Candidates are admitted to the examination room ten minutes before the start of the examination. On admission to the examination room, you are permitted to acquaint yourself with the instructions below and to read the question paper.

Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.

April 2012

Answer Book
Data Sheet
Any bilingual English language dictionary permitted
Only Casio-fx83 calculators are permitted

Final Examination for MSc

Course in Atmosphere, Oceans and Climate
Course in Data Assimilation and Inverse Modelling in Geosciences

MTMW15

Extra-tropical weather systems

Two hours

Answer **ANY TWO** questions

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100.

1. The Q-vector is defined

$$\mathbf{Q} = -\left(\frac{\partial u_g}{\partial x}\frac{\partial b'}{\partial x} + \frac{\partial v_g}{\partial x}\frac{\partial b'}{\partial y}, \frac{\partial u_g}{\partial y}\frac{\partial b'}{\partial x} + \frac{\partial v_g}{\partial y}\frac{\partial b'}{\partial y} \right).$$

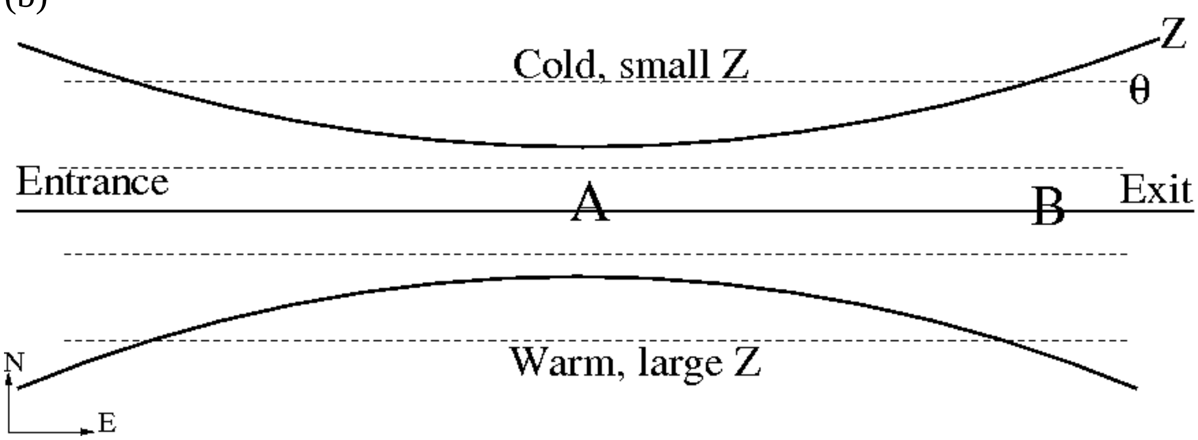
(a) Consider a natural coordinate system $(\hat{\mathbf{s}},\hat{\mathbf{n}})$ where $\hat{\mathbf{s}}$ points along a buoyancy contour and $\hat{\mathbf{n}}$ points orthogonal to this, towards the cold air. Show that the Q-vector can be written as

$$\mathbf{Q} = -|\nabla_h b'|\,\hat{\mathbf{k}} \times \frac{\partial \mathbf{v}_g}{\partial s}.$$

Give a nonmathematical description of how the direction of the Q-vector can be found using this expression.

[10 marks]

(b)



Consider this schematic of a jet streak in the Northern Hemisphere. Copy this schematic into your answer books.
The windspeed in the exit region at a height of 10 km decreases from 40 $\mathrm{ms^{-1}}$ to 20 $\mathrm{ms^{-1}}$ over a lengthscale of 500 km. The windspeed at the surface is zero.

Use typical midlatitude scalings to estimate the magnitude of $\mathbf{Q}$ and mark its direction on your schematic. State any parameter values that you need to assume.

Using the quasi-geostrophic omega equation in the form

$$N^2\nabla_h^2 w + f_0^2 \frac{\partial^2 w}{\partial z^2} = 2\nabla_h \cdot \mathbf{Q},$$

and assuming a cross-jet lengthscale of 400 km, estimate the magnitude of the vertical velocity.

Mark the regions of ascent and descent relative to the jet axis.

[20 marks]

(c) Using the quasi-geostrophic vorticity equation, $D_g\zeta_g = f_0 \frac{\partial w}{\partial z}$, and assuming $\mathbf{v_g} \cdot \nabla\zeta_g = 0$, determine the increase in relative vorticity from the diagnosed vertical stretching in the ascent region during one day. Express your answer as a multiple of f_0.

[6 marks]

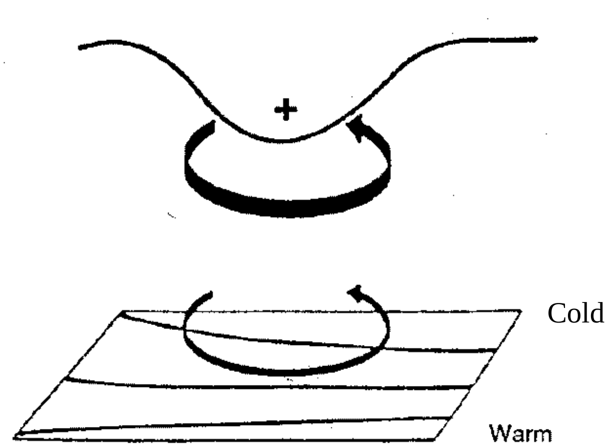
(d) Mark on your sketch the direction of **Q** in the jet entrance region.
Label the regions of ascent and descent in the jet entrance region.
Is the circulation thermally direct or indirect?

[6 marks]

(e) Discuss the various reasons why the value of the vertical velocity obtained from the quasi-geostrophic omega equation may differ from that observed for real weather systems, giving examples.

[8 marks]

2.



(a) Describe, using diagrams, the mechanism of baroclinic instability. Start from the above diagram showing a positive potential vorticity anomaly overlying a meridional temperature gradient.

In which directions do the temperature and pressure fields slope with height?

[18 marks]

(b) The figure below shows the growth rate of waves according to the Eady model as a function of both the zonal wave number, *k*, and meridional wave number, *l*, (where both wavenumbers are normalised by $1/L_R = f_0/NH$ such that the maximum growth rate for $l=0$ occurs for a wavenumber of $1.6/L_R$) where $K_R = f/NH$, *f* is the Coriolis parameter, *N* is the Brunt-Väisälä frequency, and ΔU is the difference in *U* between the ground and *H*, where *H* is the tropopause height (from James 1995).

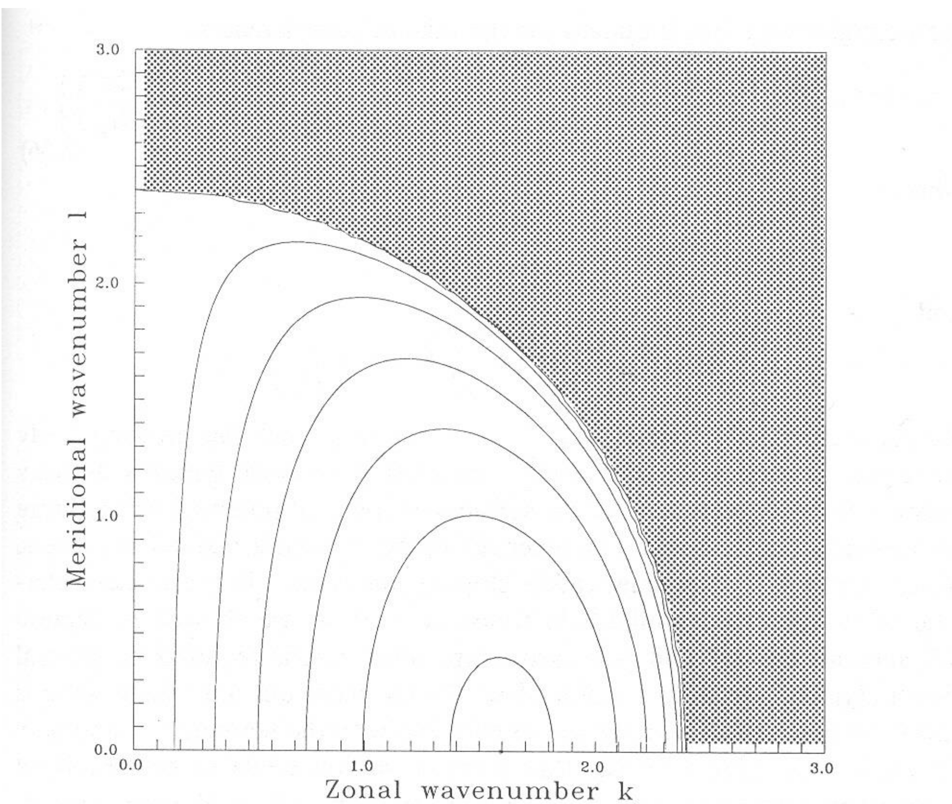


Fig. 5.17. Growth rate of waves with zonal wavenumber k and meridional wavenumber l according to the Eady model of baroclinic instability. Contour interval is 0.05 $K_R \Delta U$.

For an infinitely wide jet (*l*=0) the maximum growth rate, $\sigma_{\max}$, is given by

$$\sigma_{\max} = -\frac{0.31}{N}\frac{\partial \bar{b}}{\partial y} \text{ at } k = \frac{1.6 f_0}{NH}.$$

Use these results, in addition to material covered in the lectures, to predict qualitatively (with justification) how the maximum growth rate and size of cyclones will change under the following scenarios:
(i) Reducing the width of the upper level jet
(ii) Lowering the tropopause
(iii) Adding moisture
(iv) Adding surface friction
(v) Moving northwards from the extratropics
(vi) Strengthening the upper level jet

[24 marks]

(c) Construct a series of diagrams analogous to those in (a) that describes the process of anticyclogenesis including the distribution of vertical motions associated with the process. Does the concept of mutual amplification apply in this case?

[8 marks]

3. The Sawyer-Eliassen equation for cross-frontal circulation can be written as

$$\mathcal{N}^2 \frac{\partial^2 \psi}{\partial x^2} + \mathcal{F}^2 \frac{\partial^2 \psi}{\partial z^2} - 2\mathcal{S}^2 \frac{\partial^2 \psi}{\partial x \partial z} = 2Q_1,$$

where Q_1 is the x-component of the **Q**-vector, the coefficients are given by $\mathcal{N}^2 = N^2 + \frac{\partial b}{\partial z}$, $\mathcal{S}^2 = \frac{\partial b}{\partial x}$, and $\mathcal{F}^2 = f\left(f + \frac{\partial v_g}{\partial x}\right)$, and the streamfunction, ψ, is defined such that the cross-frontal circulation, $(u_a, w) = \left(\frac{\partial \psi}{\partial z}, -\frac{\partial \psi}{\partial x}\right)$.

(a) Give a descriptive name for each of the three coefficients

[6 marks]

(b) For a certain frontal rainband, the following data have been collected at 50°N: relative vorticity of $1\times10^{-4}\,\mathrm{s}^{-1}$, a vertical gradient in potential temperature anomaly of –2.4K km^{-1}, and vertical wind shear of 6 ms^{-1} km^{-1}.

Calculate the magnitude of each of the coefficients, stating any parameter values you have to assume.

[8 marks]

(c) If the forcing term in the Sawyer-Eliassen equation is given by $Q_1 = \hat{Q}\sin(kx + mz)$, where k and m are the horizontal and vertical wavenumbers respectively, show by substitution that a solution to the equation is given by $\psi = \hat{\psi}\sin(kx + mz)$ where

$$\hat{\psi} = \frac{2\hat{Q}}{(k^2N^2 - 2kmS^2 + m^2F^2)},$$

assuming $\mathcal{N}^2$, $\mathcal{F}^2$, and $\mathcal{S}^2$ are constants.

[10 marks]

(d) Estimate k and m from typical cross-frontal and vertical lengthscales and use these to calculate $\hat{\psi}$ given $\hat{Q} = 5 \times 10^{-13}$ s^{-3}.

[10 marks]

(e) Calculate the peak vertical velocity.

[8 marks]

(f) A similar circulation equation derived using quasi-geostrophic theory would not include the $\mathcal{S}^2$ term. What effect would this have on the estimated vertical velocity?

[4 marks]

(g) What is the primary difference between the semi-geostrophic and quasi-geostrophic equations?

[4 marks]

[End of Question paper]