You are allowed ten minutes before the start of the examination to acquaint yourself with the instructions below and to read the question paper.

Do not write anything until the invigilator informs you that you may start the examination. You will be given five minutes at the end of the examination to complete the front of any answer books used.

January 2009

Answer Book
Data Sheet

Any bilingual English language dictionary permitted
Calculators and programmable calculators are permitted

THE UNIVERSITY OF READING

MSc/Diploma
Course in Atmosphere, Oceans and Climate

Course in Mathematics
and Numerical Modelling of the Atmosphere and Oceans

PAPER MTMW11/MTMW99

Fluid dynamics of the atmosphere and oceans

Two hours

Answer **ANY TWO** questions

The marks for the individual components of each question are given in [ ] brackets. The total mark for the paper is 100

1.

(a) Let ρ be the density of a fluid. Write down equations for Eulerian and Lagrangian conservation of density and explain the physical meaning of the equations. [8 marks]

(b) The equation for hydrostatic balance is given by

$$\frac{\partial p}{\partial z} = -\rho g . \qquad (1.1)$$

Show that if the temperature does not vary with height, the *density* profile for air is of the form $\rho = \rho_0 \exp\left(-z / H\right)$, where ρ_0 is a reference density, and H is to be found. (You may find it helpful to use the ideal gas law, $p = \rho RT$.) [8 marks]

(c) Suppose air is initially at rest in a state of hydrostatic balance. A small parcel of fluid is displaced upwards by an amount δz without altering the overall pressure field. Show that, if the density of the parcel is conserved, the upward force per unit volume on the parcel will be approximately,

$$F \approx g \frac{\partial \rho}{\partial z} \delta z . \qquad (1.2)$$

[10 marks]

(d) Write down an equation of motion for the air parcel from part (c), using Newton's second law. Solve the equation of motion, using the density profile you derived in part (b) and give a physical interpretation of your solution. [14 marks]

(e) How would your solution to part (d) change if instead of using the density profile from part (b), you used a density profile such that $\frac{\partial \rho}{\partial z} > 0$? What does this imply physically? [10 marks]

2.

(a) The momentum equation for flow of a fluid relative to a co-ordinate frame rotating with uniform angular velocity Ω may be written:

$$\frac{D\mathbf{u}}{Dt} = \underset{(A)}{-2\boldsymbol{\Omega}\wedge\mathbf{u}} \quad \underset{(B)}{-\boldsymbol{\Omega}\wedge(\boldsymbol{\Omega}\wedge\mathbf{r})} \quad \underset{(C)}{+\mathbf{g}} \quad \underset{(D)}{-\frac{1}{\rho}\nabla p}$$

Briefly describe the process represented by each of the terms A, B, C, D in this equation. [12 marks]

(b) A parcel of ocean-water at a latitude of 50°N moves parallel to the Earth's surface at 0.5 m s^{-1} from west to east. Write expressions for the west-east, south-north and vertical components of term (A). Estimate values for each of these components for the parcel, which you may assume has a mass of 1 kg. [12 marks]

(c) Now suppose that the flow is slow, incompressible, inviscid and homogeneous. Starting from the full momentum equation given in part (a) show that

$$\left(\boldsymbol{\Omega}\bullet\nabla\right)\mathbf{u} = 0 \, . \qquad (2.1)$$

Describe any approximations that you make.

(You may wish to make use of the vector identity $\nabla\wedge\left(\mathbf{A}\wedge\mathbf{B}\right) = \left(\mathbf{B}.\nabla\right)\mathbf{A} - \mathbf{B}\left(\nabla.\mathbf{A}\right) - \left(\mathbf{A}.\nabla\right)\mathbf{B} + \mathbf{A}\left(\nabla.\mathbf{B}\right)$)

[16 marks]

(d) Explain the consequences of equation (2.1) for flow in a rotating tank, where the rotation axis is parallel to the *z*-axis . [10 marks]

3.

(a) A wave disturbance of the atmosphere is of the form

$$\varphi(x, y, t) = \mathrm{Re}\left[\Psi \exp\left\{i\left(kx + ly - \omega t\right)\right\}\right]. \qquad (2.2)$$

Sketch the form of the disturbance at t = 0 in the *x*-*y* plane. Mark the crests of the waves and wave-vector, $(k, l)^{\mathrm{T}}$ clearly on your diagram. [10 marks]

(b) The barotropic vorticity equation on a midlatitude β-plane is given by

$$\frac{\partial}{\partial t}\nabla_H^2\psi + J\left(\psi, \nabla_H{}^2\psi\right) + \beta\frac{\partial\psi}{\partial x} = 0, \qquad (2.3)$$

where ψ is a streamfunction and

$$J(\psi, \nabla_H{}^2\psi) = \begin{vmatrix} \dfrac{\partial\psi}{\partial x} & \dfrac{\partial\psi}{\partial y} \\ \dfrac{\partial}{\partial x}\nabla_H{}^2\psi & \dfrac{\partial}{\partial y}\nabla_H{}^2\psi \end{vmatrix}. \qquad (2.4)$$

Show that the dispersion relationship for Rossby waves is

$$\omega = Uk - \frac{\beta k}{\left(k^2 + l^2\right)}, \qquad (2.5)$$

where U is constant, zonal flow and k and l are the wave numbers in the zonal and meridional directions respectively. Describe any approximations you make. [15 marks]

(c) Calculate the phase-speed of the waves in the *x*-direction, and hence give the value of for $K^2 = k^2 + l^2$ when the West-East component of the Rossby-wave disturbance is stationary relative to the earth's surface. [10 marks]

(d) Calculate the group velocity of the stationary waves and give a physical interpretation of your answer. [15 marks]

(End of Question Paper)