

Candidates are admitted to the examination room ten minutes before the start of the examination. On admission to the examination room, you are permitted to acquaint yourself with the instructions below and to read the question paper.

Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.

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April 2014

MTMG49

Answer Book  
Data Sheet  
Any bilingual English language dictionary permitted  
Only Casio-fx83 calculators are permitted

UNIVERSITY OF READING

Boundary Layer Meteorology and Micrometeorology (MTMG49)

Two hours

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Answer **ANY TWO** questions

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100.

1.(a) The evolution of turbulent kinetic energy per unit mass,  $e$ , is given by

$$\frac{D\bar{e}}{Dt} = -\overline{u'w'} \frac{\partial \bar{u}}{\partial z} + \frac{g}{\theta_0} \overline{w'\theta'} - \frac{\partial}{\partial z} \overline{w'e} - \frac{1}{\rho} \frac{\partial}{\partial z} \overline{w'p'} - \varepsilon$$

State the name and a brief physical interpretation for each of the five terms on the right hand side of the above equation.

[10 marks]

(b) Under what conditions does the turbulent kinetic energy equation take the form

$$\varepsilon = -\overline{u'w'} \frac{\partial \bar{u}}{\partial z} + \frac{g}{\theta_0} \overline{w'\theta'} \quad (1)$$

Stating any assumptions that you make, show that an equivalent form for the surface layer is given by

$$\varepsilon = \frac{u_*^3}{\kappa z} - \frac{g}{\theta_0} u_* \theta_* \quad (2)$$

where  $\kappa = 0.4$  is the Von Kármán constant.

[8 marks]

(c) Some students carry out turbulence measurements in order to test out the prediction given by equation (2). They obtain the following values:  $u_* = 0.45 \text{ m s}^{-1}$ , surface heat flux  $H_0 = 250 \text{ W m}^{-2}$ , and surface temperature  $\theta_0 = 297 \text{ K}$ . What is the value of  $\varepsilon$  at a height of  $z = 10 \text{ m}$ ?

Conditions become overcast but the wind-speed remains the same. What value of  $\varepsilon$  would be obtained under these conditions? State your assumptions clearly.

[8 marks]

- (d) One of the students has a different method for obtaining  $\varepsilon$  using spectral analysis.

Sketch the energy spectrum  $S(f)$  as a function of frequency  $f$ . Describe the processes affecting turbulence at low, middle and high frequencies, quoting typical length scales where appropriate.

[10 marks]

- (e) The students calculate the energy spectrum  $S(f)$  as a function of frequency  $f$  for the measured velocity component  $u$ , which is shown in Table 2.1.

The form of the spectrum in the inertial sub-range is

$$S(f) = \alpha \varepsilon^{2/3} f^{-5/3}$$

where  $\alpha = 0.5$ . Explain how the students calculate a value of  $\varepsilon$  from these data, stating any assumptions they make. What value do they obtain?

[14 marks]

Data-point	Frequency $f$ [Hz]	Spectral energy density $S(f)$ [ $\text{m}^2 \text{s}^{-1}$ ]
1	0.005	30.5
2	0.05	5.61
3	0.5	0.136
4	2.5	0.0103
5	5	0.00325
6	10	0.00102

Table 2.1

- 2.(a) Sketch the profiles of potential temperature, wind speed and specific humidity in the atmospheric boundary layer over land, by day and by night, under clear sky conditions in mid-latitudes. Identify on your sketches the significant layers, their heights, and processes that occur within them.

[13 marks]

- (b) The mean winds in the Ekman model of the atmospheric boundary layer satisfy the equations:

$$-f\bar{v} = K_m \frac{\partial^2 \bar{u}}{\partial z^2} \quad f(\bar{u} - u_g) = K_m \frac{\partial^2 \bar{v}}{\partial z^2}$$

which have the solutions

$$\bar{u} = u_g [1 - \exp(-z/h) \cos(z/h)] \quad \bar{v} = u_g \exp(-z/h) \sin(z/h)$$

where  $h = (2K_m / f)^{1/2}$ . What does  $h$  represent physically? For typical values of  $h$  and  $f$  estimate a typical value of  $K_m$ . How does this value compare with the viscosity of air? Sketch a hodograph of the Ekman wind profile in the Northern hemisphere. Mark the significant features of the profile.

State any ways in which the Ekman profile differs from observed boundary layer wind profiles in neutral, barotropic conditions.

[15 marks]

- (c) Explain how, and under what conditions, the nocturnal jet is formed.

Why is the nocturnal jet not observed close to the ground or in the free atmosphere?

[8 marks]

- (d) Describe the physical and chemical processes affecting the production of ozone and evolution of ozone concentrations throughout a daily cycle in an urban area.

[14 marks]

3. (a) The surface energy balance can be written as

$$R_n - G = H + \lambda E$$

Explain the meaning of all the symbols in the above equation, paying attention to the sign convention.

The Bowen ratio,  $B$ , is defined as  $B = \frac{H}{\lambda E}$ .

Derive an expression for  $H$  in terms of  $R_n$ ,  $G$  and  $B$ .

Give typical values of  $B$  for the following surfaces: (i) rural, (ii) urban, (iii) ocean, (iv) desert.

[15 marks]

(b) The Carson model predicts that the height,  $h$ , of the convective boundary layer at time  $t$  after sunrise is given by

$$h(t) = \left( \frac{2(1+2E)}{\rho c_p \gamma} \int_0^t H_0(t) dt \right)^{1/2}$$

Describe the assumptions made in the derivation of this formula. If the initial potential temperature profile is described by  $\gamma = 4.6 \text{ K km}^{-1}$  and in the 12 hours of daylight between sunrise and sunset the time-averaged surface sensible heat flux is  $150 \text{ W m}^{-2}$ , use the model to estimate the depth of the boundary layer at sunset. You should assume reasonable values for any parameters not given explicitly.

[15 marks]

- (c) Sketch the internal boundary layer at night starting with a stable temperature profile after a cold-warm transition. Explain the factors influencing the depth of the thermal internal boundary layer.

Explain how the roughness length and friction velocity changes as air passes from a smooth (rural) to a rough (urban) surface, and draw 3 sketches of the logarithmic wind profile just before, just after, and well downstream of the change between the rural and the urban surface.

[12 marks]

- (d) Plume patterns from chimneys depend on the wind speed and on the vertical temperature profile. Sketch plume patterns from a chimney and the corresponding absolute temperature profile for (i) a convective and dry situation during daytime and (ii) a stable situation during a clear night (in the temperature profiles, include also a dry adiabat as a dashed line, for reference).

[8 marks]

[End of Question paper]