You are allowed ten minutes before the start of the examination to acquaint yourself with the instructions below and to read the question paper.

Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.

January 2010

Answer Book
Data Sheet

Any bilingual English language dictionary permitted
Calculators and programmable calculators are permitted

THE UNIVERSITY OF READING

MSc/Diploma
Course in Atmosphere, Oceans and Climate

Course in Mathematics
and Numerical Modelling of the Atmosphere and Oceans

PAPER MTMW11/MTMW99

Fluid dynamics of the atmosphere and oceans

Two hours

Answer **ANY TWO** questions

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100

1.

(a) Explain what is meant by an "inertial" frame of reference, and say why a rotating frame of reference is not an inertial frame of reference.

[6 marks]

(b) Show that the rate of change of a vector V in an inertial frame of reference is related to its rate of change in a rotating frame of reference by

$$\frac{d_A\mathbf{V}}{dt} = \frac{d_R\mathbf{V}}{dt} + \mathbf{\Omega} \wedge \mathbf{V} \qquad (1.1)$$

[12 marks]

(c) Use this result to show that the acceleration of fluid in an absolute frame of reference is related to its rate of change in a rotating frame of reference by

$$\frac{D_A\mathbf{u}}{Dt} = \underset{(A)}{\frac{D_R\mathbf{u}_R}{Dt}} + \underset{(B)}{2\mathbf{\Omega} \wedge \mathbf{u}_R} + \underset{(C)}{\mathbf{\Omega} \wedge (\mathbf{\Omega} \wedge \mathbf{r})} \qquad (1.2)$$

[12 marks]

(d) A circular depression is located at latitude 50ºN. At a distance of 450 km from its centre, the wind blows steadily around the depression, at 15 m s^{-1}. Estimate the magnitudes of terms A, B and C in equation (1.2) above.

[10 marks]

(e) Given your estimates, say why term (C) is often not explicitly seen in the equations of fluid flow on a rotating planet.

[10 marks]

2.

(a) Starting from the horizontal component of the Euler equations of motion,

$$\frac{D\mathbf{v}}{Dt} = -2\underline{\mathbf{\Omega}} \wedge \mathbf{v} - \frac{1}{\rho}\nabla p$$

deduce the magnitude of horizontal pressure fluctuations parallel to the flow trajectory using scale analysis. State any assumptions you make.

[12 marks]

(b) The geostrophic velocity for an incompressible fluid on an f-plane satisfies

$$\mathbf{v}_g = \frac{1}{\rho_0 f_0}\mathbf{k} \wedge \nabla p \,.$$

Show that this flow is (horizontal) divergence free, and has no vertical velocity.

[8 marks]

(c) Show that the typical magnitude of the ageostrophic component of the flow velocity is U^2 / fL, where U is the velocity scale for the full horizontal velocity field.

[16 marks]

(d) A Gulf Stream ring in the mid-latitude Atlantic has a radius of 75 km and typical horizontal flow speeds of 0.2 m s^{-1}. Estimate the magnitude of the vertical velocity arising from the ageostrophic flow you would expect to be associated with the system.

[14 marks]

3.

(a) Write down the three components of vorticity in Cartesian coordinates, in terms of velocity in an inertial (or absolute) frame.

[6 marks]

(b) The vorticity equation for a fluid may be written

$$\frac{\partial \underline{\zeta}}{\partial t} = -(\mathbf{u}.\nabla)\underline{\zeta} + \frac{\nabla\rho \times \nabla p}{\rho^2} + (\underline{\zeta}.\nabla)\mathbf{u} - \underline{\zeta}(\nabla.\mathbf{u})$$

(1) (2) (3) (4)

Write brief notes on the physical processes represented by each of the terms on the right hand side of this equation. When is term (2) zero? Give an example of a geophysical situation where term (2) is important in changing the vorticity.

[12 marks]

(c) Starting from the vorticity equation given in part(a), derive the barotropic vorticity equation for the vertical component of relative vorticity, ξ,

$$\frac{D\xi}{Dt} = f\frac{\partial w}{\partial z}.$$

State any assumptions or approximations that you use.

[14 marks]

(d) Boundary Layer flow encounters a gently sloping hill. Describe the pattern of vorticity you would expect to form in the vicinity of the hill, and account for it in terms of processes represented by the terms in the vorticity equation.

[18 marks]

(End of Question Paper)