The Central Role of Data Assimilation in Probabilistic Forecasting

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LSE CATS CENTRE FOR THE ANALYSIS OF TIME SERIES

Talk Outline

1. Why data assimilation has a role in probabilistic forecasting.
2. Pseudo-orbit data assimilation (PDA) ensembles.
3. Predictive Performance.

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Laplace's Demon

'An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.'

Pierre Simon Laplace, A Philosophical Essay on Probabilities (1814).

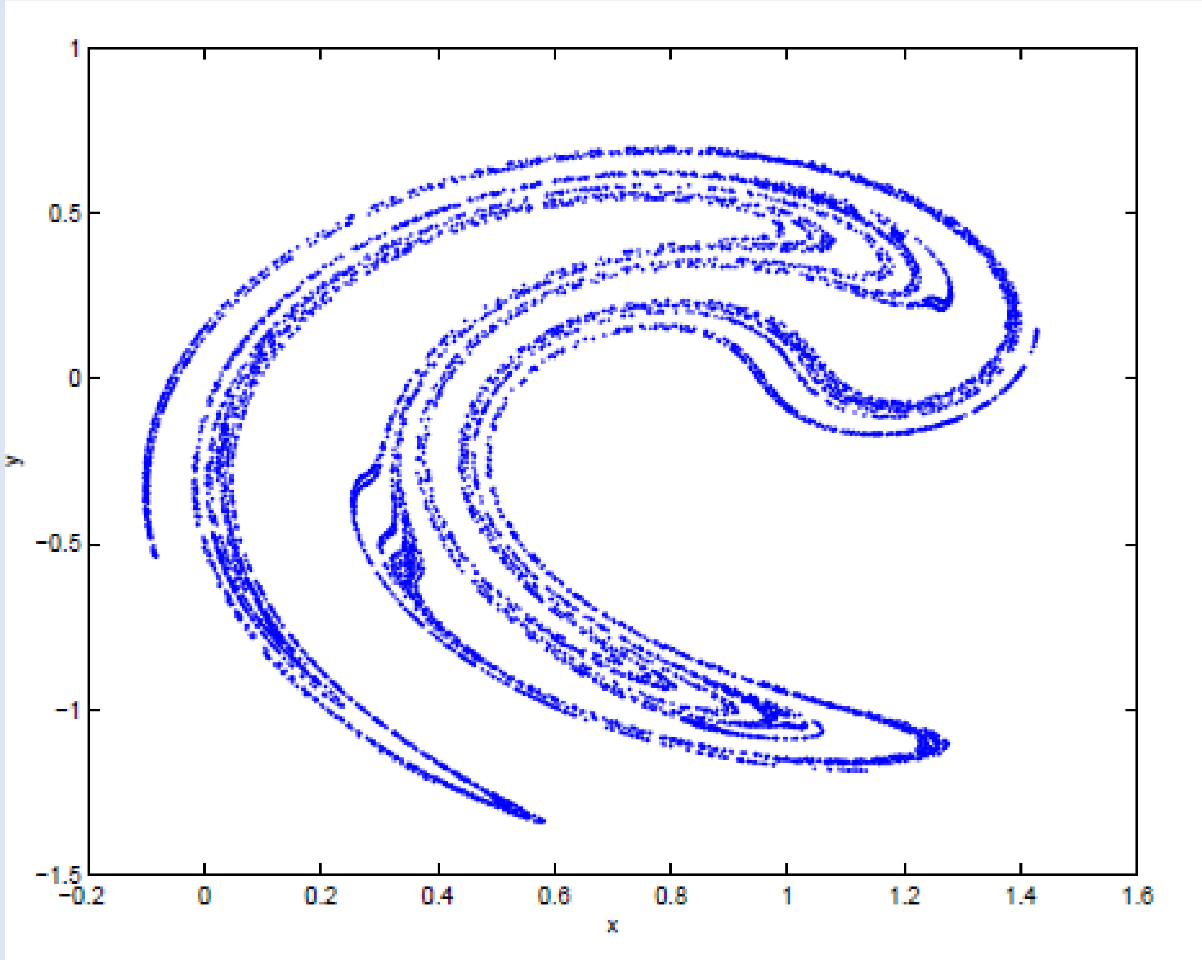
Laplace’s Demon

- Laplace sets out three required conditions to make perfect deterministic predictions of the future.
- *‘know all forces that set nature in motion’* (A perfect mathematical model of the laws of nature.)
- ‘*all positions of all items of which nature is composed’* (Exact observations of the universe.)
- ‘*vast enough to submit these data to analysis’* (Infinite computational power.)

Demon’s Apprentice (2007)

- The Demon’s Apprentice (2007) has inexact observations.
- A perfect mathematical model of the laws of nature.
- ~~Exact observations of the universe.~~
- Infinite computational power.

Ikeda Map



Two dimensional map, chaotic for chosen parameter values.

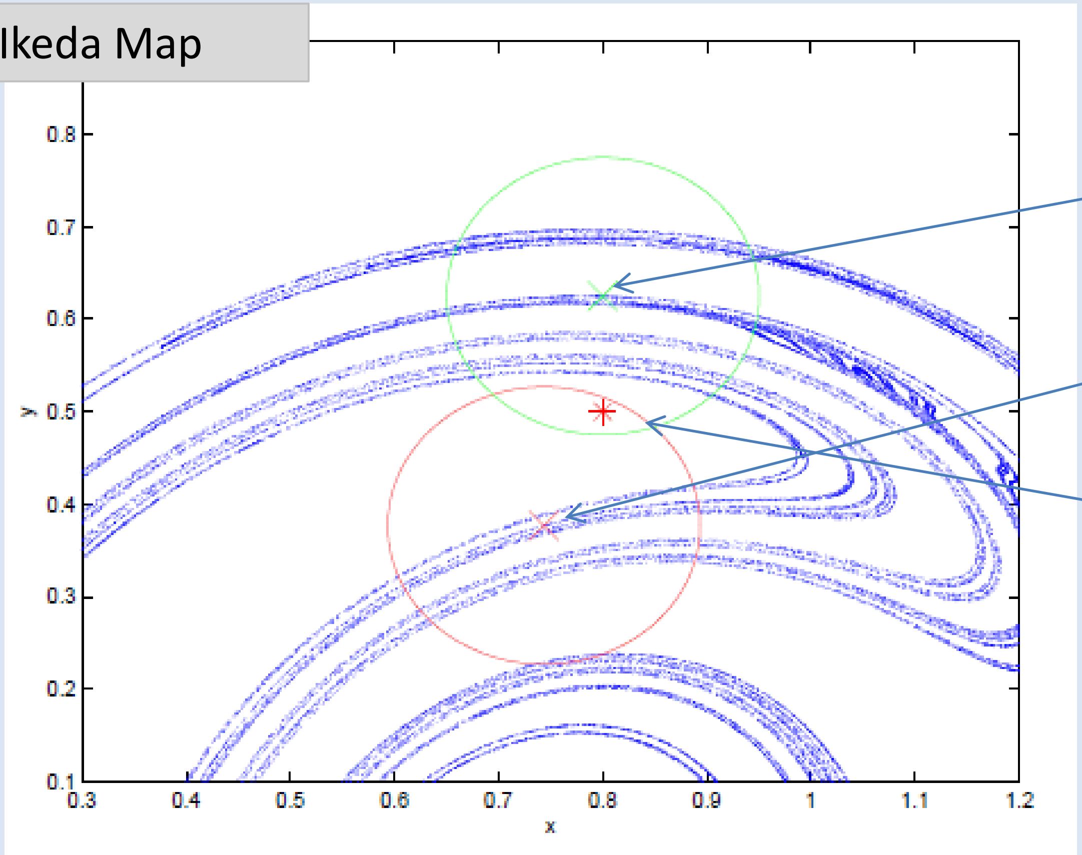
$$x_{n+1} = \gamma + u(x_n cos\phi - y_n sin\phi)$$

$$y_{n+1} = u(x_n sin\phi + y_n cos\phi)$$

Indistinguishable States

- With an imperfect observation of the initial state, many states exist that are indistinguishable from the true state (Judd, Smith,2001)

Ikeda Map



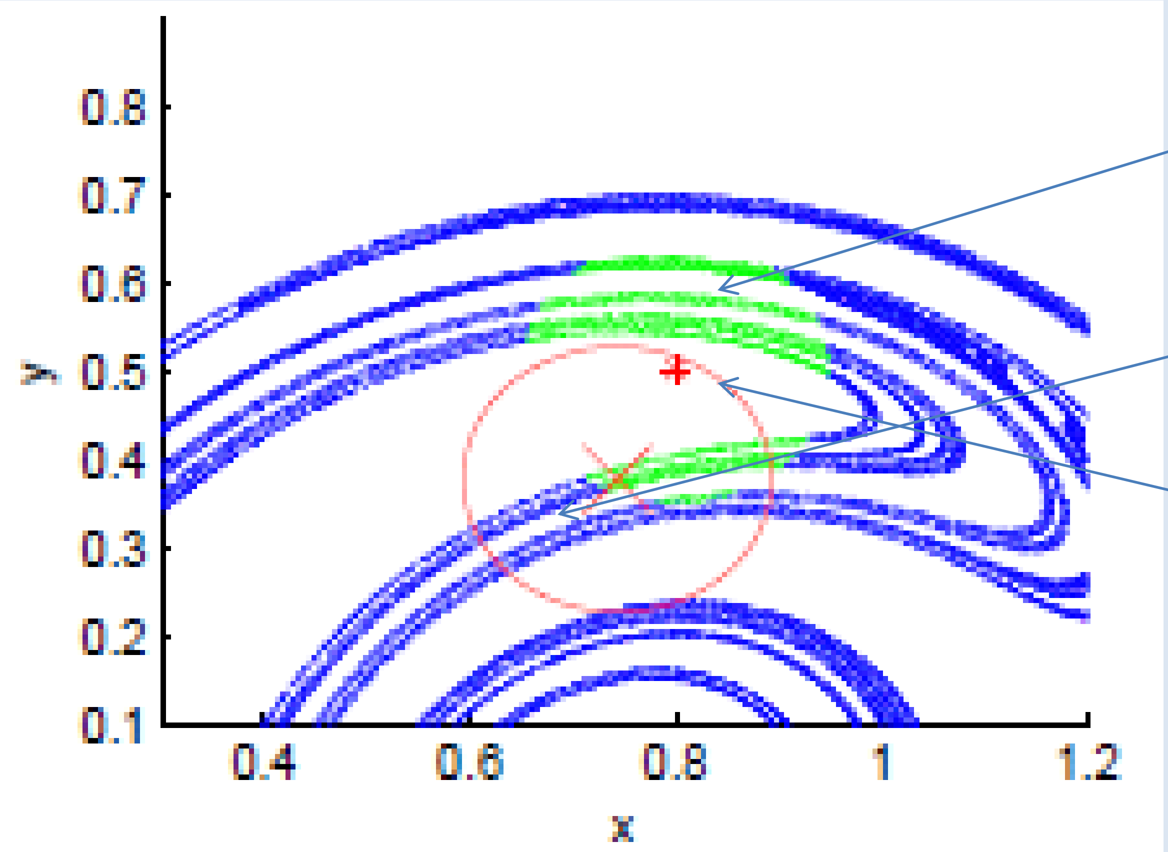
An indistinguishable state

True initial state

Observation lies within region of uncertainty for both states. They are therefore indistinguishable.

Indistinguishable States

- With an imperfect observation of the initial state, many states exist that are indistinguishable from the true initial state.



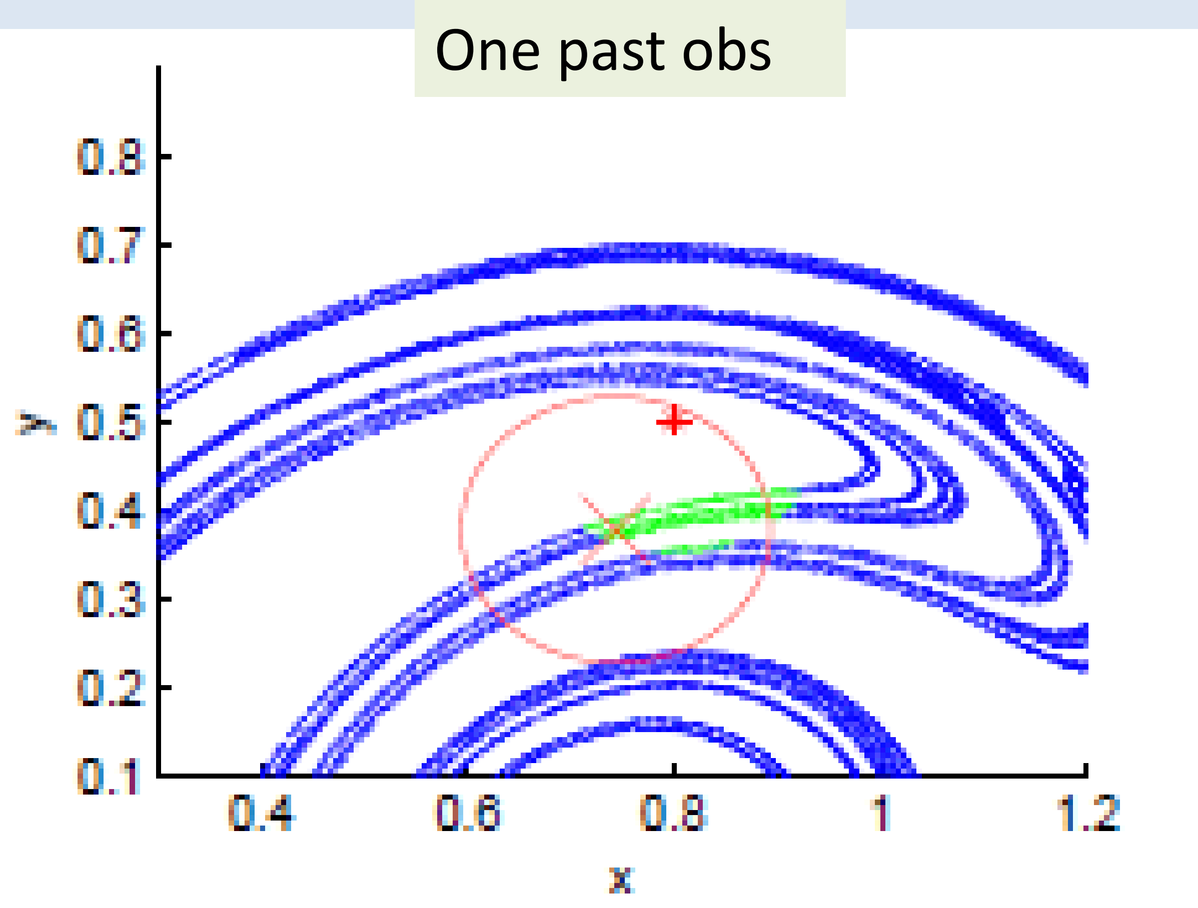
Green points: Indistinguishable states

True initial state

Observation.

Indistinguishable States

- What if we take past observations into account?

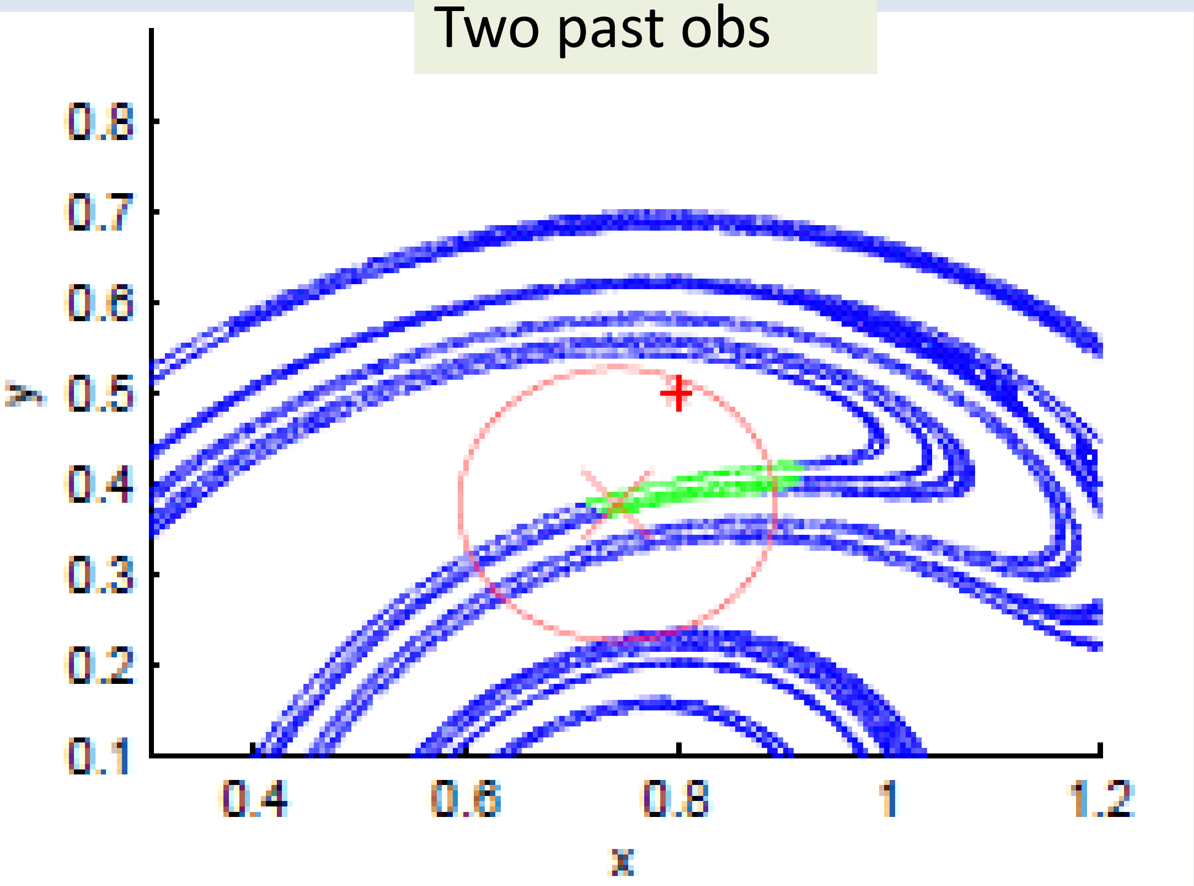


Number of indistinguishable state decreases

Indistinguishable States

- What if we take past observations into account?

Two past obs



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Indistinguishable States

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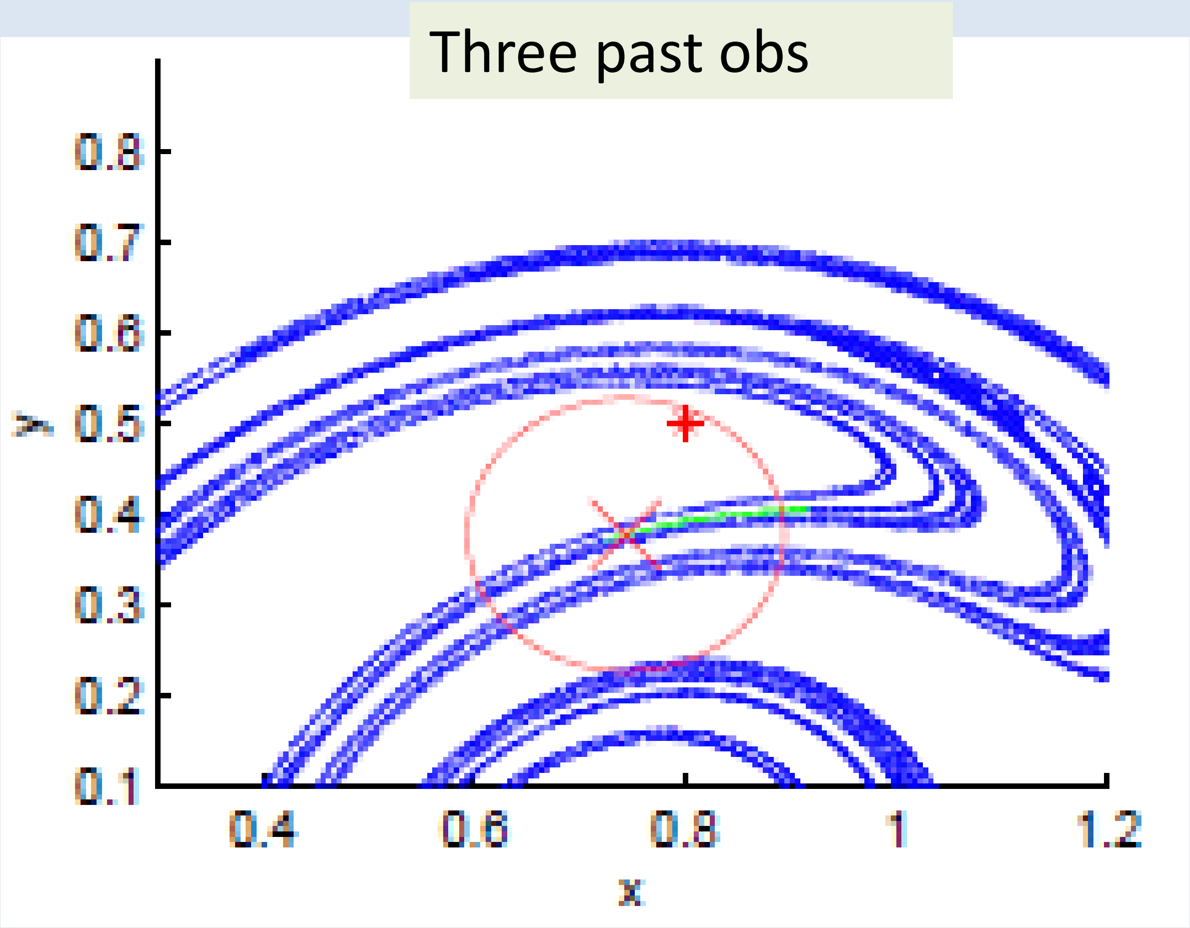

Number of indistinguishable state decreases

Even with infinite past obs, we can never narrow down to the true state (Judd, Smith 2001).

Probabilistic Forecasting

- When observations are noisy, the exact initial condition can never be recovered.
- Demon’s Apprentice can sample as many indistinguishable states as it desires.
- Laplace’s Demon can make perfect *deterministic* forecasts.
- Apprentice can make perfect *probabilistic* forecasts.

- What if we don’t have infinite computational power?

Ensemble Forecasting

- Ensemble forecasts consist of multiple point forecasts with slightly different initial conditions.
- Ensembles can be used to form probabilistic forecasts.
- How best to sample the initial conditions?
- One approach: sample around the observation (inverse noise).
- Quick and easy but initial condition ensemble inconsistent with the model dynamics.
- Data assimilation can find states consistent both with obs and model dynamics.

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Pseudo-orbit data assimilation

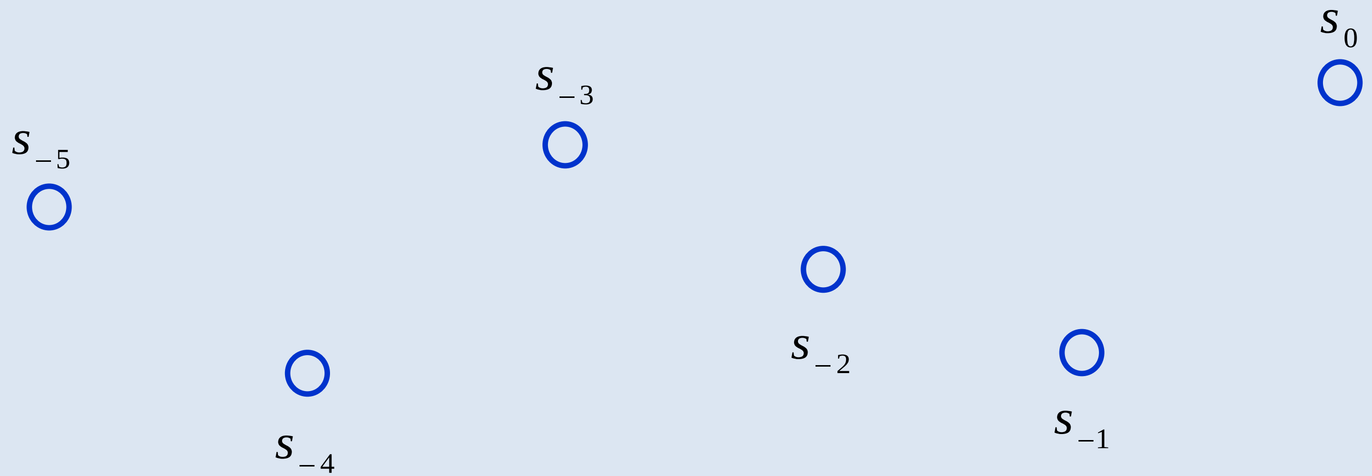
- Define the mismatch cost function c(**u**)=$\sum_{-n+1}^{0} |F(\boldsymbol{u}_t) - \boldsymbol{u}_{t+1}|$ where F(.) is the model 1 step operator.
- Use gradient descent to minimise the mismatch function.
- Using Euler approximation method, each iterative step is applied using:

$$x_i \mapsto x_i - \frac{2\Delta}{w} \times \begin{cases} -\mathcal{A}(x_i)(x_{i+1} - f(x_i)), & i = 0, \\ \begin{matrix} (x_i - f(x_{i-1})) \\ -\mathcal{A}(x_i)(x_{i+1} - f(x_i)) \end{matrix}, & 0 < i < w, \\ (x_i - f(x_{i-1})), & i = w, \end{cases}$$

$A(x_i)$ is the adjoint matrix.

Pseudo-orbit data assimilation

- Initialise **u** with a set of observations..
- As mismatch is reduced, **u** approaches a model trajectory.



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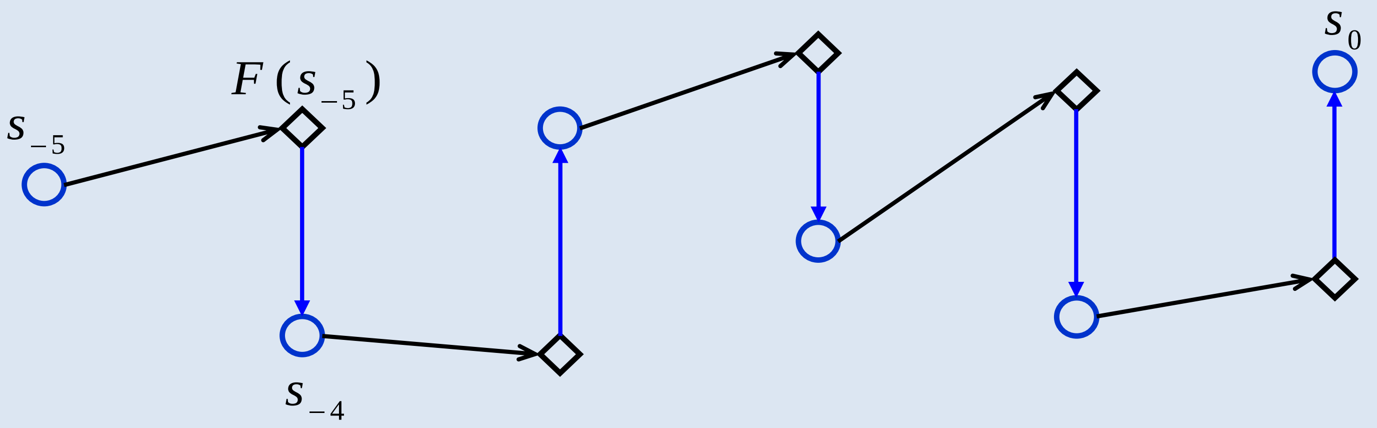

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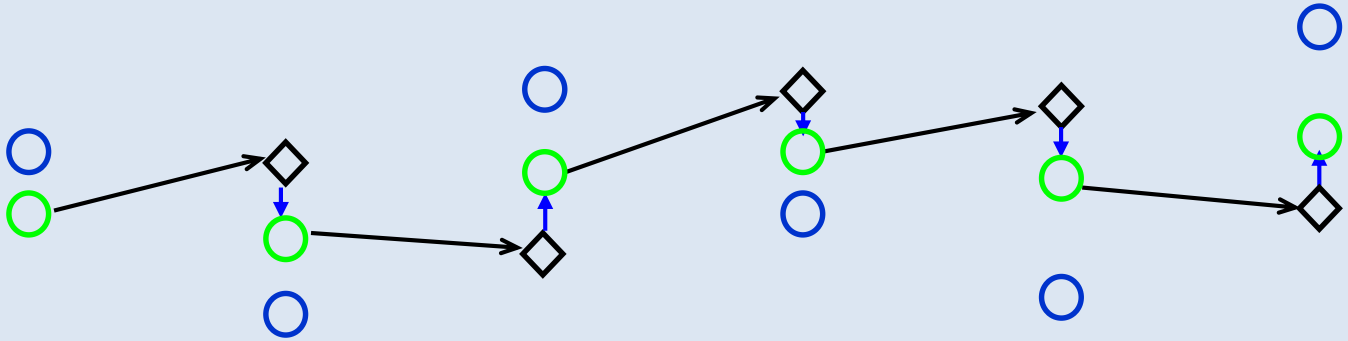

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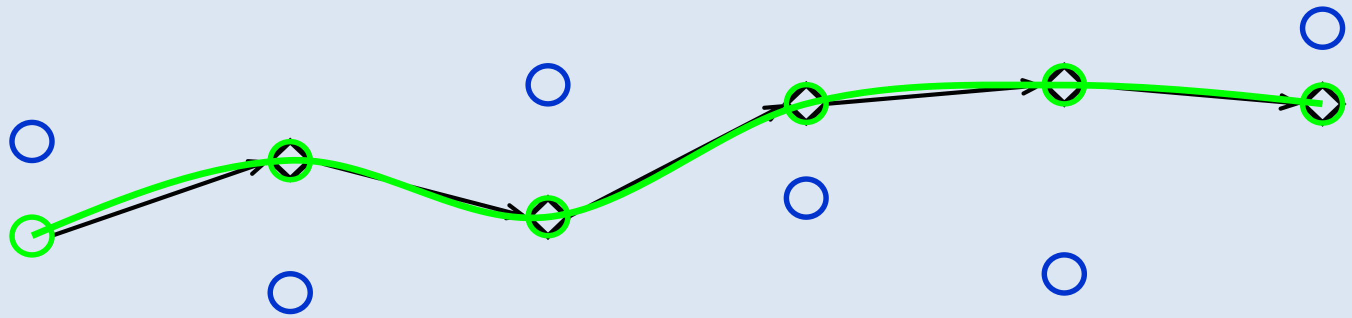

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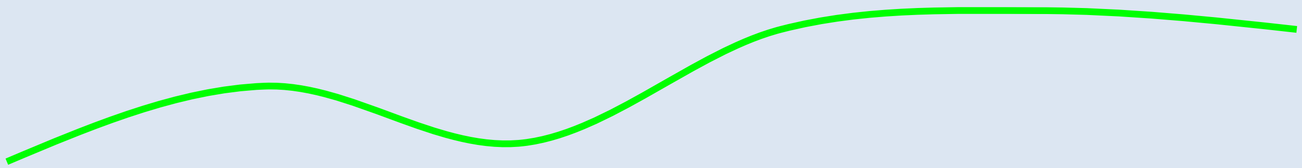
PDA Ensembles

- Use PDA on the observations to find a reference pseudo-orbit.
- Renoise the referencee pseudo-orbit by adding random perturbations from the noise distribution.
- Use PDA to find new pseudo-orbits
- Collect initial condition.



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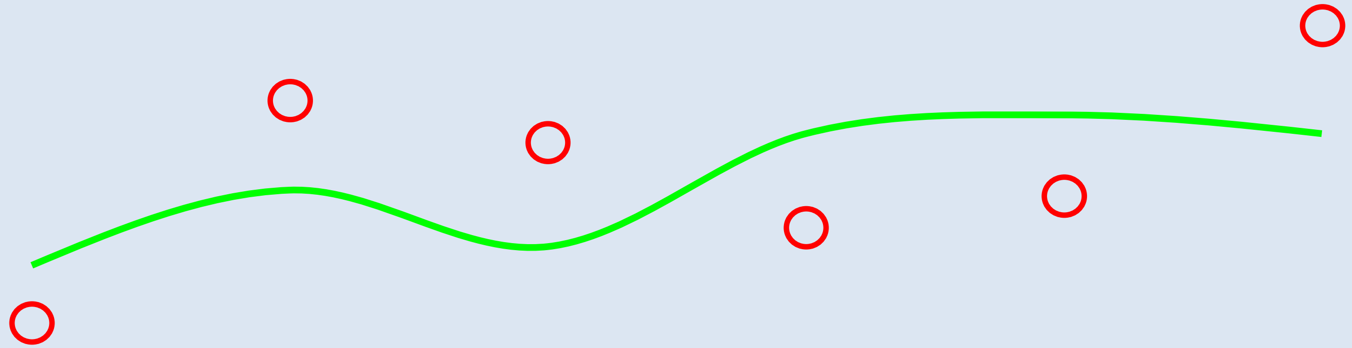

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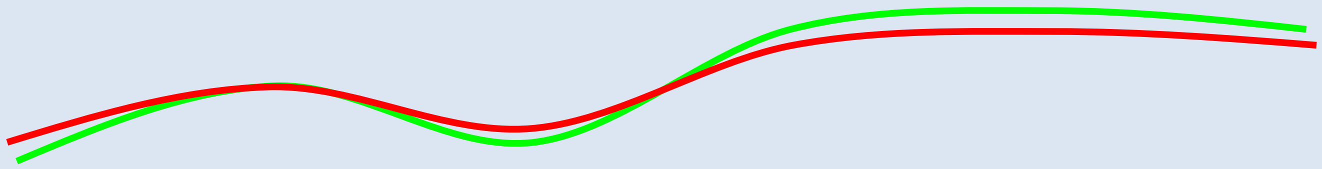

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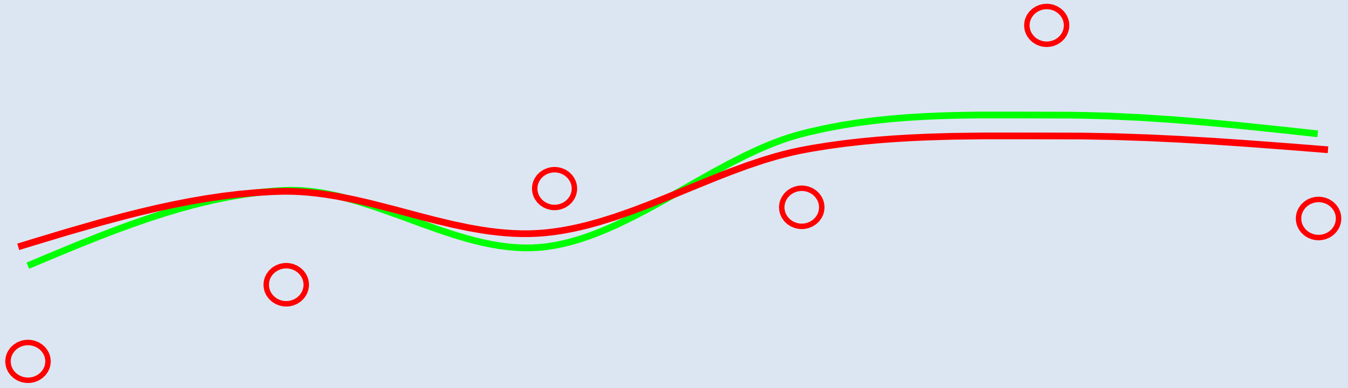

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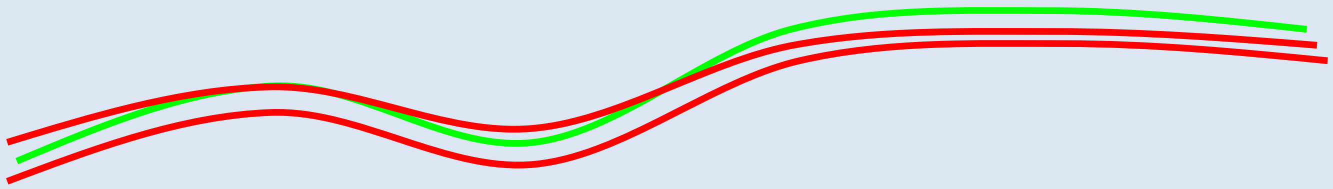

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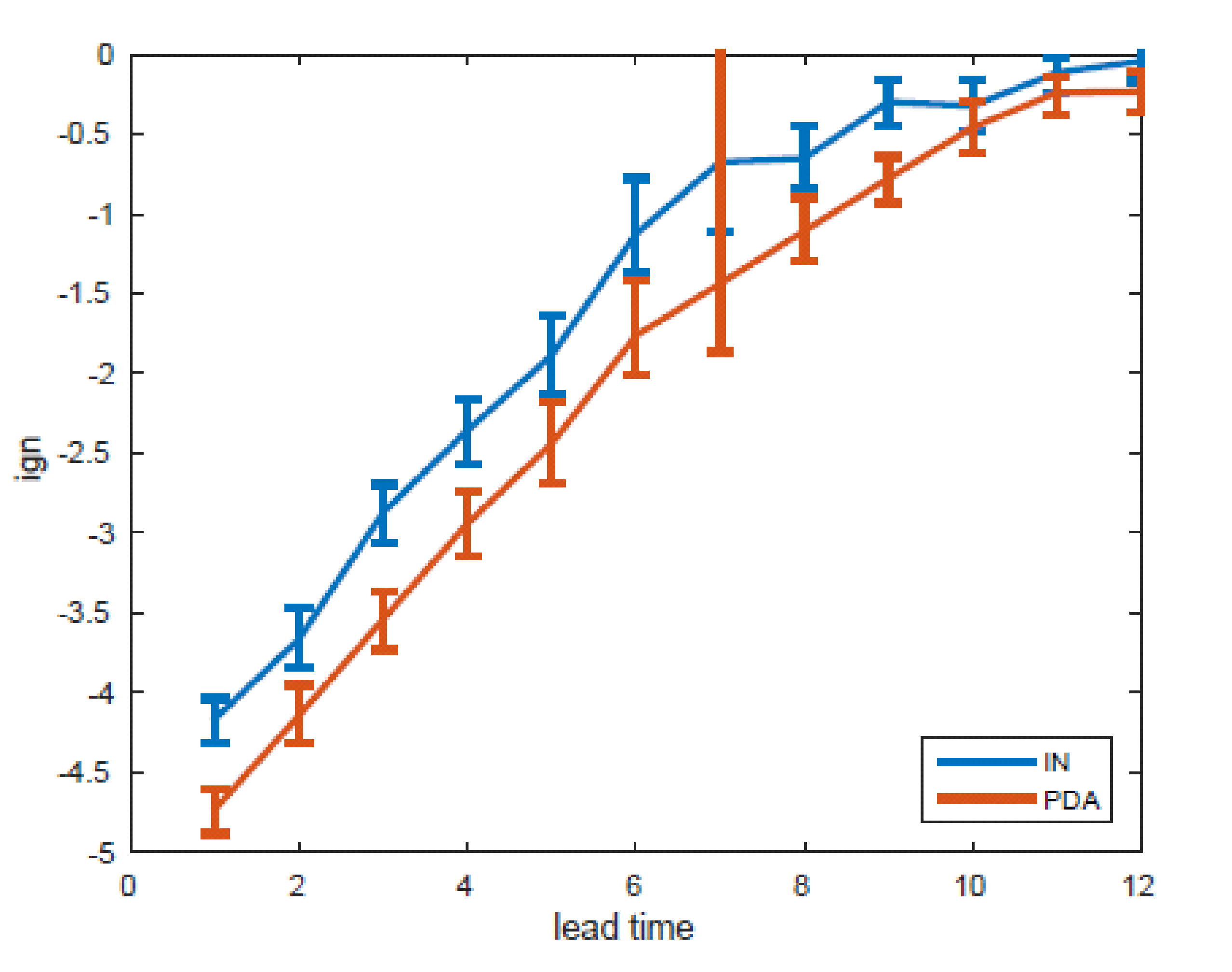
Predictive Performance

- Make probabilistic forecasts of the Ikeda Map based on:
- 16 member inverse noise ensembles.
- 16 member PDA ensembles (assimilation window of 4 steps).
- Convert ensembles into forecast densities using kernel dressing and blending (Broecker and Smith, 2008).
- Forecasts take the form $\mathrm{p(x)} = \alpha p_m(x) + (1-\alpha) p_{clim}(x)$
- Evaluate forecasts using ignorance score - ign=$-\log_2(p(y))$.

Predictive Performance



- PDA ensembles (red) result in significantly better mean ignorance score than inverse noise (blue).

PDA in practice

- PDA has been shown to be effective in both low and high dimensional systems.
- Has been demonstrated in a high dimensional weather model (NOGAPS).
- PDA will feature in the NCAR Data Assimilation Research Testbed (DART).

Conclusions

- With observational noise, forecasts of nonlinear systems should be probabilistic.
- Data assimilation can help find initial states consistent with both observations and the model.
- PDA ensembles can result in more skillful probabilistic forecasts.

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‘The farther back you can look, the further ahead you are likely to see’
Winston Churchill