Candidates are admitted to the examination room ten minutes before the start of the examination. On admission to the examination room, you are permitted to acquaint yourself with the instructions below and to read the question paper.

Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.

April 2011

Answer Book
General Data Sheet
Figures for question 2 included
Any bilingual English language dictionary permitted
Only Casio-fx83 calculators are permitted

THE UNIVERSITY OF READING

MSc Examination for Courses in Sciences

Extra-tropical Weather Systems

MTMW15

2 hours

Answer **ANY TWO** questions

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100.

1. (a) The quasi-geostrophic (QG) thermodynamic equation may be written

$$\frac{D_g b'}{Dt} + N^2 w = 0 .$$

Describe the physical meaning of the two terms in this equation.

[6 marks]

(b) Assuming that N^2 is constant, derive the following relation

$$f_0 \frac{\partial w}{\partial z} = -\frac{D_g}{Dt}\left(f_0 \frac{\partial}{\partial z}\left(\frac{b'}{N^2} \right) \right).$$

[10 marks]

(c) Using the result from (b), along with the QG equation for absolute geostrophic vorticity,

$$\frac{D_g \zeta_g}{Dt} = f_0 \frac{\partial w}{\partial z},$$

prove that the following quantity (the quasi-geostrophic potential vorticity (PV))

$$q = \zeta_g + \frac{f_0}{N^2} \frac{\partial b'}{\partial z},$$

is conserved following the geostrophic flow

[6 marks]

(d) Express the two terms from part (c) in terms of the geostrophic streamfunction (ψ_g) and constant parameters alone.

[8 marks]

Question 1 continues overleaf

Question 1 continued

(e) Using the result from (d), determine what fraction of a QG PV *perturbation* is associated with a relative vorticity (ξ_g) anomaly if the perturbation has the natural scaling

$$\frac{NH}{f_0 L} = 1$$

[10 marks]

(f) Consider mid-latitude flow with the following geostrophic streamfunction

$$\psi_g = \psi_0 \, \mathrm{e}^{-z/H} \sin\left(\frac{2\pi}{L} x\right)$$

Prove that this flow is horizontally non-divergent and calculate its geostrophic relative vorticity (ξ_g)

[10 marks]

2. In his 1949 paper, Eady provided a workable example of a simplified atmospheric system featuring baroclinic instability.

(a) Provide a concise description of the Eady model, underlining the basic assumptions and features (domain geometry, spatial/temporal scales of interest, basic state properties, etc.). Illustrate your answer with schematic diagrams where appropriate. Credit will be awarded to candidates demonstrating the ability to give an overview of the main physical and mathematical properties of the Eady model and its meteorological implications.

[15 marks]

(b) The evolution equation of the QGPV in the interior of the fluid is:

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\left(\nabla^2\psi'+\frac{f_0^2}{N^2}\frac{\partial^2\psi'}{\partial z^2}\right)=0 \qquad (1)$$

whereas the evolution equation for the temperature at the boundaries $z=0,H$ is:

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\frac{\partial\psi'}{\partial z}-\frac{\partial\psi'}{\partial x}\frac{\partial\bar{u}}{\partial z}=0 \qquad (2)$$

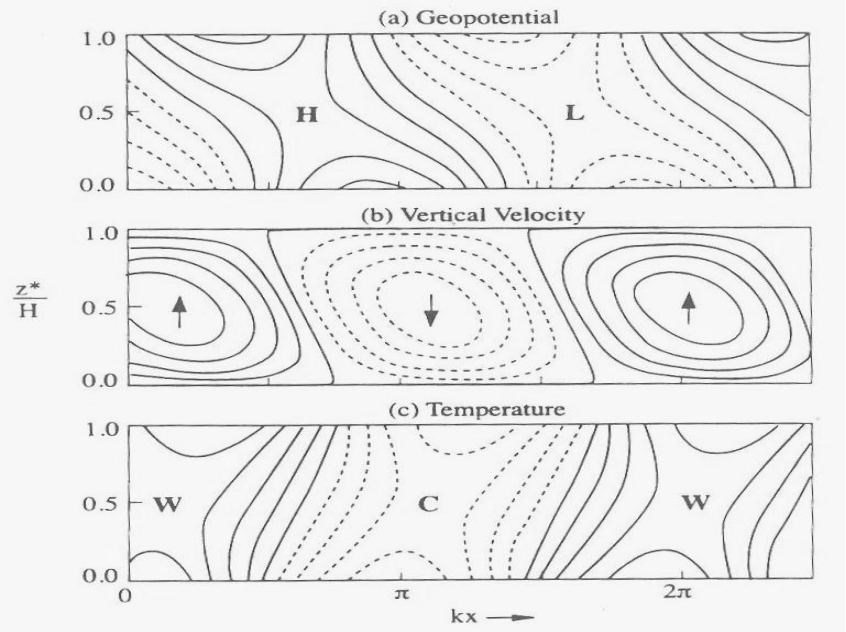
where $\bar{u}=\Lambda z$. Using the usual modal expansion, show under which conditions unstable waves can develop and discuss the results you obtain. What is the typical horizontal spatial scale of the unstable waves? Is there link between their vertical spatial scale and *H*?

[20 marks]

Question 2 continues overleaf

Question 2 continued

(c) The figure below shows the structure of the most unstable Eady Wave. (i) Deduce the sign of the zonal and vertical averages of $v'T'$ and of $w'T'$. What is the impact of the wave in terms of large scale heat transport? (ii) Is baroclinic instability redistributing heat? (iii) Considering the $w'T'$ term, and how density depends on temperature, what is the impact of its non-vanishing average value in terms of energetics? (iv) How is the centre of mass of the system moving? (v) What happens if the centre of mass is lowered?



[15 marks]

3.

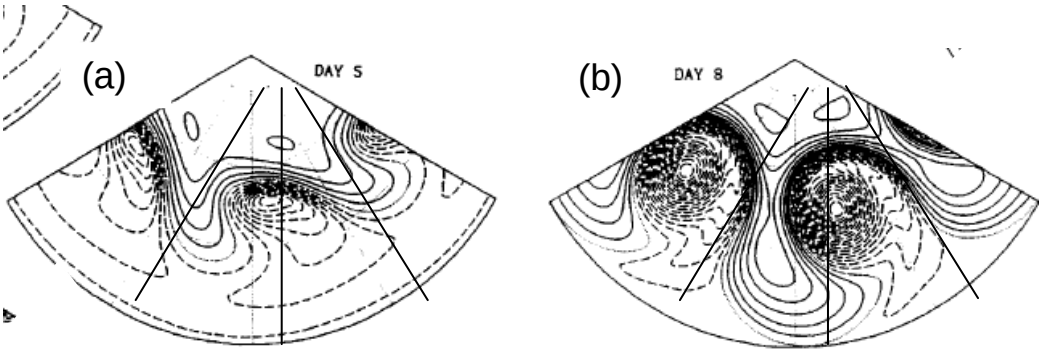


Figure 1: Baroclinic lifecycle: Surface pressure (contours every 4 hPa with the 1000 hPa contour dotted; dashed contours represent pressure below 1000 hPa) at day 5 (a) and day 8 (b) of the simulation of an extratropical cyclone. Lines of constant longitude are drawn every 30 degrees. Extracted from Thorncroft et al. (1993).

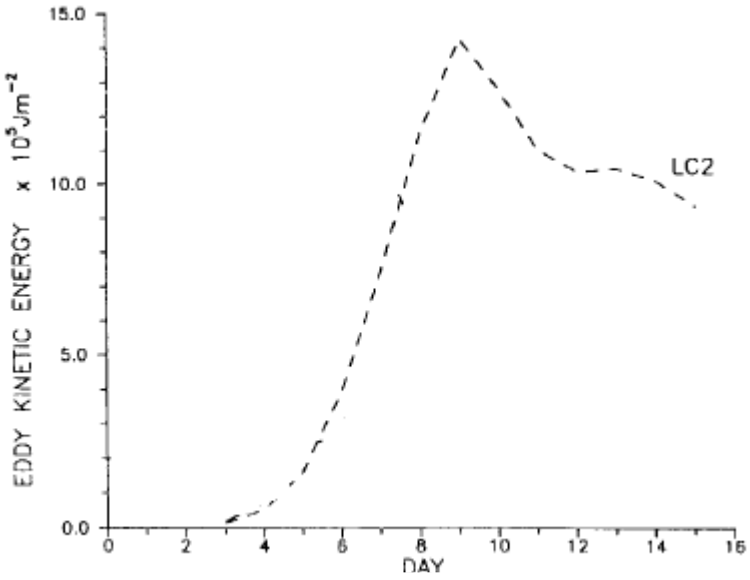


Figure 2: Evolution of the eddy kinetic energy. Extracted from Thorncroft et al. (1993).

Question 3 continues overleaf

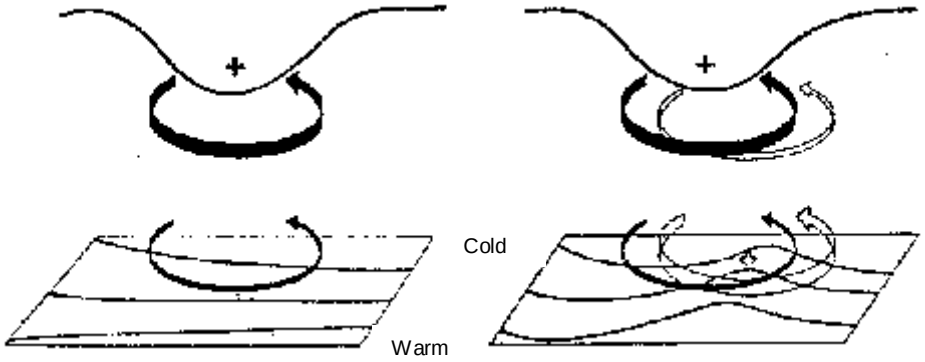
Question 3 continued

(a) The Eady theory of baroclinic instability predicts that the maximum growth rate of mid-latitude disturbances, $\sigma_{\max}$, is given by

$\sigma_{\max} = -\frac{0.31}{N}\frac{\partial \bar{b}}{\partial y}$ at wavenumber $k = \frac{1.6 f_0}{NH}$. Using typical midlatitude scalings for the latitude at which the disturbance is occurring (45°N) calculate its maximum growth rate and wavelength. Compare these results with the estimates that can be deduced from the information supplied by figures 1-2, which show output from an idealised dry model simulation of an extratropical cyclone (a so-called baroclinic lifecycle). [12 marks]

(b) The evolution of the eddy kinetic energy has 3 stages: initial exponential growth followed by approximately linear growth and finally saturation and decay.
Consider each stage separately - can the Eady theory explain the behaviour or not (explain your reasoning)? [10 marks]

(c) Consider this figure taken from Holton (1992), which illustrates the potential vorticity view of cyclogenesis at an early (left) and later (right) time



Based on this diagram, where a PV anomaly in the upper troposphere is represented together with surface temperature isolines, qualitatively explain the process by which a PV anomaly on the tropopause can initiate cyclogenesis in a baroclinic flow.

[12 marks]

Question 3 continues overleaf

Question 3 continued

(d) An "edge" wave trapped at the surface has the following geostrophic streamfunction:

$$\psi_g = \psi_0 \sin k(x-ct) \exp\left(\frac{-Nkz}{f_0}\right)$$

From this expression, derive an expression for the meridional wind (v_g) and the buoyancy perturbation (b') associated with this wave.

[6 marks]

(e) Unstable baroclinic waves feature a positive poleward heat flux, so that

$$\frac{k}{2\pi} \int_0^{2\pi/k} v_g b' \, dx > 0$$

Using this relation, explain whether or not the edge wave from part (b) will be able to grow unstably.

[10 marks]

(End of Question Paper)