Candidates are admitted to the examination room ten minutes before the start of the examination. On admission to the examination room, you are permitted to acquaint yourself with the instructions below and to read the question paper.

Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.

April 2010

Answer Book
Data Sheet
Figures for Question 3 included
Any bilingual English language dictionary permitted
Calculators and programmable calculators are permitted

THE UNIVERSITY OF READING

MSc/Diploma
Course in Weather, Climate and Modelling

Course in Mathematics
and Numerical Modelling of the Atmosphere and Oceans

Paper MTMW15

Extra-tropical Weather Systems

Two hours

Answer **ANY TWO** questions

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100

1. *"Rossby Potential Vorticity Problem"*
If we neglect the terms describing the effect of the planetary curvature, the evolution equation for the total vorticity $\zeta_a = \zeta_r + f$ can be written as:

$$\frac{\partial \zeta_a}{\partial t} + \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} \right)\zeta_r + w\frac{\partial}{\partial z}\zeta_r + v\frac{df}{dy} = -\zeta_a\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - f\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) +$$

(1) (2) (3) (4) (5) (6)

$$-\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z} \right) + \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x} \right) + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \qquad (1)$$

(7) (8) (9)

(a) explain the physical meaning of each term 1-9 in equation (1).
[15 marks]

(b) Perform a scale analysis for each term described in a) using the following characteristic magnitudes for synoptic motions:

$$U,V \approx 10ms^{-1}; W \approx 10^{-2}ms^{-1}; L \approx 10^6 m; H \approx 10^4 m; \Delta p \approx 1kPa; \rho \approx 1Kgm^{-3}$$
$$\Delta\rho \approx 10^{-2}Kgm^{-3}; \tau \approx 10^5 s; f_0 \approx 10^{-4}s^{-1}; \beta \approx 10^{-11}s^{-1}m^{-1}; Fx, Fy \approx 10^{-5}ms^{-2}$$

Which terms give the leading contributions? (Hint: the horizontal divergence term will prove to be the largest from the scaling analysis but is actually of the same order of magnitude as the Eulerian time derivative. Think about possible cancellations.).
[10 marks]

(c) In the absolute vorticity equation given in part (a), identify those terms which are consistent with
the leading order treatment obtained in part (b). Hence derive the following approximate evolution equation:

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} \right)\zeta_a = -f\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \qquad (2)$$

[5 marks]

Question 1 continues overleaf

Question 1 continued

(d) Let's continue from equation (2) and consider the case of a homogeneous, QG, barotropic, incompressible fluid ($\vec{\nabla}.\vec{u}=0$, $\partial u/\partial z=\partial v/\partial z=0$) of variable depth $h(x,y)=z_2(x,y)-z_1(x,y)$ where z_2, z_1 are the heights of the upper and lower boundaries. Using the incompressibility conditions gives:

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\zeta_a=\zeta_a\frac{\partial w}{\partial z}$$

where the relative vorticity has been retained on the right-hand side. Show that by integrating from the lower to the upper boundary of the fluid you obtain (Remember that the flow is constrained between the two boundaries with no-slip boundary conditions.):

$$h\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\zeta_a=\zeta_a\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)h$$

[5 marks]

(e) Rearrange the equation obtained in d) to show that this implies QGPV conservation:

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\left(\frac{\zeta_a}{h}\right)=0$$

[10 marks]

(f) How is e) modified if the motion is purely horizontal?

[5 marks]

2. *“Rossby-Charney-Eliassen Problem”*
The inviscid barotropic vorticity equation reads as follows:

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\zeta_r+v\frac{df}{dy}=0 \qquad (1)$$

Let’s consider perturbations from a zonal basic state with no background vorticity.

(a) Using the usual expansion: $u=\overline{u}+u';v=v';\zeta_r=\zeta_r{}'$, where $\overline{u}$ is a constant, and the beta plane approximation, show that the perturbation from of equation (1) can be written as:

$$\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)\nabla^2\psi'+\beta\frac{\partial\psi'}{\partial x}=0 \qquad (2)$$

where ψ' is the perturbation streamfunction. Explain also the physical meaning of the streamfunction and write down explicitly the streamfunction of the basic state.

[15 marks]

(b) Using the usual modal expansion in *x*,*y*, and *t*, derive the dispersion relation formula and write down explicitly the phase velocity of the Barotropic Rossby wave.

[10 marks]

(c) Under what conditions does the x-component of the phase velocity vanish? Also, comment on the x-component of the phase velocity relative to the basic flow. In the frame of reference of the the basic flow, in which direction (backwards or forwards) would the wave crests be going along x?

[10 marks]

Question 2 continues overleaf

Question 2 continued

(d) If we consider a domain with variable height $h(x,y) = z_2(x,y) - z_1(x,y) = H - z_1(x,y)$, where H is a constant, we find that equation (2) becomes (Charney and Eliassen, 1949):

$$\left(\frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x}\right)\nabla^2\psi' + \beta\frac{\partial\psi'}{\partial x} = -\frac{f_0}{H}\overline{u}\frac{\partial z_1}{\partial x} \qquad (3)$$

We assume that the topography can be written as $z_1(x,y) = h_0\cos(px)\cos(qy)$. Describe the stationary solutions ($\partial/\partial t = 0$) of equation (3) in terms of waves. What is the structure of the resulting streamfunction? Is there a critical wavenumber leading to divergence in the response? Comment on this.

[10 marks]

3. *“Eady Problem”*

In his 1949 paper, Eady provided a workable example of a simplified atmospheric system featuring baroclinic instability. Credit will be awarded to candidates demonstrating the ability to give an overview of the main physical and mathematical properties of the Eady model and its meteorological implications.

(a) Provide a concise description of the Eady model, underlining the basic assumptions and features (domain geometry, spatial/temporal scales of interest, basic state properties, etc.). Illustrate your answer with schematic diagrams where appropriate.

[15 marks]

(b) The evolution equation of the QGPV in the interior of the fluid is:

$$\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)\left(\nabla^2\psi'+\frac{f_0^2}{N^2}\frac{\partial^2\psi'}{\partial z^2}\right)=0 \qquad (1)$$

whereas the evolution equation for the temperature at the boundaries $z=0,H$ is:

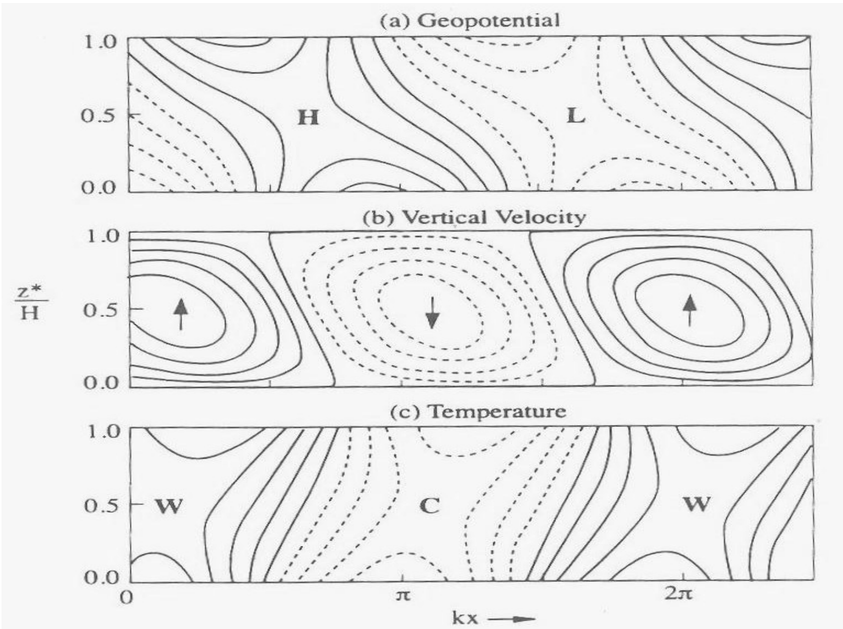
$$\left(\frac{\partial}{\partial t}+\overline{u}\frac{\partial}{\partial x}\right)\frac{\partial\psi'}{\partial z}-\frac{\partial\psi'}{\partial x}\frac{\partial\overline{u}}{\partial z}=0 \qquad (2)$$

where $\overline{u}=\Lambda z$. Using the usual modal expansion, show under which conditions unstable waves can develop and discuss the results you obtain. What is the typical horizontal spatial scale of the unstable waves? Is there link between their vertical spatial scale and *H*? [20 marks]

Question 3 continued overleaf

Question 3 continued

(c) The figure below shows the structure of the most unstable Eady Wave. (i) Deduce the sign of the zonal and vertical averages of $v'T'$ and of $w'T'$. What is the impact of the wave in terms of large scale heat transport? (ii) Is baroclinic instability redistributing heat? (iii) Considering the $w'T'$ term, and how density depends on temperature, what is the impact of its non-vanishing average value in terms of energetics? (iv) How is the centre of mass of the system moving? (v) What happens if the centre of mass is lowered?



[15 marks]

(End of Question Paper)