You are allowed ten minutes before the start of the examination to acquaint yourself with the instructions below and to read the question paper.

Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.

January 2011

Answer Book
General Data Sheet
Any bilingual English language dictionary permitted
Only Casio-fx83 calculators are permitted

UNIVERSITY OF READING

MSc/Diploma in Atmosphere, Oceans and Climate

MSc in Mathematics and Numerical Modelling of the Atmosphere and Oceans

PAPER MTMW11/MTMW99
Fluid Dynamics of the Atmosphere and Oceans

Two hours

Answer **ANY TWO** questions

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100

1. (a) The momentum equation for flow of a fluid relative to a coordinate frame rotating with uniform angular velocity $\mathbf{\Omega}$ may be written

$$\underset{(1)}{\frac{D\mathbf{u}}{Dt}} = \underset{(2)}{-2\mathbf{\Omega}\wedge\mathbf{u}} \quad \underset{(3)}{-\mathbf{\Omega}\wedge\ \mathbf{\Omega}\wedge\mathbf{r}} \quad \underset{(4)}{+\mathbf{g}} \quad \underset{(5)}{-\frac{1}{\rho}\nabla p}.$$

Briefly describe the process described by each of the terms in this equation.

[15 marks]

(b) A parcel of ocean water at a latitude of 45^{o}N is moving parallel to the earth's surface, from north to south at a speed of 0.5 ms^{-1}. Write expressions for the west-east, south-north and vertical components of term (2). Calculate approximate values for each of these components for the parcel, which you may assume has a mass of 1 kg.

[15 marks]

(c) The parcel undergoes an inertial oscillation. Assuming you may treat the motion of the parcel as a particle motion, show that the particle velocity is of the form

$$u = A\cos Ct + B\sin Ct \ \ \mathrm{ms}^{-1},$$
$$v = -A\sin Ct + B\cos Ct \ \ \mathrm{ms}^{-1},$$

where A, B and C are constants to be found.

Hence calculate the radius and period of the oscillation. (The period of an inertial oscillation is the time taken to traverse a complete circle).

[20 marks]

2. (a) Rossby waves in uniform flow U parallel to the x-axis on a β-plane are solutions of the barotropic vorticity equation of the form

$$\varphi(x,y,t) = \mathrm{Re}\left[\Psi \exp\left(i\left(kx + ly - \omega t\right)\right)\right].$$

Sketch the form of the disturbance at t = 0 in the x-y plane. Mark the crests of the waves and wave-vector, $(k, l)^{\mathrm{T}}$ clearly on your diagram.

[10 marks]

(b) A Rossby wave at 40ºN has wavelength 4000 km, with crests running at 130º to the west to east (x-) axis. Calculate the x- and y-components of the wave vector $(k, l)^T$.

[10 marks]

(c) The dispersion relationship for Rossby waves is

$$\omega = Uk - \frac{\beta k}{k^2 + l^2}.$$

Use this relationship to obtain an expression for the phase-speed in the x-direction of a Rossby wave. Suppose the wave described in part (b) is stationary relative to the Earth's surface and calculate the size of the zonal flow, U. You may use the formula $\beta = \dfrac{2\Omega\cos\phi_0}{a}$.

[15 marks]

(d) Obtain a formula for the group velocity for a general Rossby wave in uniform flow U parallel to the x-axis on a β-plane. Explain the physical significance of group velocity, and how it differs from phase velocity.

[15 marks]

Turn over

3. (a) Explain the terms “streamline” and “pathline”, saying how you would calculate them. In what circumstances are streamlines and pathlines similar, and in what circumstances do they differ? (Pathlines are sometimes known as trajectories.)

[14 marks]

(b) The momentum equation for an incompressible fluid in an inertial frame can be written

$$\frac{D\mathbf{u}}{Dt} \;=\; \mathbf{g} \;\; -\frac{1}{\rho}\nabla p \;\; +\nu\nabla^2\mathbf{u}\,.$$

From the momentum equation, derive Bernoulli’s theorem:

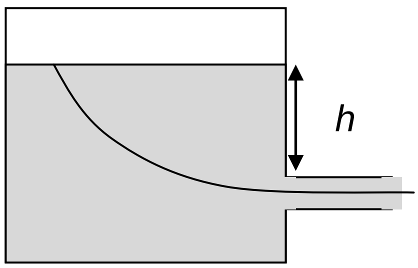
$$\mathbf{u}\cdot\nabla\left(\frac{1}{2}\mathbf{u}\cdot\mathbf{u}+\Phi+\frac{p}{\rho}\right)=0.$$

Explain the assumptions you have to make. You may assume the vector identity:

$$\nabla\;\mathbf{A.B} \;=\; \mathbf{B.}\nabla\;\mathbf{A}+\;\mathbf{A.}\nabla\;\mathbf{B}+\mathbf{B}\wedge\;\nabla\wedge\mathbf{A}\;+\mathbf{A}\wedge\;\nabla\wedge\mathbf{B}$$

[16 marks]

(c)



Waste water flows into a large tank at a rate of $1\text{x}10^{-4}$ m^3s^{-1} and out of a thin exit pipe of cross sectional area $4\text{x}10^{-5}$ m^2 . In steady state, the surface of the water in the large tank is at rest. Estimate how high above the pipe is the water in the tank (height h in the figure). Hint: use Bernoulli on the illustrated streamline.

[20 marks]

(End of Question Paper)