Candidates are admitted to the examination room ten minutes before the start of the examination. On admission to the examination room, you are permitted to acquaint yourself with the instructions below and to read the question paper.

Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.

January 2013 **MTMW11/99**

Answer Book
Data Sheet
Any bilingual English language dictionary permitted
Only Casio-fx83 calculators are permitted

UNIVERSITY OF READING

Fluid Dynamics of the Atmosphere and Oceans (MTMW11/99)

Two hours

Answer **ANY TWO** questions

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100.

1. The mass continuity equation can be written:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \underline{u}) = 0$$

a) Show what this equation implies for the total mass contained within a fixed volume when the velocity across the boundary of the volume is zero.

[8 marks]

b) Calculate the rate of change in density *following a fluid parcel* at an instant when the density is 1.2 kg m^{-3} and the velocity divergence is 10^{-5} s^{-1}.

[7 marks]

c) Draw the streamlines associated with the steady flow with velocity components $u = \alpha x$ and $v = -\alpha y$.

[6 marks]

d) Small, unsteady velocity perturbations are now added to this flow. Describe briefly how two neighbouring trajectories can differ from the streamlines, using a sketch, and the implications for fluid behaviour.

[5 marks]

e) The dispersion relation for barotropic Rossby waves is:

$$\omega = Uk - \frac{\beta k}{(k^2 + l^2)}$$

where *U* is a uniform zonal flow and *k* and *l* are the wavenumbers in the zonal and meridional directions respectively.

Explain what the parameter *β* means.

[3 marks]

f) Derive an expression for the group velocity of Rossby waves from the dispersion relation and show that its zonal component is greater than the zonal phase speed of the waves.

[14 marks]

g) Discuss the implications of difference in group and phase speeds for the behaviour of Rossby waves in the mid-latitude westerlies.

[7 marks]

2. a) Show that a fluid parcel displaced upwards by distance δz differs in potential density from its environment (of potential density ρ_e) by:

$$\delta\rho_\theta = -\frac{\partial \rho_e}{\partial z}\delta z$$

stating carefully any approximations that you make.

[10 marks]

b) When is the environmental profile statically stable? Why?

[5 marks]

c) The vertical momentum equation can be written:

$$\frac{Dw}{Dt} = -g\,\frac{\rho_\theta{}'}{\rho_\theta} - \frac{1}{\rho}\frac{\partial p'}{\partial z}$$

Neglecting the effects of pressure perturbations, show that the frequency of oscillation, *N*, of disturbed fluid parcels is given by:

$$N^2 = -\frac{g}{\rho_e}\frac{\partial \rho_e}{\partial z}$$

[12 marks]

d) Disturbances to a shallow layer of fluid initially at rest can be described by the linearised shallow water equations:

$$\frac{\partial u}{\partial t} = -g\,\frac{\partial h'}{\partial x}$$

$$\frac{\partial v}{\partial t} = -g\,\frac{\partial h'}{\partial y}$$

$$\frac{\partial h'}{\partial t} = -H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

where *H* is the average fluid depth and h' is a perturbation to it. Show that waves of the form $h' = h_0 e^{i(kx+ly-\omega t)}$ obey the dispersion relation:

$$\omega^2 = gH(k^2 + l^2)$$

[12 marks]

e) Calculate the angular frequency, ω, for an external gravity wave with a total wavelength of 200m on top of a dense fluid layer with average depth 20m. How does it compare with the fastest internal gravity wave frequencies in the atmosphere?

[11 marks]

3. The vorticity equation for a rotating fluid can be written in the form:

$$\underset{(1)}{\frac{\partial \underline{\zeta}}{\partial t}} = \underset{(2)}{-\left(\underline{u}.\nabla\right)\underline{\zeta}} \underset{(3)}{-\underline{\zeta}\left(\nabla.\underline{u}\right)} + \underset{(4)}{\left(\underline{\zeta}.\nabla\right)\underline{u}} + \underset{(5)}{\frac{\nabla\rho\times\nabla p}{\rho^2}} \qquad \text{(Eqn. 3.1)}$$

where $\underline{\zeta}$ is the absolute vorticity vector, but $\underline{u}$ is the velocity relative to a rotating frame of reference.

a) Give a physical description of each of the 5 terms in Eqn (3.1) and sketch the two mechanisms changing vorticity via term (4).
[10 marks]

b) Consider a laboratory experiment where a tank contains water which rotates with it at rate Ω about a vertical axis. The water is now disturbed by stirring gently for a short time while still rotating. Show that the full vorticity equation reduces in this situation to:

$$\left(\underline{\Omega}.\nabla\right)\underline{u} \approx 0 \qquad \text{(Eqn. 3.2)}$$

stating carefully your reasoning and approximations made.
[18 marks]

c) Describe what happens to the distribution of dye that is dropped into the rotating tank and compare it with the evolution of dye when the fluid is not rotating. Use Equation 3.2 to explain why there is a difference in the two experiments.
[10 marks]

d) Under scaling appropriate for large-scale atmospheric flow, the vorticity equation is well approximated by:

$$\frac{D\xi}{Dt} = f_0 \frac{\partial w}{\partial z}$$

Where ξ is the vertical component of relative vorticity and f_0 is the value of Coriolis parameter at a reference latitude.

Estimate the vertical velocity in the mid-troposphere associated with the spin-up of cyclone vorticity from $0.1 f_0$ to $0.4 f_0$ in one day.
[12 marks]

[End of Question paper]