On admission to the examination room, you should acquaint yourself with the instructions below. You <u>must</u> listen carefully to all instructions given by the invigilators. You may read the question paper, but must <u>not</u> write anything until the invigilator informs you that you may start the examination.

You will be given five minutes at the end of the examination to complete the front of any answer books used.

DO NOT REMOVE THIS QUESTION PAPER FROM THE EXAM ROOM.

January 2015 **MTMW11/98/ 2014/15 A001**

Answer Book
Data Sheet

Any non-programmable calculator is permitted

UNIVERSITY OF READING

Fluid Dynamics of the Atmosphere and Oceans (MTMW11/98)

Two hours

Answer **ANY TWO** questions

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100

1. (a) Define the following terms for fluid flow: trajectory; streamline; and streakline. Make sure that your definitions carefully distinguish between these three terms.

[10 marks]

(b) Any two-dimensional (horizontal) velocity field can be split into two terms:

$$\mathbf{v} = \mathbf{k} \times \nabla\psi + \nabla\chi$$

where $\mathbf{v} = (u, v)$ and $\mathbf{k}$ is a unit vector normal to the plane of motion.

Derive expressions for the divergence and the vorticity in terms of the fields y and c, and explain their physical significance

If the horizontal flow is non-divergent, what is the significance of lines of constant y?

Explain briefly why the largest gradients of c near the tropopause are found in the tropics.

[15 marks]

(c) This Eulerian form of the mass conservation equation is:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

Assuming that $\mathbf{u} = (u, 0, 0)$ for simplicity, show that the mass conservation equation can be written in the alternative Lagrangian form:

$$\frac{D\rho}{Dt} + \rho \frac{\partial u}{\partial x} = 0$$

Hence write down the corresponding Lagrangian expression when all three velocity components are non-zero. <u>What is the physical significance of the first term in the above expression?</u>

[15 marks]

(d) A meteorological station observes a westerly wind (from the west) of 10 ms^{-1}, and records a temperature fall of 2K in 24 hours. Charts reveal that the temperature gradient in the eastward direction is 3×10^{-6} K m^{-1}. Estimate the Eulerian and Lagrangian rates of change of temperature at the station, in units K s^{-1}.

[10 marks]

2. (a) A fluid flow consists of a circular vortex of uniform vorticity χ and radius R, surrounded by fluid of zero vorticity. Calculate the flow speed around the edge of the vortex by using Stokes's Theorem (in standard notation):

$$\iint (\nabla \times \mathbf{u}) \cdot d\mathbf{S} = \oint \mathbf{u} \cdot d\mathbf{l}$$

Now calculate the speed of the flow outside the vortex at radius $r > R$. Hence sketch the flow speed as a function of radius between $r = 0$ and $r = 3R$.

[10 marks]

(b) Kelvin's circulation theorem can be written:

$$\frac{DC}{Dt} = -\oint \frac{dp}{r}$$

where C is the circulation, and where the integral is around a closed material line of fluid.

Assuming that the fluid is an ideal gas and that the material line lies on an isentropic surface (constant potential temperature surface), show that the integral vanishes and $DC/Dt = 0$.

[15 marks]

(c) The linearized barotropic vorticity equation on a β-plane can be written (in the usual notation) as:

$$\frac{\partial \xi}{\partial t} + U\frac{\partial \xi}{\partial x} + \beta v = 0$$

Explain what this equation means physically. Assuming small-amplitude Rossby waves with the form $e^{i(kx-\omega t)}$, show that their zonal phase speed is:

$$c = U - \frac{\beta}{k^2}$$

[15 marks]

(d) Calculate the wavelength of Rossby waves that are stationary relative to the Earth's surface when the average zonal flow is 20 m s^{-1} at 40º N. Outline briefly how Rossby waves are generated in the atmosphere.

[10 marks]

3. The vorticity equation for a rotating fluid can be written in the form:

$$\frac{\partial}{\partial t}\boldsymbol{\zeta} = -(\mathbf{u}\cdot\nabla)\boldsymbol{\zeta} - \boldsymbol{\zeta}(\nabla\cdot\mathbf{u}) + (\boldsymbol{\zeta}\cdot\nabla)\mathbf{u} + \frac{\nabla\rho\times\nabla p}{\rho^2}$$

(1) (2) (3) (4) (5)

where $\boldsymbol{\zeta}$ is the **absolute** vorticity vector and $\mathbf{u}$ is the relative velocity with respect to the rotating frame of reference.

(a) Write brief notes on the physical interpretation of each of the 5 terms in the above equation.

[10 marks]

(b) Consider a laboratory experiment where a tank contains water, which rotates with the tank at a rate Ω about a vertical axis (no flow relative to the tank). The water is now stirred gently for a short time while the tank is still rotating. Show that in this situation the above vorticity equation reduces to:

$$(\boldsymbol{\Omega}\cdot\nabla)\mathbf{u} \approx 0$$

Hint: consider the values of each of the terms in the vorticity equation before and after the disturbance.

[25 marks]

(c) What would you expect the distribution of dye to look like when it is dropped into the tank before the disturbance and (with dye of a different colour) after the disturbance?

[5 marks]

(d) For large-scale atmospheric motion the vorticity equation can be approximated as:

$$\frac{D\xi}{Dt} = f_0 \frac{\partial w}{\partial z}$$

where ξ is the vertical component of relative vorticity and f_0 is the value of the Coriolis parameter at a reference latitude. Estimate the vertical velocity in the mid troposphere associated with the spin-up of a cyclone from an initial relative vorticity of 0.1 f_0 to a relative vorticity of 0.4 f_0 in one day.

[10 marks]

(End of Question Paper)