On admission to the examination room, you should acquaint yourself with the instructions below. You must listen carefully to all instructions given by the invigilators. You may read the question paper, but must not write anything until the invigilator informs you that you may start the examination.

You will be given five minutes at the end of the examination to complete the front of any answer books used.

DO NOT REMOVE THE QUESTION PAPER FROM THE EXAM ROOM.

January 2016 **MTMW11/MTMW98 2015/16 A001**

Answer Book
Data Sheet
Any bilingual English language dictionnary permitted
Any non-programmable calculator permitted

UNIVERSITY OF READING

FLUID DYNAMICS OF THE ATMOSPHERE AND OCEANS (MTMW11/MTMW98)

Two hours

Answer **ANY TWO** questions.

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100.

1. (a) Explain what is meant by the term "inertial frame". Explain further why a second frame of reference, which rotates relative to the inertial frame, cannot itself be an inertial frame.

[8 marks]

(b) Given that the rate of change of the position vector $\mathbf{r}$ viewed from a rotating frame can be related to its rate of change viewed from an inertial frame by

$$\frac{D_A\mathbf{r}}{Dt} = \frac{D_R\mathbf{r}}{Dt} + \mathbf{\Omega} \times \mathbf{r}$$

show that the acceleration vectors are related by

$$\frac{D_A\mathbf{u}_A}{Dt} = \frac{D_R\mathbf{u}}{Dt} + 2\mathbf{\Omega} \times \mathbf{u} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}) \qquad (1)$$

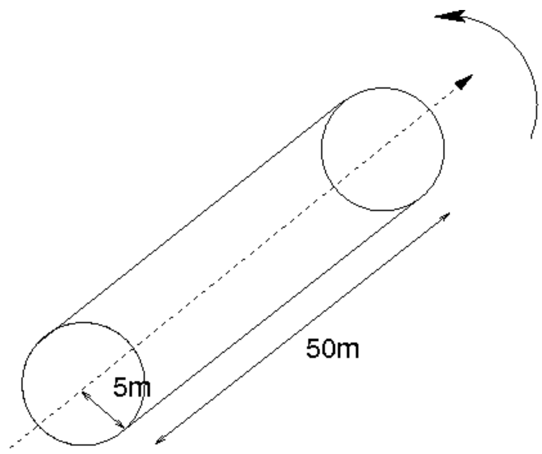
where $\mathbf{u}$ is the velocity viewed from a rotating frame and $\mathbf{u}_A$ is the velocity viewed from an inertial frame.

[12 marks]

(c) Make estimates of typical magnitudes of the three terms on the right hand side of Eq. 1 within a mid-latitude weather system, clearly stating the assumptions made.

[18 marks]

(d) Consider a cylindrical space station of length 50 m and radius 5 m in deep space. An artificial gravity field can be created by rotating the space station.



Which term in Eq. 1 is responsible for the artificial gravity? How quickly must the station be rotated about its long axis to produce an Earth-type gravity? Why would a rotation about the long axis be the most convenient choice?

[12 marks]

2. (a) Consider the primitive equations for incompressible horizontal flow on an f-plane

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$
$$\frac{Dv}{Dt} + fu = -\frac{1}{\rho}\frac{\partial p}{\partial y}$$

With reference to the above equations, explain what is meant by the term "geostrophic velocity" and write down expressions for the x and y components of the geostrophic flow.

[7 marks]

(b) Show that the geostrophic velocity has zero horizontal divergence and hence explain why it has no associated vertical velocity.

[7 marks]

(c) Show that the ageostrophic components of the horizontal flow, u_a and v_a satisfy the equations

$$\frac{Du}{Dt} = fv_a$$

$$\frac{Dv}{Dt} = -fu_a$$

Hence, by performing a scale analysis show that the ageostrophic velocity scales as $Ro\,U_g$ where U_g is a scale for the geostrophic flow and Ro is a number that you should derive and name.

[11 marks]

(d) For a regime in which Ro is small, typical pressure fluctuations may be scaled as $P = \rho_0 fUL$. Obtain a scaling for typical values of the vertical velocity that is valid for the same regime.

[5 marks]

(e) The length of the day on the planet Jupiter is 10 hr and its atmosphere is comprised of a mixture of gases with a gas constant of $R = 3600$ J kg^{-1}K^{-1}. The great red spot is an anticyclonic weather system on Jupiter at a latitude of $22°$, and is around 10^4 km across with wind speeds of order 50 m s^{-1}. You may also assume a temperature of around 125 K. For this weather system:

(i) Comment on the validity of geostrophic balance.

[5 marks]

(ii) Estimate a typical ageostrophic wind speed.

[3 marks]

(iii) Estimate the ratio of the pressure variations across the system to the mean pressure.

[5 marks]

(iv) There is uncertainty about the vertical extent of the circulation. If we were able to take measurements of vertical velocity within the great red spot to within an accuracy of 1 ms^{-1}, would that help to resolve the uncertainty about its depth?

[7 marks]

3. (a) Rossby waves in a uniform flow U parallel to the x-axis on a β-plane are solutions of the barotropic vorticity equation comprising the real

part of

$$\varphi = \Psi \exp\left[i\left(kx + ly - \omega t\right)\right]$$

Sketch the form of the disturbance at $t = 0$ in the xy plane. Mark the crests of the waves and the wave vector (k, l) clearly on your sketch.

[10 marks]

(b) Such a Rossby wave at $50°$N has a wavelength of 3500 km, with crests running at an angle of $120°$ to the west-to-east x axis. Calculate the x and y components of the wave vector, k and l.

[10 marks]

(c) The dispersion relation for Rossby waves is

$$\omega = Uk - \frac{\beta k}{k^2 + l^2}$$

Use this relationsip to obtain an expression for the phase speed of the wave in the x direction. What must be the strength of the background zonal flow U if the wave described in part (b) is stationary relative to the Earth's surface? You may wish to use the expression

$$\beta = \frac{2\Omega\cos\phi_0}{a}$$

in constructing your answer.

[15 marks]

(d) Determine a formula for the group velocity for a general Rossby wave in uniform zonal flow on a β-plane. Explain the physical significance of the group velocity, making clear how that differs from the phase velocity.

[15 marks]

[End of Question Paper]