Question 1

You need to use the provided tephigram in answering this question; please hand in the tephigram with your answers. In the rest of this question you may assume that the atmosphere is in hydrostatic balance and that L is constant at $2.5 \times 10^6\,\mathrm{J\,kg^{-1}}$.

On our Reading University Atmospheric Observatory, Mike measures a surface temperature of 21°C, a surface pressure of 980hPa, and a relative humidity of 50%.

(a) (i) Determine the wet-bulb potential temperature θ_w for a surface air parcel?
(ii) Use two different ways of finding the local vapour mixing ratio (from the tephigram as well as from a direct calculation from the given measurements). **[7]**

(b) (i) What temperature would an air parcel have it it were lifted adiabatically from the surface to 600hPa?
(ii) Estimate how much latent heat would have been released per unit mass in this process?
(iii) What is the (dry) potential temperature difference between the parcel at the surface and the lifted parcel at 600hPa?
(iv) Explain the physics of why the dry potential temperature of the parcel at 600hPa is larger than at the surface, despite this being an adiabatic lifting process. **[8]**

Assume that at the same time a radiosonde measures a constant temperature lapse rate of 7.5°C per 100hPa for the lowest 500hPa.

(c) Determine the lifting condensation level (LCL) as well as the level of free convection (LFC) of a surface air parcel. **[8]**

The convective inhibition (CIN) is defined as

$$\mathrm{CIN} = R\int_{\mathrm{LFC}}^{\mathrm{SFC}} (T_e - T_p)\,\frac{\mathrm{d}p}{p},$$

where the integral is taken between the level of free convection and the surface.

(d) (i) What do T_e and T_p represent in the above equation?
(ii) Explain the physical meaning of convective inhibition.
(iii) Why is it an important quantity in predicting convection? **[7]**

(e) Using the above equation for CIN, estimate, with help of the tephigram, the convective inhibition for the surface parcel described above. **[8]**

Temperature lapse rate is usually expressed in Kelvin per kilometre.

(f) (i) Derive the general relationship between the lapse rate as expressed in Kelvin per Pascal ($\mathrm{d}T/\mathrm{d}p$) and the lapse rate expressed in Kelvin per metre ($\mathrm{d}T/\mathrm{d}z$).
(ii) For the measured lapse rate of 7.5°C per 100hPa, what is the lapse rate in Kelvin per kilometre near the surface? **[7]**

(g) Why does lower part of the atmosphere usually become very stable on cloudless nights? **[5]**

Question 2

Consider a closed vessel of 1 litre of dry air at 20°C and 1000hPa. Now, water vapour at the same temperature is introduced through a valve, until the vapour is just saturated. Then the vessel is sealed off. In the rest of this question you may assume that the latent heat of evaporation is constant at $L = 2.5 \times 10^6\,\mathrm{J\,kg^{-1}}$.

(a) For the dry air/vapour mixture, what is
(i) the pressure
(ii) the density
(iii) the specific humidity? **[7]**

We now cool the vessel down to 0°C.

(b) (i) What is the volume of the condensed water?
(ii) How much heat (in Joules) needs to be extracted from the vessel to cool it down from 20°C to 0°C? (Here, you may ignore the volume of the condensed water compared to that of the vessel.) **[9]**

(c) The presence of liquid water changes the heat capacity of a mixture. Explain physically how and why the heat capacity of the liquid/vapour/dry air mixture is different from that of dry air. What strategy could you use to calculate the heat capacity of the mixture? **[6]**

In practice we need condensation nuclei in the vessel for the water to condense on. If the water condenses in drops, its saturation vapour pressure becomes a function of temperature T as well as drop radius r. This saturation vapour pressure is given by the Kelvin equation

$$e_s(T,r) = e_s(T,\infty)\,\exp\left(\frac{2\gamma}{\rho_l R_v T r}\right),$$

where $e_s(T,\infty)$ is the usual saturation vapour pressure over flat surfaces, which only depends on the temperature, γ is the surface tension of water, ρ_l is the density of liquid water, and R_v is the specific gas constant for water vapour.

(d) Explain the physics of why surface tension increases the saturation vapour pressure around a spherical drop. **[7]**

(e) (i) From the Kelvin equation, derive an expression for the radius (the so-called Kelvin radius) below which the Kelvin effect becomes strong.
(ii) What is this radius in μm?
(iii) Explain how the Kelvin effect normally makes it impossible for drops to form without a condensation nucleus. **[8]**

(f) (i) Assume the vapour is saturated for a drop of 1μm radius. Now introduce a further drop of 5μm radius in that same vapour; calculate the supersaturation in % with respect to the drop of 5μm radius.
(ii) Explain the effect this situation will have on the growth of each of the two drops. **[8]**

(g) Drops are negatively buoyant and will fall down. Why, then, do clouds stay up in the sky? **[5]**

Question 3

Consider a single slab atmosphere at temperature T_a overlying a planet's surface at temperature T_e. Assume that the atmosphere is fully transparent in the short-wave and fully absorbing in the long-wave part of the spectrum. Assume that the irradiance S of the sun is a constant at $S = 350\,\mathrm{W\,m^{-2}}$. Assume that the planet's surface has an albedo of $\alpha = 0.4$

(a) (i) With help of a schematic and using radiative energy budgets, derive that at equilibrium $\sigma T_a^4 = S(1-\alpha)$ and that $T_e = 2^{1/4} T_a$. Here, σ is the Stefan–Boltzmann constant.
(ii) With the given values, what is the surface temperature T_e of the planet? **[7]**

Now add another slab of atmosphere, at temperature T_b, on top of the existing one.

(b) (i) What will the new surface temperature T_e be at radiative equilibrium?
(ii) What are the implications of this result for global warming? **[8]**

(c) Describe, qualitatively, what will happen to T_b, T_a, and T_e if we allow a constant upward enthalpy flux (for example, due to dry convection) to occur from layer a to layer b? **[8]**

Enthalpy H is defined as $H = U + pV$.

(d) State the name and SI units of each of the symbols in this equation. **[4]**

(e) (i) Explain the role of enthalpy in processes where matter flows into a thermodynamic system.
(ii) Explain why the latent heat of evaporation L is equal to the specific enthalpy difference between vapour and liquid. **[8]**

Rain is responsible for a net latent heating of the atmosphere.

(f) What is the heating rate, in $\mathrm{W\,m^{-2}}$, corresponding to a rainfall rate of $5\,\mathrm{mm\,day^{-1}}$? You may assume that the latent heat of evaporation is constant at $L = 2.5 \times 10^6\,\mathrm{J\,kg^{-1}}$. **[7]**

(g) Assume the heating, calculated above, is uniformly distributed throughout the depth of the atmosphere. What is the temperature change in $\mathrm{K\,day^{-1}}$ corresponding to this heating. (Assume a surface pressure of 1000hPa. Also, if you did not find a sensible value in question (3f) choose a value that seems reasonable to you.) **[8]**