

Estimating atmosphere-ocean forecast error cross-covariances for strongly coupled 4D-Var data assimilation

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Introduction

- strongly coupled atmosphere-ocean data assimilation solves the assimilation problem for a single coherent atmosphere-ocean state
- requires specification of the relationship between the errors in the atmosphere and ocean model forecasts
 - non-trivial ...
- we've been using an idealised strongly coupled 4D-Var system to investigate the nature and structure of coupled atmosphereocean forecast error cross-correlations



Strongly coupled incremental 4D-Var

- control vector contains both atmosphere and ocean model variables
- allows for cross-covariances between atmosphere and ocean background errors

$$\mathbf{B} = \begin{pmatrix} \mathbf{B}_{\mathrm{AA}} & \mathbf{B}_{\mathrm{AO}} \\ \mathbf{B}_{\mathrm{OA}} & \mathbf{B}_{\mathrm{OO}} \end{pmatrix}$$

- atmosphere obs can influence ocean analysis & vice versa
- more balanced analysis state

first guess $\mathbf{x}_{0}^{(0)} = \mathbf{x}_{b}$ non-linear trajectory computed using **coupled** model $\mathbf{x}_{i}^{(k)} = m(t_{i}, t_{0}, \mathbf{X}_{0}^{(k)})$ innovations $\mathbf{d}_{i}^{(k)} = \mathbf{y}_{i} - h(\mathbf{x}_{i}^{(k)})$ perturbation first guess $\delta \mathbf{x}_{i}^{(k)} = 0$ \mathbf{v} TL of **coupled** model: $J^{(k)}$ ADJ of **coupled** model: $\nabla J^{(k)}$ update $\mathbf{x}_{0}^{(k+1)} = \mathbf{x}_{0}^{(k)} + \delta \mathbf{x}_{0}^{(k)}$

single minimisation process

Ensemble of 4D-Vars



Estimate background error covariance from a ensemble of perturbed strongly coupled 4D-Var analyses $y+\delta^{\circ}$

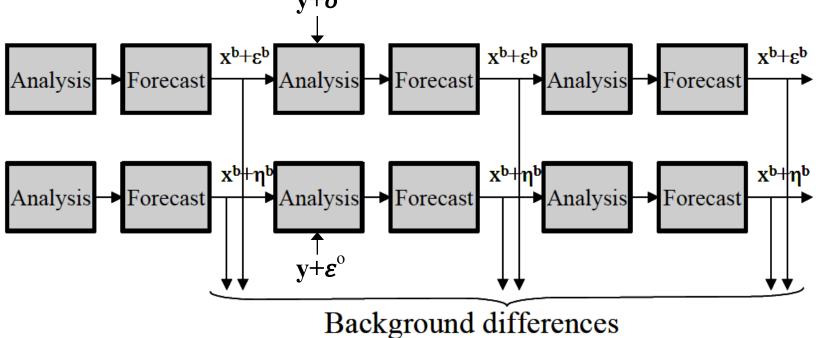


Figure 3: Schematic illustration of the analysis-ensemble method of generating fields of background difference. adapted from Fisher 2003

Differences between pairs of background fields have same **correlation** structure as background error (but twice the variance)

Idealised system



single-column, coupled atmosphere-ocean model

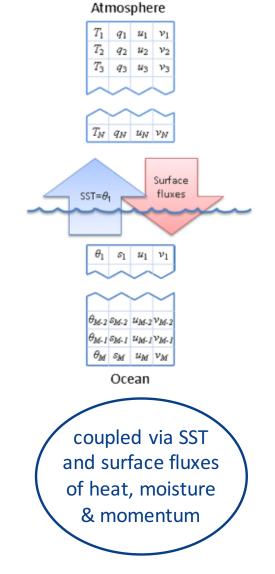
Atmosphere

simplified version of the ECMWF single column model adiabatic component + vertical diffusion (no convection) 4 state variables on 60 model levels (surface to ~0.1hPa) forced by large scale horizontal advection

Ocean

K-Profile Parameterisation (KPP) mixed-layer model based on the scheme of Large et al¹
4 state variables on 35 model levels (1-250m)
forced by short and long wave radiation at surface

1. DOI: 10.1029/94RG01872



Identical twin experiments



- 12 hour assimilation window, 3 outer-loops, 8 cycles
- experiments repeated using data for June 2013 & Dec 2013 (point is 188.75°E, 25°N, N Pacific Ocean)
- 'true' initial state is coupled model forecast initialised using ERA Interim and Mercator Ocean data
- initial background state is a perturbed coupled model forecast
- 3 hourly observations are generated by adding random noise to 'truth'
- error covariance matrices **B** and **R** are diagonal (same for all cycles)
- ensemble of 500 members generated by perturbing initial background state and observations
- average pairs over a several assimilation cycles to increase effective ensemble size



Comments

- each cycle starts at either 12 UTC or 00 UTC which corresponds to the early hours of the morning and early afternoon local time.
- one set of error correlations from 12 hour forecast ensembles from day to-night and one set of error correlations from 12 hour forecast ensembles from night-to-day.
- allows comparison of day-night plus summer-winter error correlations.

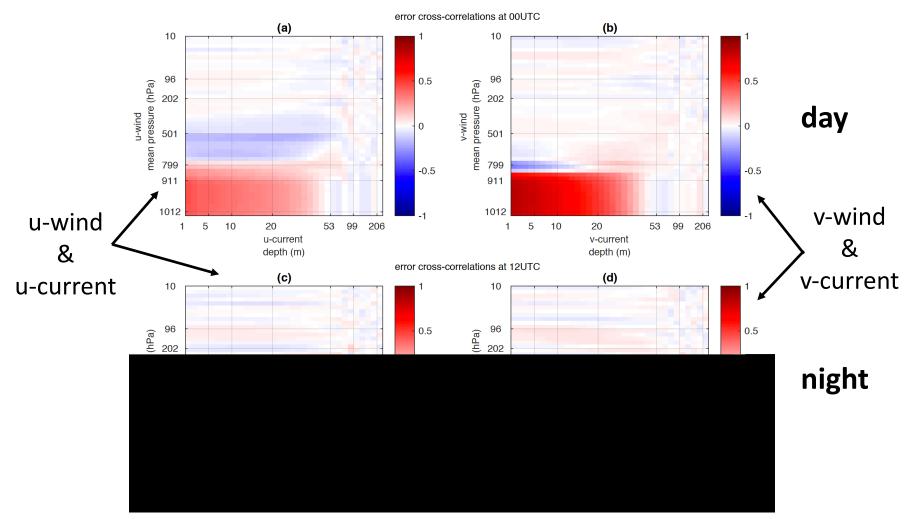


Questions

- how do the atmosphere-ocean error cross correlation structures vary between summer and winter, and between day and night?
- where are the atmosphere-ocean error cross-correlations strongest?
- can we explain our results by considering the underlying model physics, forcing and known atmosphere-ocean feedback mechanisms?
- what happens if we use a smaller ensemble?

December case:



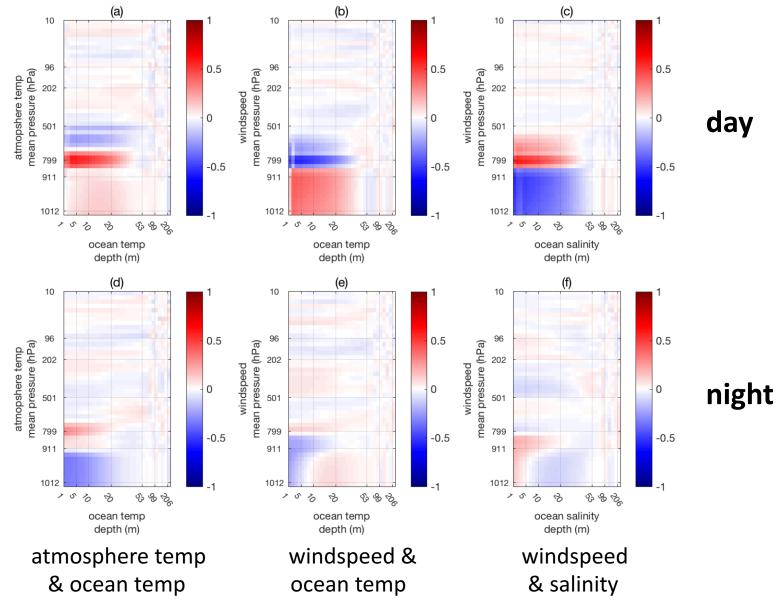


atmosphere wind-ocean current error cross-correlations

December case:

University of Reading

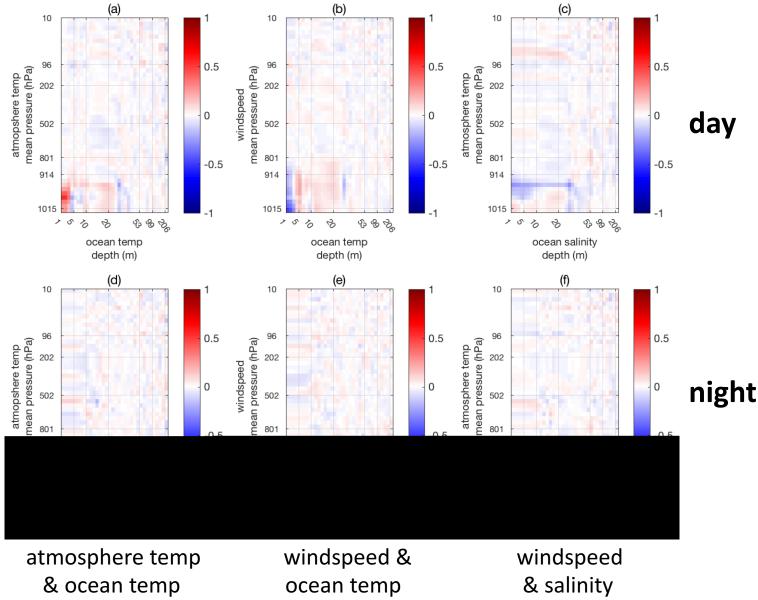
atmosphere-ocean error cross-correlations



June case:



atmosphere-ocean error cross-correlations



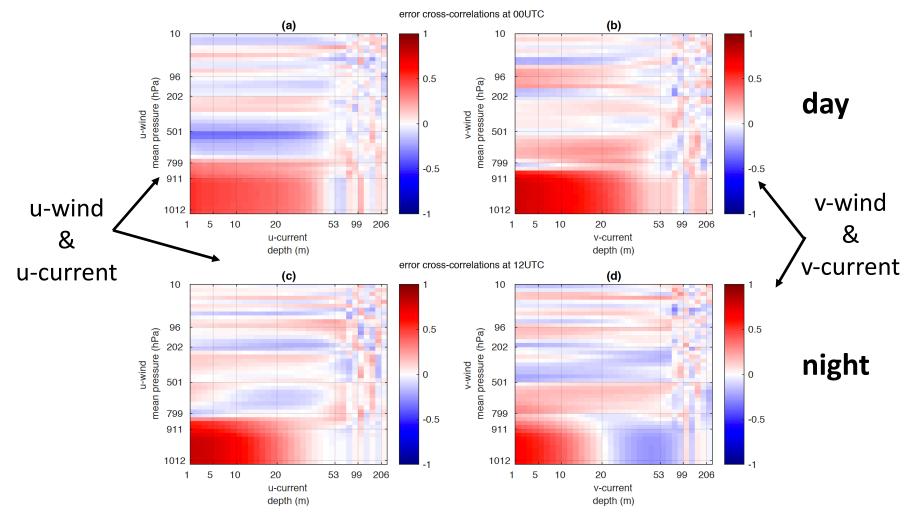


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December case: 50 members

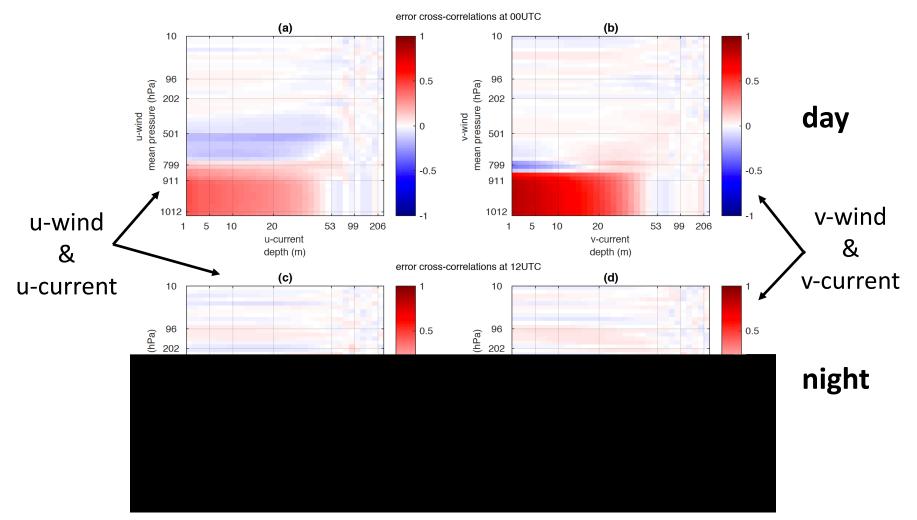




atmosphere wind-ocean current error cross-correlations

December case: 500 members





atmosphere wind-ocean current error cross-correlations



Localisation

- statistics of smaller ensemble may be sufficient if combined with an appropriate vertical localisation method (Schur product)
- construct multivariate localisation function using a univariate function (Gaspari & Cohn) - apply block by block
- need to define co-ordinate for distance, correlation length-scale:
 - atmosphere and ocean model components use different vertical co-ordinate systems
 - very different length-scales in atmosphere and ocean

Localisation



Distance between a given pair of atmosphere and ocean model levels (in metres)

$$d(z_a(i), z_o(j)) = z_a(i) + z_o(j)$$

where $z_a(i)$ is the height at atmosphere model level *i* and $z_o(j)$ is depth at ocean model level *j*.

Rescale

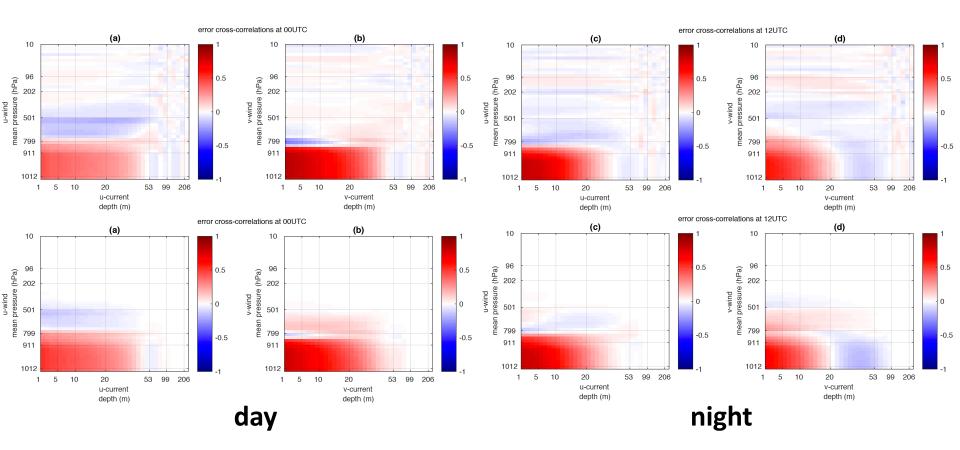
$$\widehat{d}(z_a(i), z_o(j)) = \left(\frac{z_a(i)}{L_a} + \frac{z_o(j)}{L_o}\right)$$

where L_a and L_o define the height of the atmosphere boundary layer and depth of the ocean boundary layer.

Localisation results



atmosphere wind-ocean current error cross-correlations



top: 500 member ensemble **bottom:** 50 member ensemble + localisation with L_a = 7000m, L_o = 75m;



Summary

- significant variation in atmosphere-ocean error cross correlation structures between summer and winter, and between day and night.
- strongest cross-correlations are in near surface atmosphere-ocean boundary, beyond this atmosphere-ocean errors appear to be mostly uncorrelated.
- results can be explained using knowledge of underlying model physics, forcing and known atmosphere-ocean feedback mechanisms.
- with a limited ensemble size ensemble error correlations are noisy.
- vertical localisation with a scaled distance dependent localisation function offers a potential solution.



Questions?