Candidates are admitted to the examination room ten minutes before the start of the examination. On admission to the examination room, you are permitted to acquaint yourself with the instructions below and to read the question paper.

Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.

April 2009

Answer Book
Data Sheet
Figures for Question 2 included
Any bilingual English language dictionary permitted
Calculators and programmable calculators are permitted

THE UNIVERSITY OF READING

MSc/Diploma
Course in Weather, Climate and Modelling

Course in Mathematics
and Numerical Modelling of the Atmosphere and Oceans

Paper MTMW15

Extra-tropical Weather Systems

Two hours

Answer **ANY TWO** questions

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100

Figures for Question 2:

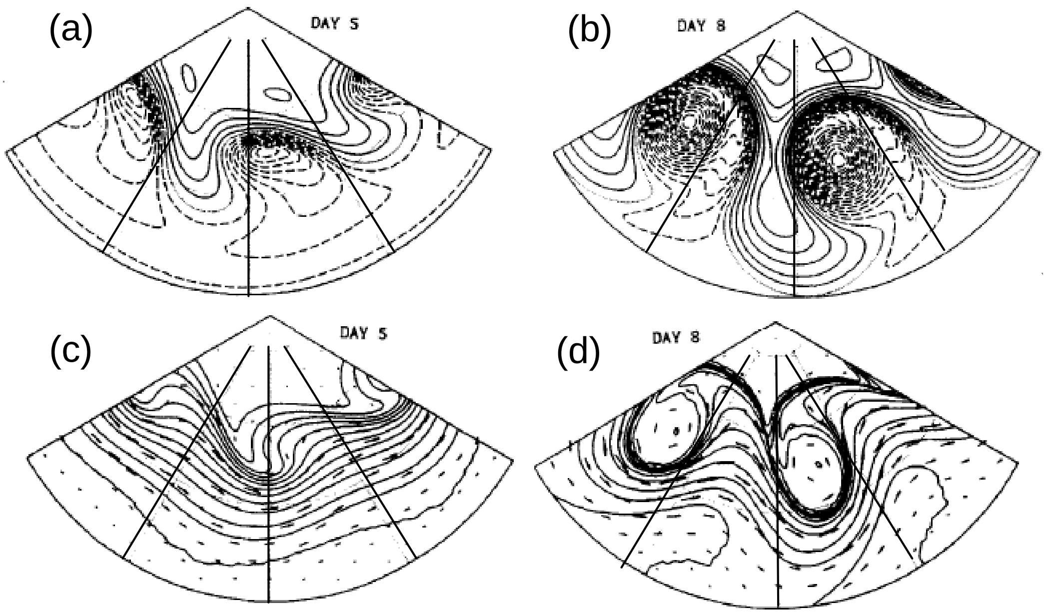

Figure 1: Baroclinic lifecycle: (a) and (b) show surface pressure (contours every 4 hPa with the 1000 hPa contour dotted; dashed contours represent pressure below 1000 hPa). (c) and (d) show potential temperature on the 2PVU surface (contours drawn every 5 K from 290K to 350K going equatorward, relative flow windspeed vectors are also plotted). Lines of constant longitude are drawn every 30 degrees. Extracted from Thorncroft et al. (1993).

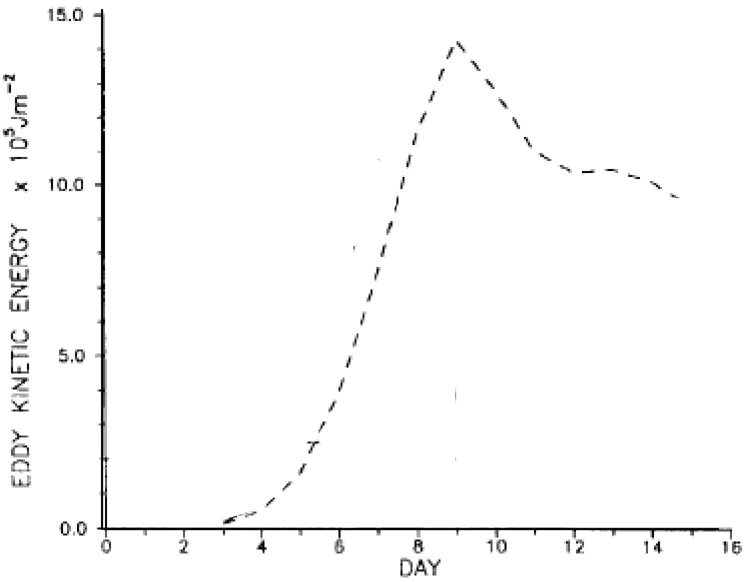

Figure 2: Evolution of the eddy kinetic energy. Extracted from Thorncroft et al. (1993).

Turn over

1.

(a) The forcing term of the quasi-geostrophic Omega equation can be expressed in terms of a Q-vector given by

$$\mathbf{Q} = -|\nabla_h b'|\,\hat{\mathbf{k}} \times \frac{\partial \mathbf{v}_g}{\partial s}$$

in natural coordinates.
Consider a jet entrance region with cold air to the north and warm air to the south.
Determine the direction of the Q-vector and mark this on a sketch.
Indicate regions of ascent and descent predicted by the Omega equation.

[10 marks]

(b) Describe the so-called cancellation problem that can arise during the use of the thermal advection/vorticity advection form of the omega equation.

Give an example of a situation in which the cancellation problem occurs and illustrate your answer with a sketch.

[12 marks]

(c) The quasi-geostrophic vorticity and thermodynamic equations on an f-plane are

$$D_g \zeta_g = f_0 \frac{\partial w}{\partial z} + \nabla \times \mathbf{F}$$
$$D_g b' = -N^2 w + \dot{b}$$

where **F** is the frictional acceleration in the horizontal direction and $\dot{b}$ is the rate of change of buoyancy due to diabatic processes.
Derive the quasi-geostrophic potential vorticity equation including the frictional and diabatic terms starting from these equations.
State any assumptions you make.

[12 marks]

Question 1 continued overleaf

Question 1 cont'd.

(d) If the mesoscale can be defined such that the Rossby number is unity, estimate the Rossby number for each of the following three systems and thus determine whether they can be considered mesoscale: frontal rainbands, hurricanes and severe mid-latitude thunderstorms.

[8 marks]

(e) Use the following approximate equation for the generation of vertical vorticity in a severe cumulonimbus cloud to estimate the vertical velocity needed to increase the vertical component of relative vorticity to $10^{-4}s^{-1}$ in 10 minutes.

$$\frac{\partial \xi}{\partial t} = \left(\frac{d\bar{u}}{dz}\right)\frac{\partial w}{\partial y}$$

where $\bar{u}$ is the mean inflow to the storm which decreases from $20ms^{-1}$ at the surface to $5ms^{-1}$ at 3km and the horizontal storm dimension is 20km. [8 marks]

2. Use the supplied figures showing output from an idealised dry model simulation of an extratropical cyclone (a so-called baroclinic lifecycle) to inform your answers to this question. Figure 1 shows the surface pressure and potential temperature on the tropopause (2 PVU surface) after 5 days and 8 days of model integration. Figure 2 shows the evolution of the eddy kinetic energy (a measure of intensity) of the cyclone.

(a) Show using schematics how in general the structure of baroclinic systems can be determined by considering a vertical coupling between a temperature wave on the tropopause and surface level disturbances.

Does the cyclone in Fig. 1 have a baroclinically unstable structure at
(i) 5 days?
(ii) 8 days?
Give your reasoning. [14 marks]

(b) The Eady theory of baroclinic instability predicts that the maximum growth rate of cyclones, $\sigma_{\max}$, is given by $\sigma_{\max} = -\frac{0.31}{N}\frac{\partial \overline{b}}{\partial y}$ at wavenumber $k = \frac{1.6 f_0}{NH}$.
Using typical midlatitude scalings for the latitude at which the cyclone is occurring (50^{o}N) calculate the maximum growth rate and wavelength of the cyclone and compare the predicted wavelength with a wavelength estimated from figure 1. [14 marks]

(c) The evolution of the eddy kinetic energy has 3 stages: initial exponential growth followed by approximately linear growth and finally saturation and decay.
Consider each stage separately - can the Eady theory explain the behaviour or not (explain your reasoning)?
Compare the predicted maximum growth rate obtained in (b) with a growth rate estimated from figure 2 [12 marks]

(d) This model does not represent the effects of moisture. Explain how and why you would expect the cyclone evolution to change if moisture were included [10 marks]

3.

(a) The frontal surface can be defined as the slope of surfaces of either constant potential temperature, θ_e, or constant absolute momentum, $M = fx + v$. Derive expressions for these slopes and show that they can be expressed as

$$\left(\frac{dz}{dx}\right)_M = -\frac{F^2}{S^2}$$

$$\left(\frac{dz}{dx}\right)_{\theta_e} = -\frac{S^2}{N_s^{\ 2}}$$

where $F^2 = f\left(f + \frac{\partial \overline{v}}{\partial x}\right) = f\overline{\zeta}$, $S^2 = \frac{\partial b'}{\partial x} = f\frac{\partial \overline{v}}{\partial z}$, and $N_s^{\ 2} = \frac{g}{\theta_0}\frac{\partial \theta_e}{\partial z}$.

[10 marks]

(b) The dispersion relation for small amplitude perturbations of the form $\psi = \psi_0 e^{i(kx+mz)+\sigma t}$ to a geostrophically balanced front is

$$\sigma^2 = \frac{-F^2 + 2S^2\alpha - N_s^{\ 2}\alpha^2}{1+\alpha^2} \qquad (1)$$

where $\alpha = k/m$ is the aspect ratio and $\alpha << 1$ if the hydrostatic approximation is made.

Identify the criterion for the solutions to be unstable (making the hydrostatic approximation) and show that this can be expressed as $F^2 < S^4/N_s^{\ 2}$.

Show that this criterion can be expressed as $\mathrm{Ri} < \frac{f}{\overline{\zeta}}$ where Ri is the Richardson number given by $\mathrm{Ri} = N_s^{\ 2}\left(\frac{\partial \overline{v}}{\partial z}\right)^{-2}$ (assume $f/\overline{\zeta} > 0$).

[12 marks]

Question 3 continued overleaf

Question 3 cont'd

(c) Determine the aspect ratios of the short- and long-wave instability cut-offs (α_1 and α_2 respectively) by finding the roots of equation (1) (again making the hydrostatic approximation).

Prove that α_1 and α_2 are real and satisfy $\alpha_1 > \alpha_{\theta_e}$ and $\alpha_2 < \alpha_M$ where α_{θ_e} and α_M are the magnitudes of the slopes of the θ_e and M surfaces given in (a). [12 marks]

(d) By calculating the Richardson number for the following data show that the atmosphere is unstable to conditional symmetric instability (CSI): $\partial \overline{v} / \partial z = 10^{-2} \mathrm{s}^{-1}$, $\overline{\zeta} = 0.7 f$, and $N_s^2 = 5 \times 10^{-5} \mathrm{s}^{-2}$.

[6 marks]

(e) Calculate the growth rate of the most unstable CSI roll (using the data in (d)) if the atmosphere is hydrostatic. [10 marks]

(End of Question Paper)