Candidates are admitted to the examination room ten minutes before the start of the examination. On admission to the examination room, you are permitted to acquaint yourself with the instructions below and to read the question paper.

Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.

January 2014 **MTMW11/99**

Answer Book
Data Sheet
Any bilingual English language dictionary permitted
Only Casio-fx83 calculators are permitted

UNIVERSITY OF READING

Fluid Dynamics of the Atmosphere and Oceans
(MTMW11/MTMW99)

Two hours

Answer **ANY TWO** questions

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100

1. (a) Explain what the following two expressions represent for a fluid:

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

[5]

(b) For a fluid in motion explain what is meant by a trajectory, a streamline and a streakline. Take care that your explanations distinguish clearly between these three entities.

[10]

(c) A 2-D velocity field can be split into two terms:

$$\boldsymbol{v} = \boldsymbol{k} \times \nabla\psi + \nabla\chi$$

Show that: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \nabla^2\chi$

And that: $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \nabla^2\psi$

Under what conditions would the streamlines of the velocity field coincide with contours of constant ψ?

[10]

(d) Assume that the equations governing the behaviour of an incompressible fluid flow, with no variation of velocity in the vertical, are given by:

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

$$\frac{Dv}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{D\rho}{Dt} = 0$$

By referring to each of the terms individually, explain what each of these equations means physically.

[10]

(e) For small perturbations on a shallow layer of the ocean, the equations in part (d) can be reduced to the form:

$$\frac{\partial u}{\partial t} = -g\frac{\partial h'}{\partial x}$$

$$\frac{\partial h'}{\partial t} = -H\frac{\partial u}{\partial x}$$

where we have assumed no variation in the y-direction. By assuming a wavelike solution for h' show that the phase speed of the wave is $c_p = \sqrt{gH}$, and that the group speed is $c_g = \sqrt{gH}$. Explain whether such waves are dispersive or non-dispersive.

[15]

2. (a) Euler's equation in a rotating frame of reference can be written in the form:

$$\frac{D\boldsymbol{u}}{Dt} = -2\boldsymbol{\Omega} \times \boldsymbol{u} - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{r}) + \boldsymbol{g} - \frac{1}{\rho}\nabla p$$

Explain the physical significance of all the terms in this equation. Estimate the ratio of the second and third terms on the right of the equality sign for the Earth.

[10]

(b) Draw a diagram to illustrate why it is possible to simplify Euler's equation by introducing an effective gravity.

[5]

(c) Show by using the first law of thermodynamics in the form: $ds = c_p \frac{dT}{T} - R\frac{dp}{p}$, that the potential temperature θ of an air parcel is given by:

$$\theta = T\left({}^{p}/_{p_0}\right)^{-R/c_p}$$

Explain briefly why the quantity θ is useful in meteorology.

[15]

(d) When natural co-ordinates are used to describe horizontal motion, the component of the momentum equation normal to the velocity at any point is:

$$\frac{V^2}{R} + fV = -\frac{1}{\rho}\frac{\partial p}{\partial n}$$

Explain what each of the terms in this equation represents physically, and draw sketches to illustrate their meaning for both cyclonic and anti-cyclonic circulation in the northern hemisphere.

[10]

(e) Show by defining geostrophic velocity that $\frac{V^2}{R} + fV - fV_g = 0$
If $V_g = 10\,ms^{-1}$ at $|R| = 500km$, find V for a cyclone at 50°N in the northern hemisphere.

[10]

3. (a) State briefly how Rossby waves are generated in the atmosphere.

[5]

(b) Consider a situation where the atmosphere is initially zonally symmetric and then air parcels are displaced by a Rossby wave. Assuming that, for such a wave, the vorticity equation can be approximated as

$$\frac{D\xi}{Dt} + \beta v = \frac{D}{Dt}(\xi + f_0 + \beta y) = 0$$

show that the Rossby wave will be expected to propagate westwards with respect to the undisturbed atmospheric flow. Draw a schematic diagram to illustrate why the propagation is westward.

[10]

(c) Consider air flow over a shallow hill. Assume that the barotropic vorticity equation applies to this situation:

$$\frac{D}{Dt}\left(\xi + \frac{fh}{D}\right) = 0$$

where h is the height of the hill above ground level and D the depth of the troposphere. Using this equation describe the changes in relative vorticity that you will expect as air flows over the hill. Make clear where you will expect to find the largest value of $|\xi|$ and give its value at that point together with its sign.

[10]

(d) Kelvin's circulation theorem can be written:

$$\frac{dC}{dt} = -\oint \frac{dp}{\rho}$$

where the integral is around the boundary of a closed material contour with circulation C. Show that if the circuit lies on an isentropic surface, the integral vanishes and $\frac{dC}{dt} = 0$. (Assume that the fluid is an ideal gas.)

[15]

(e) Explain briefly why the quantity $\frac{\boldsymbol{\zeta}\cdot\nabla\theta}{\rho}$ is of fundamental importance in geophysical fluid dynamics. When this quantity is plotted on an isentropic surface, what do the isolines (contours) of equal value represent?

[10]

(End of Question Paper)