Candidates are admitted to the examination room ten minutes before the start of the examination. On admission to the examination room, you are permitted to acquaint yourself with the instructions below and to read the question paper.

Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.

April 2013 **MTMW15**

Answer Book
Data Sheet
Any bilingual English language dictionary permitted
Only Casio-fx83 calculators are permitted

UNIVERSITY OF READING

Extratropical weather systems (MTMW15)

Two hours

Answer **ANY TWO** questions

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100.

1.

(a) Describe with the aid of sketches, the horizontal shear and horizontal deformation mechanisms of frontogenesis indicating regions of ascent and descent.

[8 marks]

(b) Frontal surfaces can be defined as surfaces of constant momentum $M = f_0 x + v_g$, where x is distance in the cross frontal direction and v_g is the along front component of geostrophic velocity.

Derive an expression for the slope of the frontal surface, in terms of geostrophic absolute vorticity and vertical shear of the geostrophic wind, and estimate the slope of a typical front.

[9 marks]

(c) The following two equations apply to a two-dimensional front with flow $\mathbf{u} = (u_g + u_a, v_g, w)$

$$\frac{D}{Dt}\left(\frac{\partial b'}{\partial x}\right) = Q_1 - \mathcal{S}^2\left(\frac{\partial u_a}{\partial x}\right) - \mathcal{N}^2\frac{\partial w}{\partial x}$$

$$\frac{D}{Dt}\left(f\frac{\partial v_g}{\partial z}\right) = -Q_1 - \mathcal{F}^2\frac{\partial u_a}{\partial z} - \mathcal{S}^2\frac{\partial w}{\partial z}$$

where

$\mathcal{F}^2 = f\left(f + \frac{\partial v_g}{\partial x}\right) = f\zeta_g$, $\mathcal{S}^2 = \frac{\partial b'}{\partial x} = f\frac{\partial v_g}{\partial z}$, $\mathcal{N}^2 = \frac{g}{\theta_0}\frac{\partial \theta}{\partial z} = N^2 + \frac{\partial b'}{\partial z}$ and Q_1 is the x-component of the **Q**-vector.

Explain the physical significance of each of $\mathcal{F}^2, \mathcal{S}^2, \mathcal{N}^2$ and explain why the left-hand sides of the two equations should be equal.

Use these equations to derive the Sawyer-Eliassen equation:

$$\mathcal{N}^2\frac{\partial^2\psi}{\partial x^2} - 2\mathcal{S}^2\frac{\partial^2\psi}{\partial x\partial z} + \mathcal{F}^2\frac{\partial^2\psi}{\partial z^2} = -2Q_1,$$

where ψ is the ageostrophic streamfunction.

[14 marks]

(d) Sketch a typical solution for ψ in the x-z plane for a cold front with Q_1>0, indicating the background state.

[5 marks]

(e)
$$Q_1 = -\left(\frac{\partial u_g}{\partial x}\frac{\partial b'}{\partial x} + \frac{\partial v_g}{\partial x}\frac{\partial b'}{\partial y}\right)$$
where the first term on the right hand side is a geostrophic stretching deformation and the second term is a geostrophic shearing deformation.

Consider a flow given by
$$u_g = \Lambda z\,,$$
$$v_g = \gamma x + \Lambda z$$
where Λ and γ are positive constants.

Show that Q_1 does not have a geostrophic stretching deformation component.

Calculate an expression for the geostrophic shearing deformation component of Q_1.

It can be shown that frontogenesis occurs if
$$Q_1 \frac{\partial b'}{\partial x} > 0.$$
State whether frontogenesis is occurring and give your reasoning (assume Northern hemisphere).

[8 marks]

(f) In a mathematical sense, the Sawyer-Eliassen equation is soluable if it is elliptic and so $\mathcal{F}^2\mathcal{N}^2 - \mathcal{S}^4 > 0$. The Ertel-Rossby geostrophic potential vorticity of the basic state is given by the expression
$$PV = \frac{1}{\rho}\left(\zeta_g \frac{\partial \theta}{\partial z} - \frac{\partial v_g}{\partial z}\frac{\partial \theta}{\partial x}\right).$$
Show that the ellipticity condition is equivalent to *fPV*>0.

[6 marks]

2.

(a) Describe the basic state of the Eady model of baroclinic instability. Sketch the growth rate curve and discuss its interpretation

(Recall that the maximum growth rate $\sigma = -\frac{0.31}{N}\frac{\partial \bar{b}}{\partial y}$ at wavenumber $k = \frac{1.6 f_0}{NH}$ and the shortwave cut off is at $k = \frac{2.4 f_0}{NH}$).

[12 marks]

(b) Stating your assumptions (and using typical midlatitude values where required), estimate the optimal wavelength and e-folding time for the growth of a cyclone at 50^{o}N with a vertical gradient in potential temperature of 4K/km and vertical windshear of 8 ms^{-1}/km.

How does you result compare to the values you expect for typical extratropical cyclones? Give at least one reason why the Eady model may not predict reasonable values.

[16 marks]

(c) Show that the shortwave cutoff implies that instability only occurs if $H < \frac{2.4 f_0}{Nk}$, i.e. the troposphere is sufficiently shallow.

A theory for the height of the tropopause states that it is determined by the criterion that the atmosphere is not unstable for Eady waves of any wavelength. Calculate *k* for the largest wave that would fit in a latitude circle at 50 ^{o}N (assuming the same basic state as in part (b)), then calculate the value of *H* for which this wave becomes stable.

Is this a reasonable number for the tropopause height?

[10 marks]

(d) If Eady waves are not assumed to be infinite in the meridional direction, but instead confined to a region of width L, the shortwave cutoff occurs at $\kappa = \dfrac{2.4 f_0}{NH}$ where $\kappa^2 = k^2 + \dfrac{\pi^2}{L^2}$.

Find H for L=1500km (a typical value) using the theory in (c).

Discuss whether this is now reasonable explanation for the tropopause height.

[6 marks]

(e) Considering the Eady model or otherwise, give at least two reasons why cyclones develop preferentially at the start of the storm tracks over the western Pacific and western Atlantic oceans.

[6 marks]

3.

(a) If the effects of heating and friction are included, the quasi-geostrophic thermodynamic and vorticity equations are given by

$$D_g b' + N^2 w = \dot{b}$$

$$D_g \zeta_g = f_0 \frac{\partial w}{\partial z} + \hat{\mathbf{k}} \cdot (\nabla \times \mathbf{F})$$

where $\dot{b}$ is the time rate of change of buoyancy and **F** is the frictional acceleration in the horizontal direction.

If there is a frictional force at $z = 0$ in the form of a linear drag given by

$$\mathbf{F} = -c\mathbf{v_g},$$

and assuming no vertical motion ($w = 0$ everywhere), show that

$$D_g \zeta_g = -c\xi_g.$$

Neglecting vorticity advection, estimate the time taken for the surface vorticity to decrease to half its initial value for $c = 10^{-5}\,\mathrm{s}^{-1}$.

[10 marks]

(b)

Derive the following equation for the rate of change of quasi-geostropic potential vorticity, q, including frictional and diabatic terms, starting from the quasi-geostrophic thermodynamic and vorticity equations in (a):

$$D_g q = f_0 \frac{\partial}{\partial z}\left(\frac{\dot{b}}{N^2}\right) + \hat{\mathbf{k}} \cdot (\nabla \times \mathbf{F})$$

where

$$q = f_0 + \xi_g + f_0 \frac{\partial}{\partial z}\left(\frac{b'}{N^2}\right).$$

State any assumptions you make.

[8 marks]

(c) Explain with the aid of diagrams how potential vorticity is generated by instantaneous latent heat release in the atmosphere (ignore frictional effects).

[12 marks]

(d) The heating rate, H, associated with a midlatitude convective system has the form

$$\mathcal{H} = \mathcal{H}_0 \sin^2 Az$$

where $\mathcal{H}_0$ and A are constants.
Assume a peak heating rate of 4 Kday^{-1} at 4 km height and typical midlatitude scalings where required (state your assumptions).

Calculate A and use this to calculate the rate of change of q at 2km height (ignoring frictional effects).

Find the change in q over one day and express it in terms of f_0.

[15 marks]

(e) Explain qualitatively how potential vorticity will evolve in the lower troposphere when there is surface friction
(i) ignoring diabatic processes,
(ii) including diabatic processes
for a localized region of heating arising from the release of conditional instability.

[5 marks]

[End of Question paper]