind balance that includes the vorticity term.

[6 marks]

(b) By assuming a horizontal sinusoidal pressure perturbation field, derive the relationship between pressure perturbations and vorticity.

Similarly, by assuming a horizontal sinusoidal perturbation field in potential temperature, derive the relationship between the vertical gradient of vorticity and buoyancy.

[10 marks]

(c) Consider a cold-cored cut off low pressure system associated with a tropopause depression. Considering your answer to (b) sketch the wind field associated with this low at both the surface and tropopause level.

[6 marks]

(d) Using the quasi-geostrophic thermodynamic equation,

$$\frac{D_g}{Dt} b' + N^2 w = 0,$$

and vorticity equation,

$$\frac{D_g}{Dt} \zeta_g = f_0 \frac{\partial w}{\partial z},$$

derive the equation for the total geostrophic derivative of quasi-geostrophic potential vorticity, q, in adiabatic frictionless flow where

$$q = \zeta_g + f_0 \frac{\partial}{\partial z}\left(\frac{b\prime}{N^2}\right).$$

State any assumptions you make.

What can you infer from your equation about changes in q following the flow?

[12 marks]

(e) The cut off low (described in part (c)) has a maximum potential temperature anomaly of 20 K and height anomaly of 700 m at a height of about 8 km. The anomalies are 700 km across in the horizontal. Assume typical midlatitude values for other parameters as required.

Stating your assumptions, estimate the pressure anomaly, buoyancy, and relative geostrophic vorticity at 8 km height.

Assuming that the buoyancy anomaly is zero at the surface, estimate the value of the quasi-geostrophic potential vorticity.

[Note that an anomaly in geopotential height, Z', on a pressure surface is related to geostrophic streamfunction by $\psi_g = gZ'/f_0$].

[16 marks]

2. (a) Describe the basic state of the Eady model of baroclinic instability and briefly discuss its main results in relation to the behaviour of observed cyclones.

You should include a sketch of and discussion of the growth rate curve in your answer and consider typical values for the maximum growth rate of cyclones, and the wavelength at which this occurs.

Note that the maximum growth rate and shortwave cutoff in this model occur for wavenumbers, k, of $1.6 f_0/NH$ and $2.4 f_0/NH$, respectively.

[24 marks]

(b) What does the shortwave cutoff imply for the tropopause height, H, for cyclones intensifying through baroclinic instability?

[4 marks]

(c) A theory for the height of the tropopause states that H is determined by the criterion that the atmosphere is not unstable for Eady waves of any wavelength. Calculate k for the largest wave that will fit on a latitude circle at 45^{o}N.

Calculate the value of H for which this wave becomes stable.

Is this a reasonable value for the height of the tropopause?

[10 marks]

(d) If Eady waves are not assumed to be infinite in the meridional direction, but instead confined to a region of width L, the shortwave cutoff occurs at

$$K_{\text{cutoff}} = \frac{2.4 f_0}{NH},$$

where

$$K_{\text{cutoff}} = \left(k^2 + \frac{\pi^2}{L^2}\right)^{1/2}.$$

For what value of L would the tropopause height be 10 km according to the theory in (c).

Discuss whether this is a reasonable theory for the tropopause height.

[12 marks]

3. The Omega equation (written in terms of vertical velocity) is given by

$$N^2 \nabla_h^2 w + f_0^2 \frac{\partial w}{\partial z^2} = S,$$

where S is a source term that can be written in two different ways:

$$S = f_0 \frac{\partial}{\partial z}\left(\mathbf{v}_g \cdot \nabla \xi_g\right) - \nabla_h^2\left(\mathbf{v}_g \cdot \nabla b'\right) + f_0 \beta \frac{\partial v_g}{\partial z} \quad (1),$$

or

$$S = 2\boldsymbol{\nabla} \cdot \mathbf{Q} \quad (2),$$

where in (2)

$$\mathbf{Q} = -|\nabla_h b'| \mathbf{k} \times \frac{\partial \mathbf{v}_g}{\partial s}$$

and s is distance along a buoyancy contour.

(a) Give the interpretation of each of the 3 terms on the right hand side in equation (1) and explain what they indicate about vertical motion.

[12 marks]

(b) Give an illustrative example (including a labelled sketch) of how equation (2) can be used to predict the locations of the ascent and descent associated with a jet exit region.

[14 marks]

(c) Using your answer to (b) explain why cyclones that track from the southern side of a jet entrance region to the northern side of a jet exit region typically intensify rapidly.

[4 marks]

(c) Describe an advantage of equation (2) over equation (1) as the source term for the Omega equation.

[4 marks]

(d) In the presence of latent heating the source term of the Omega equation is modified by the addition of extra term

$$\nabla_h^2 \dot{b},$$

where

$$\dot{b} = \frac{g}{\theta_0}\dot{\theta}$$

and $\dot{\theta}$ is a latent heating rate.

Consider a midtropospheric (3 km altitude) region of heating of 2 Kh^{-1} and a width of 300 km. Assuming this extra term is the only source term in the Omega equation and assuming typical midlatitude values where required (state the values assumed), estimate the value of the source term.

From this estimate the midtropospheric ascent rate attributable to this heating.

[12 marks]

(e) Considering the quasi-geostrophic vorticity equation

$$\frac{D_g}{Dt}\zeta_g = f_0\frac{\partial w}{\partial z},$$

estimate the change in geostrophic absolute vorticity beneath the heating region over 12 hours (neglecting the advection of vorticity).

[4 marks]

(End of Question Paper)