On admission to the examination room, you should acquaint yourself with the instructions below. You must listen carefully to all instructions given by the invigilators. You may read the question paper, but must not write anything until the invigilator informs you that you may start the examination.

You will be given five minutes at the end of the examination to complete the front of any answer books used.

DO NOT REMOVE THIS QUESTION PAPER FROM THE EXAM ROOM.

April 2015 **MTMW15/MT4YG 2014/15 A001**

Answer Book
Data Sheet

Any bilingual English language dictionary permitted
Any non-programmable calculator is permitted

UNIVERSITY OF READING

Extratropical weather systems (MTMW15)

Two hours

Answer **ANY TWO** questions

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100

1.

(a) Starting from these expressions for the vertical component of relative vorticity and hydrostatic balance

$$\xi_g = \nabla_h^2 \psi_g$$
$$f_0 \frac{\partial \psi_g}{\partial z} = b',$$

where $\psi_g$is the geostrophic streamfunction, derive the thermal wind balance relationship linking the vertical variation of relative vorticity to the buoyancy anomaly.

[4 marks]

(b) Consider a warm anomaly at the ground. Describe two mechanisms that can generate such a surface anomaly.

[4 marks]

(c) If the circulation associated with this warm anomaly decays with increasing height show, using your answer to (a), that this circulation must be cyclonic (assume Northern hemisphere).

[6 marks]

(d) Consider a thermal wave at the ground composed of alternating warm and cold anomalies propagating on a background temperature gradient typical of that of the Northern Hemisphere. The temperature anomalies are advected by the winds that they induce.

Draw sketches to demonstrate how this occurs. State the direction in which the thermal wave propagates.

[10 marks]

(e) A mathematical expression for the structure of the geostrophic streamfunction associated with the wave is given by

$$\psi_g = \psi_0 \sin(ly)\sin\big(k(x-ct)\big)\exp\left(-\frac{\kappa N z}{f_0}\right),$$

where ψ_0 is a constant, *k* and *l* are the horizontal wavenumbers, $\kappa^2 = k^2 + l^2$, and *c* is the phase speed.

Calculate the percentage reduction in amplitude of the wave at 10 km height given that the thermal anomalies have a wavelength of 1000 km. Assume typical midlatitude values for parameters where required and state your assumptions.

[9 marks]

(f) Derive expressions for the buoyancy and meridional wind component associated with this wave.

On a vertical cross-section (with axes of *z* and *kx*) sketch contours showing the potential temperature and meridional wind structure.

Explain why this wave does not lead to net polewards heat transport.

[13 marks]

(g) Given that the thermal wave has an amplitude of 10 K evaluate the magnitude of the associated pressure perturbation at the ground.

[4 marks]

2.

(a) Sketch the potential temperature and circulation associated with an upper-tropospheric positive potential vorticity anomaly, indicating the variation with both height and horizontal distance.

[6 marks]

(b) The potential vorticity anomaly relative to a resting background state is related to the geostrophic streamfunction, ψ_g, by

$$q' = \nabla_h^2 \psi_g + f_0^2 \frac{\partial}{\partial z}\left(\frac{1}{N^2}\frac{\partial \psi_g}{\partial z}\right).$$

Consider an anomaly of magnitude $q' = 2 \times 10^{-4}$ s^{-1} and horizontal and vertical scales of $L = 500$ km and $H = 5$ km respectively. Using scaling arguments and typical midlatitude values for parameters where required, estimate ψ_g and so the magnitudes of the wind and potential temperature anomalies associated with this potential vorticity anomaly.

[16 marks]

(c) The quasi-geostrophic Omega equation is given by

$$N^2 \nabla_h^2 w + f_0^2 \frac{\partial^2 w}{\partial z^2} = f_0 \frac{\partial}{\partial z}\left(\mathbf{v_g} \cdot \nabla_h \xi_g\right) - \nabla_h^2\left(\mathbf{v_g} \cdot \nabla_h b'\right) + f_0 \beta \frac{\partial v_g}{\partial z}$$

where the three source terms are on the right hand side of the equation and the subscript *h* indicates horizontal components.

If the potential vorticity anomaly defined in (b) is moving East explain, using the Omega equation and considering the differential vorticity advection source term, why vertical motion is expected and where ascent and descent are likely to occur relative to the anomaly.

[10 marks]

(d) The quasi-geostrophic horizontal momentum equations are

$$D_g v_g + \beta y u_g + f_0 u_{ag} = 0$$
$$D_g u_g - \beta y v_g - f_0 v_{ag} = 0,$$

where $D_g$is the Lagrangian time derivative given by $D_g = \frac{\partial}{\partial t} + u_g \frac{\partial}{\partial x} + v_g \frac{\partial}{\partial y}$ and the subscripts g and ag refer to geostrophic and ageostrophic wind components respectively.

Using these equations and the continuity equation derive the following equation for quasi-geostrophic absolute vorticity:

$$D_g \zeta_g = f_0 \frac{\partial w}{\partial z}.$$

[10 marks]

(e) Assume that the vertical motion in (c) can be expressed as

$$w = w_0 \sin\left(\frac{\pi z}{H}\right)$$

where the maximum vertical velocity, $w_0 = 20$ cm s^{-1} and H is as defined in (b). Using the quasi-geostrophic vorticity equation estimate the geostrophic Lagrangian rate of change of absolute vorticity at the ground.

How long (in hours) would it take to spin up absolute vorticity of magnitude equal to the typical value of planetary vorticity in the midlatitudes?

[8 marks]

3.

(a) Consider a front aligned along the y-axis. Show the circumstances under which the absolute momentum, $M = fx + v_g$, is conserved.

In a moist environment the frontal surface can be defined as the slope of surfaces of either equivalent potential temperature, θ_e, or absolute momentum, M. Derive expressions for these slopes and show that they can be expressed as

$$\left(\frac{dz}{dx}\right)_M = -\frac{\mathcal{F}^2}{\mathcal{S}^2}$$
$$\left(\frac{dz}{dx}\right)_{\theta_e} = -\frac{\mathcal{S}^2}{\mathcal{N}_s{}^2}$$

where $\mathcal{F}^2 = f\left(f + \frac{\partial \bar{v}}{\partial x}\right) = f\bar{\zeta}$, $\mathcal{S}^2 = \frac{\partial b'}{\partial x} = f\frac{\partial \bar{v}}{\partial z}$, and $\mathcal{N}_s^2 = \frac{g}{\theta_0}\frac{\partial \bar{\theta}_e}{\partial z}$ and the overbar indicates the basic state flow which is independent of time and direction y.

[16 marks]

(b) The dispersion relation for small amplitude perturbations of the form $\psi = \psi_0 e^{i(kx+mz)+\sigma t}$ to a geostrophically balanced front is

$$\sigma^2 = \frac{-\mathcal{F}^2 + 2\mathcal{S}^2\alpha - \mathcal{N}_s^2\alpha^2}{1+\alpha^2}$$

where $\alpha = {}^{k}/_{m}$ is the aspect ratio and $\alpha \ll 1$ if the hydrostatic approximation is made.

Identify the criterion for the solutions to be unstable (making the hydrostatic approximation) and show that this can be expressed as

$$\mathcal{F}^2 < \frac{\mathcal{S}^4}{\mathcal{N}_s^2}$$

Show that this criterion can be expressed as

$$\mathrm{Ri} < \frac{f}{\bar{\zeta}}$$

where Ri is the Richardson number given by $\mathrm{Ri} = \mathcal{N}_s^2 \left(\frac{\partial \bar{v}}{\partial z}\right)^{-2}$ (assume $f/\bar{\zeta} > 0$).

[12 marks]

(c) By calculating the Richardson number for the following data from a certain frontal rainband determine whether the atmosphere is unstable to conditional symmetric instability (CSI):

$$\frac{\partial \bar{v}}{\partial z} = 10^{-2}\text{s}^{-1},\ \bar{\zeta} = 0.7f, \text{ and } \mathcal{N}_s^2 = 5 \times 10^{-5}\text{s}^{-2}.$$

[6 marks]

(d) By consideration of the observed properties of convection, state why it is reasonable to take $f = 0$ in the growth rate equation in (b) if we want to apply it to extratropical convection.

[4 marks]

(e) Write down the growth rate equation for $f = 0$ and hence deduce the convective instability criterion.

Why does this growth rate equation suggest that the linear theory is inappropriate for describing atmospheric convection?

[8 marks]

(f) Which processes, if included in the theory, could improve its predictions?

[4 marks]

(End of Question Paper)