Candidates are admitted to the examination room ten minutes before the start of the examination. On admission to the examination room, you are permitted to acquaint yourself with the instructions below and to read the question paper.

Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.

January 2012 **MTMW11**
MTMW99

Answer Book
Data Sheet
Any bilingual English language dictionary permitted
Only Casio-fx83 calculators are permitted

UNIVERSITY OF READING

FLUID DYNAMICS OF THE ATMOSPHERE AND OCEANS (MTMW11/MTMW99)

Two hours

Answer **ANY TWO** questions

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100.

1.

The shallow water equations describe the horizontal fluid motion in a layer:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -g\frac{\partial h}{\partial x}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu = -g\frac{\partial h}{\partial y}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} = -h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

(a) Name all the variables appearing in the equation set and state their units (dimensions). What physical principal underlies each equation?

[9 marks]

(b) Use the Buckingham pi-theorem to deduce how many independent non-dimensional parameters characterise the solutions.

[10 marks]

(c) Apply scale analysis to show that two of the non-dimensional parameters are: $\frac{U}{fL}$ and $\frac{U^2}{gH}$. What names are these parameters given?

[12 marks]

(d) A laboratory experiment is conducted in a wide cylindrical tank. The depth of the water in the tank (when at rest), H=0.1m. The tank is made to rotate at a constant rate. When the fluid is stirred and then left alone, vortices are observed with characteristic lengthscale, L.

The scientist now increases the water depth to 0.2m and then rotates the tank at the same rate. Use the principal of dynamical similarity to estimate the lengthscale of the vortices that emerge if all other experimental details are the same.

[10 marks]

(e) Now consider a large-scale current in the mid-latitude ocean. If there is a northward current with speed $0.1 ms^{-1}$ and width 200km, calculate the difference in sea surface height across the current.

[9 marks]

2.

A circular vortex patch of radius R has uniform vorticity, ξ, surrounded by an environment with zero vorticity.

(a) Use Stokes' theorem to calculate the speed of flow around the edge of the patch.

[7 marks]

(b) Calculate the speed of the flow induced by the vortex outside the patch at radius r > R. Hence sketch flow speed as a function of radius between r=0 and 3R.

[7 marks]

(c) Kelvin's circulation theorem can be written:

$$\frac{dC}{dt} = -\oint \frac{dp}{\rho}$$

where the integral is around the boundary of a closed material contour with circulation C. p is pressure and ρ is density. Show that if the circuit lies on an isentropic surface, the integral vanishes and dC/dt=0. You may assume that the fluid is an ideal gas.

[10 marks]

(d) The linearized barotropic vorticity equation on a β-plane is:

$$\frac{\partial \xi}{\partial t} + U\frac{\partial \xi}{\partial x} + v\beta = 0$$

Briefly define ξ and β.

[4 marks]

(e) Assume small amplitude Rossby waves with the form $e^{i(kx-\omega t)}$. Stating any assumptions, show that their zonal phase speed is:

$$c = U - \frac{\beta}{k^2}$$

[12 marks]

(f) Calculate the wavelength of Rossby waves that are stationary relative to the earth's surface when the average zonal flow is 20ms^{-1} at 40N.

[10 marks]

3.

Geostrophic wind is related to pressure perturbations by:

$u_g = -\frac{1}{f\rho_r}\frac{\partial p\prime}{\partial y}$ and $v_g = \frac{1}{f\rho_r}\frac{\partial p\prime}{\partial x}$

(a) Stating the approximations necessary, show how geostrophic wind can be expressed entirely in terms of geostrophic streamfunction:
$\psi_g = \frac{p\prime}{f\rho_r}$

[4 marks]

(b) Work out the direction of rotation around a maximum in streamfunction.

[8 marks]

(c) Sketch the streamlines of the flow described by $\psi_g = \alpha xy$ marking the direction of the flow with arrows. What is the vorticity of this flow?

[10 marks]

(d) Under the anelastic approximation, hydrostatic balance can be written:

$$\frac{\partial}{\partial z}\left(\frac{p'}{\rho_r}\right) = g\frac{\theta'}{\theta_r}$$

where θ is potential temperature. Use this to derive thermal wind balance:

$$\frac{\partial u_g}{\partial z} = -\frac{g}{f\theta_r}\frac{\partial \theta'}{\partial y}$$

[6 marks]

(e) Given that the zonal jetstream at 35N has a strength of $50ms^{-1}$ at an altitude of 10km, but the surface wind is only $10ms^{-1}$, calculate the meridional gradient in potential temperature.

[8 marks]

(f) In a climate change projection for 2080, the surface temperature increases by 5K at 75N but only 2K at 20N. Estimate the change in vertical wind shear in the lower troposphere, stating your assumptions.

[14 marks]

[End of Question paper]