

**On admission to the examination room, you should acquaint yourself with the instructions below. You must listen carefully to all instructions given by the invigilators. You may read the question paper, but must not write anything until the invigilator informs you that you may start the examination.**

**You will be given five minutes at the end of the examination to complete the front of any answer books used.**

**DO NOT REMOVE THE QUESTION PAPER FROM THE EXAM ROOM.**

---

**April 2016**

**MTMG49/ 2015/6/ A001**

**Answer Book**

**Data Sheet**

**Any bilingual English language dictionary permitted**

**Any non-programmable calculator permitted**

**UNIVERSITY OF READING**

**Boundary Layer Meteorology and Micrometeorology (MTMG49)**

Two hours

---

**Answer ANY TWO questions**

The marks for the individual components of each question are given in [] brackets. The total mark for the paper is 100.

1. (a) The turbulence kinetic energy budget is

$$\underbrace{\frac{\partial \bar{e}}{\partial t}}_1 = -\underbrace{\overline{u'w'}}_2 \underbrace{\frac{d\bar{u}}{dz}}_3 + \underbrace{\frac{g}{\theta_0} \overline{w'\theta'}}_3 \underbrace{+T}_4 \underbrace{-\varepsilon}_5$$

where  $T$  represents the transport term.

Describe each of the terms (1, 2, 3, 4 and 5) in the budget, explaining the corresponding physical processes.

State which terms in the turbulence kinetic energy budget can be neglected in stable conditions.

[12 marks]

- (b) The mean wind shear in the surface layer in stable conditions is given by

$$\frac{d\bar{u}}{dz} = \frac{u_*}{\kappa z} \left( 1 + 5 \frac{z}{L} \right)$$

where  $L = -u_*^3 / (\kappa \frac{g}{\theta_0} \overline{w'\theta'})$ . Assuming that the flow is steady, use the TKE budget to estimate the dissipation rate at 3 m if  $u_* = 0.1 \text{ m s}^{-1}$  and  $\overline{w'\theta'} = -10^{-2} \text{ m s}^{-1} \text{ K}$ . State any assumptions you make.

[12 marks]

- (c) The following measurements were made during daytime over a large expanse of short grass for a 30-minute period:

Height (m)	Potential temperature (K)	Wind speed ( $\text{m s}^{-1}$ )
2	288.5	2.60
4	288.1	2.96

The Richardson number,  $Ri$  is defined as

$$Ri = \frac{\frac{g}{\theta_0} \frac{d\bar{\theta}}{dz}}{\left(\frac{d\bar{u}}{dz}\right)^2}$$

Show that at a height of 3 m, the Richardson number  $Ri$  can be estimated as approximately -0.21. Is the flow turbulent or not? Why?

[6 marks]

(d) The mean wind shear in an unstable surface layer can be written

$$\frac{d\bar{u}}{dz} = \frac{u_*}{\kappa z} \left(1 - 16 \frac{z}{L}\right)^{-1/4}$$

Hence estimate the *friction velocity*  $u_*$  and the *surface sensible heat flux*  $H = \rho c_p \overline{w'\theta'}$  for the case (c) above. You may make use of the fact that in unstable conditions  $Ri \approx z/L$ .

[10 marks]

(e) For the case (c) above, state the primary mechanism for (i) turbulence generation at 3 m, (ii) turbulence destruction at 3 m, and (iii) turbulence generation at 50 m, in each case giving your reasons.

[10 marks]

2. (a) Describe the role of turbulence in the atmospheric boundary layer, explaining the turbulence closure problem and the main purpose of a *turbulence closure scheme*.

[10 marks]

- (b) The Ekman model of boundary layer flow is defined by the following equations:

$$-f(\bar{v} - v_g) = K_m \frac{\partial^2 \bar{u}}{\partial z^2}, \quad f(\bar{u} - u_g) = K_m \frac{\partial^2 \bar{v}}{\partial z^2}$$

Explain the meaning of all the symbols. State the three assumptions that have been used to obtain these simplified equations. State the boundary conditions required for  $u$  and  $v$  at the top and bottom of the atmospheric boundary layer.

[12 marks]

- (c) The solutions for the Ekman layer when  $v_g = 0$  are

$$\bar{u} = u_g [1 - \exp(-z/h) \cos(z/h)], \quad \bar{v} = u_g \exp(-z/h) \sin(z/h)$$

with  $h = (2K_m/f)^{1/2}$ . (i) Explain what  $h$  represents physically,

giving your reasons. (ii) Sketch the hodograph of the solution with  $u$  against  $v$  through the boundary layer. (iii) Calculate the angle between the geostrophic and surface winds (note that the geostrophic wind in the above solution is aligned with  $x$ ). (iv) Discuss how these model predictions compare with observations.

[14 marks]

- (d) Explain the physical processes that control turbulence in the boundary layer following nightfall, under clear skies. How can the Ekman equations (presented in (b)) be changed to model these conditions? Sketch on a hodograph the evolution of the wind speed and direction at two suitably chosen heights indicating the key features of the evolution.

[14 marks]

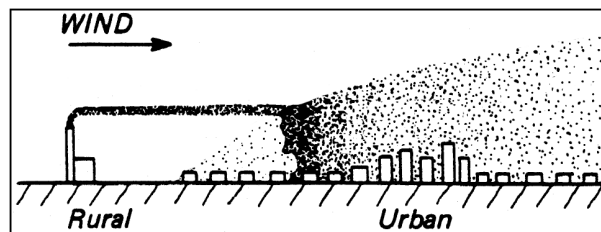
3. (a) The surface energy budget equation is

$$R_n - G = H + \lambda E$$

Sketch the typical diurnal evolution, in cloudless conditions, of the four components of this equation over (i) a sub-tropical desert, and (ii) an ocean at the same latitude. For each component, describe briefly the cause of any differences in terms of the nature of the two surfaces, including reference to their typical daytime *Bowen ratio*.

[14 marks]

- (b) The schematic below shows the dispersion of smoke emitted over rural grassland as it encounters an urban area on a cloudless night in light wind conditions.



Describe, with the use of sketch diagrams where appropriate, the two main ways (i.e. dynamical and thermodynamical) that the properties of the rural and urban surfaces in this situation lead to the observed distribution of smoke, including a discussion of the *internal boundary layer*.

[14 marks]

- (c) The steady state distribution of the mean concentration  $C$  in the plume of a continuously emitted pollutant dispersing over homogeneous flat terrain is given by

$$C(x, y, z) = \frac{Q}{2\pi\sigma_y\sigma_z\bar{u}} \exp\left(-\frac{y^2}{2\sigma_y^2} - \frac{z^2}{2\sigma_z^2}\right)$$

where  $\bar{u}$  is the mean wind velocity (aligned in the  $x$  direction),  $Q$  is the emission rate (in  $\text{kg s}^{-1}$ ) and  $\sigma_y$  and  $\sigma_z$  are the plume widths in the

horizontal ( $y$ ) and vertical ( $z$ ) directions, defined by  $\sigma_y^2 = 2K_y x / \bar{u}$  and  $\sigma_z^2 = 2K_z x / \bar{u}$ . The plume centreline corresponds to  $y = 0, z = 0$ .

- (i) If the boundary layer has stable stratification, would you expect  $K_y < K_z, K_y \approx K_z$  or  $K_y > K_z$ ? And if the boundary layer has unstable stratification? Justify your answers.

[10 marks]

- (ii) Assuming that  $Q = 10^{-3} \text{ kg s}^{-1}, \bar{u} = 5 \text{ m s}^{-1}, K_y = 10 \text{ m}^2 \text{ s}^{-1}$  and  $K_z = 1 \text{ m}^2 \text{ s}^{-1}$ , calculate the pollutant concentration  $C$  along the plume centreline at a distance  $x = 100 \text{ m}$  downstream from the virtual source ( $x = 0$ ). Repeat the calculation for similar conditions, but 20 m away from the plume centreline in the vertical direction.

[12 marks]

[End of Question Paper]