Control of gravity waves in the Met Office convective-scale 4DVAR system

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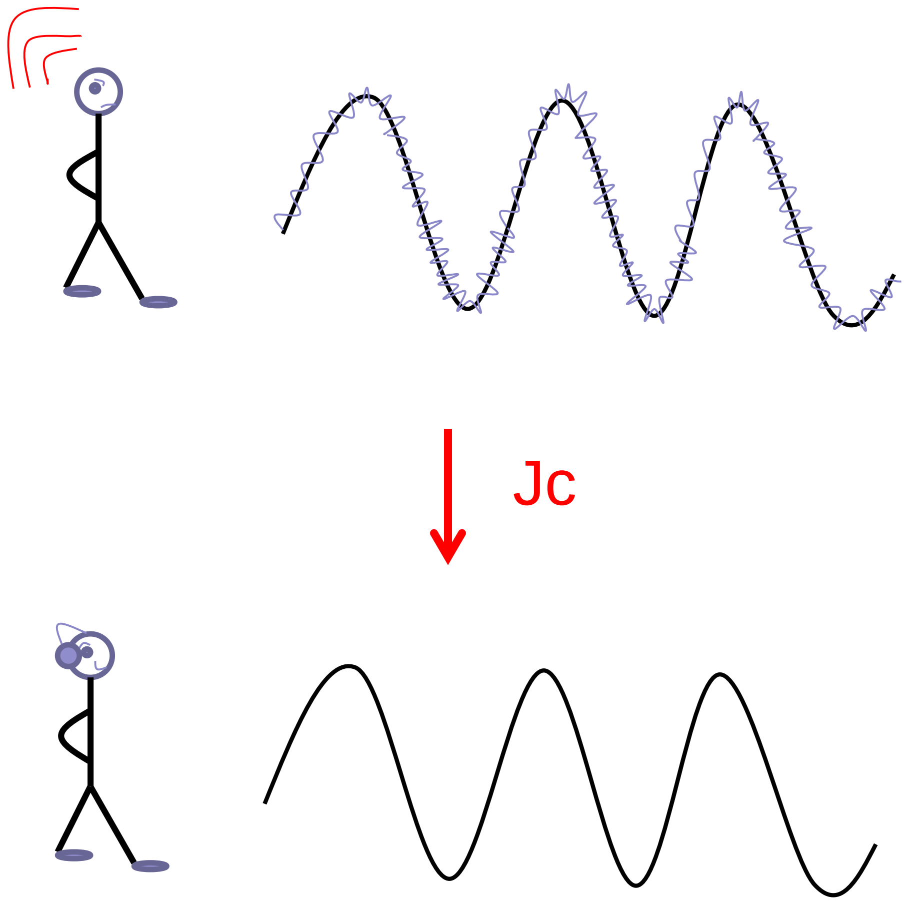
OVERVIEW

- Incremental 4Dvar formulation
- PF model and gravity waves
- Digital filter
- UKV 4DVAR implementation
- Sensitivity experiments
- Conclusions



Incremental 4D-VAR

- Based on the formulation of Rawlins et al. 2006
- Uses *S* a non-linear simplification operator with tangent linear approximation **S**
- 4DVAR Cost function, using the simplified increments (notation avoids sums)

$$J(\delta\mathbf{w}) = \frac{1}{2}(\delta\mathbf{w} - \delta\mathbf{w}^b)^T\mathbf{B}^{-1}(\delta\mathbf{w} - \delta\mathbf{w}^b) + \frac{1}{2}(\underline{\mathbf{y}} - \underline{\mathbf{y}^o})^T\underline{\mathbf{R}^{-1}}(\underline{\mathbf{y}} - \underline{\mathbf{y}^o})$$

- In the minimization a CVT (Control Variable Transform) is used.
- The **B** matrix becomes the Identity
- CVT uses $\mathbf{U}_a$, the vertically adaptive grid transform (AG; Piccolo and Cullen, 2011)

Perturbation Forecast model

- Used in the Met Office variational assimilation methods, to capture the main synoptic evolution of errors (cheaply).
- Use approximations instead of linearising the discrete equations as a tangent-linear model.
- PF model increments are hydrostatic - gravity waves permitted.
- OBS different from background can lead to unrealistic deviation from geostrophic balance. Particularly from $\mathbf{U}_a$.
- Use of a Jc term to control high-frequency oscillations.

The digital filter

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- Non recursive digital filter, for low-frequency motions:

$$\delta \mathrm{x}(t_{(l+m)/2})^{DF} = \sum_{k=l}^{m} \alpha_k \delta \mathrm{x}(t_k)$$

  l and m indicate the time window which the filter spans.

  α_k, the (real) filter coefficient.

- For high frequency motions:

$$\gamma_k = \begin{cases} \alpha_k & \text{if } k \neq (l+m)/2, \\ \alpha_k - 1 & \text{if } k = (l+m)/2 \end{cases}$$

  Given a desired filter response and a finite window, the method provides suitable α_k values.

- Jc term depends on amplitude of high frequency waves in the middle of the window.

The digital filter

- The contribution of individual waves at the given time depends on their phase.
- A term depending on the field tendencies is applied, which are 90 degrees out of phase.
- Technically the filter used has the form :

$$J_{DF} = \lambda_{DF} \left\| \sum_{k=l}^{m} \gamma_k \delta \mathbf{w}(t_k) + \sum_{k=l}^{m} \gamma_k \delta \mathbf{w}^g(t_k) \right\|_{(w)}$$

- Use a penalty term.
- Using a quadratic norm based on pressure increments to measure the size of the filter increments. The elastic term of total energy is used, depends on pressure increments.

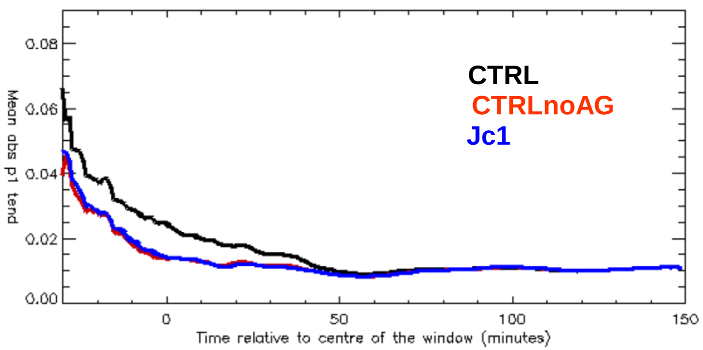
UKV 4DVAR implementation

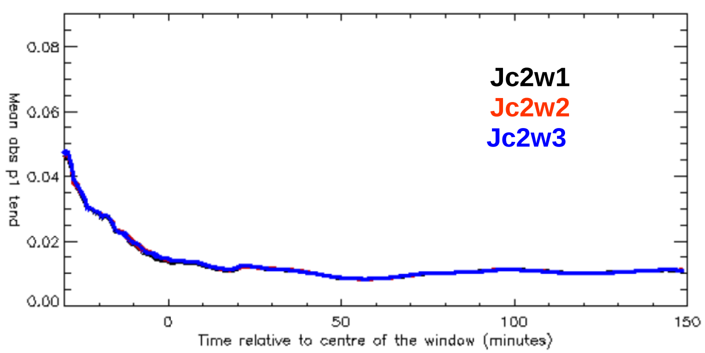
- Based on previous project for 2012 Olympics (Ballard et al., 2016 QJRMS **142**, 472-487).
- Improve post-processing products in 0-6hr period.
- Hourly updates to t+12 – potential benefit in severe weather.
- Observation cut-off 45 mins
- Perturbation Forecast model resolution 3.3km
- Doppler radial winds every 10mins
- AMVs, wind profiler, SEVIRI radiances every 15mins
- Surface rain rate from radar every 15mins (for LHN)
- Other data mostly hourly

Sensitivity tests for Jc in M.O. environment

- A digital filter uses a time window in order to find the spurious high frequency waves to cut-off.
- Different parameterizations for the weights of the Jc term are tested:
  - low (Jc1)
  - medium (Jc2)
  - high (Jc3)
- Using Jc2 different time windows are tested:
  - 56 minutes (Jc2w1)
  - 40 minutes (Jc2w2)
  - 20 minutes (Jc2w3)

High frequency motions damping

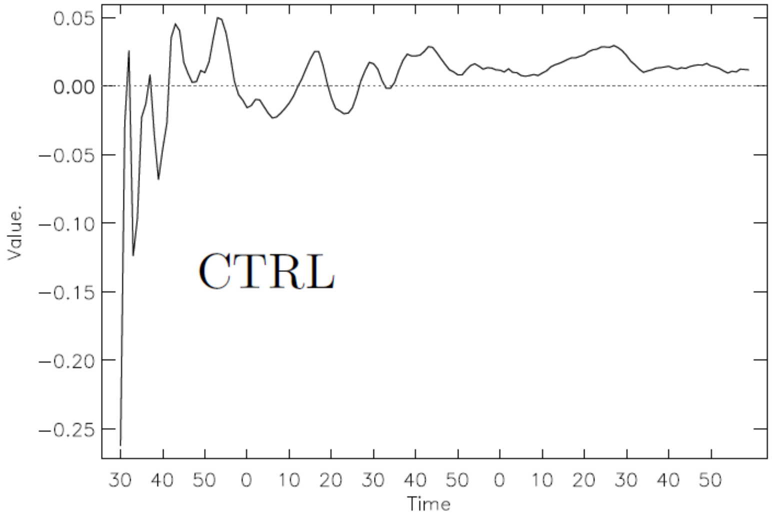


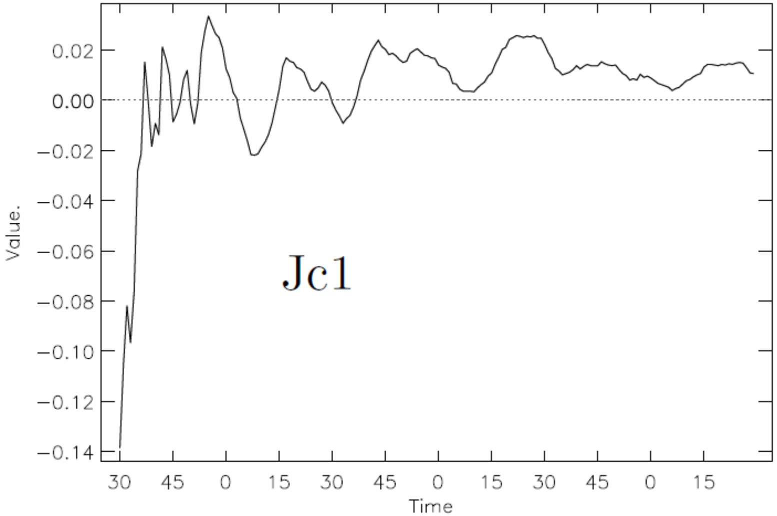


DA window

- Metric: mean absolute pressure tendency at level 1.
- Run at 16Z using the LBC at 12UTC.
- CTRL: run with AG and without Jc term.
- CTRLnoAG: run without AG and without Jc term.
- Jc1: run with AG and with Jc1.
- CTRLnoAG and Jc1 have less gravity waves than CTRL for the first 80 minutes.
- No differences are visible using different Jc approaches
- Damping of spurious gravity waves is achieved also using low weight for the Jc term and short window.

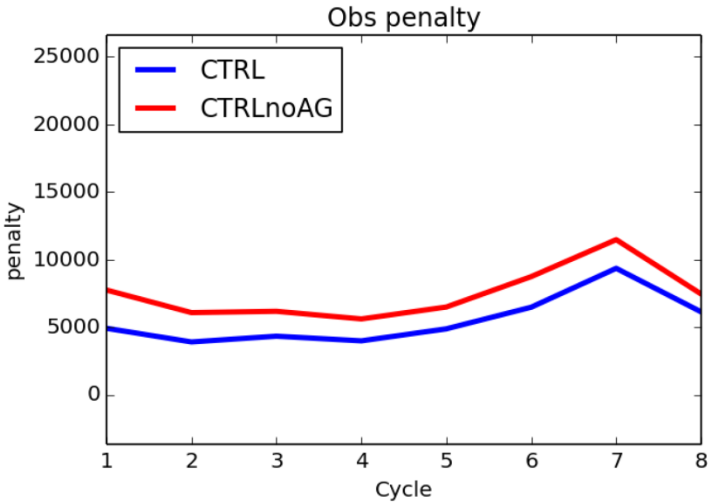
Grid point gravity waves





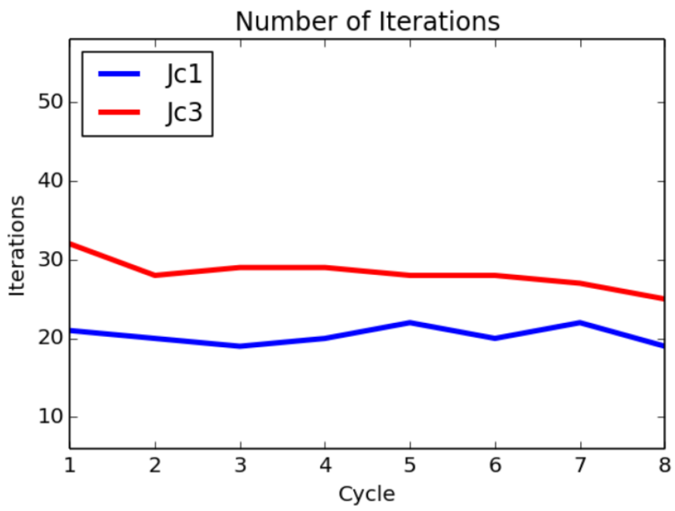
- Timeseries of pressure tendency at the same point.
- Strong impact from the imbalanced initial condition in CTRL.
- CTRLnoAG: less impact, some noise in the first 10 minutes.
- Jc1: impact comparable with CTRLnoAG and smoother.

Impact on the observation penalty term



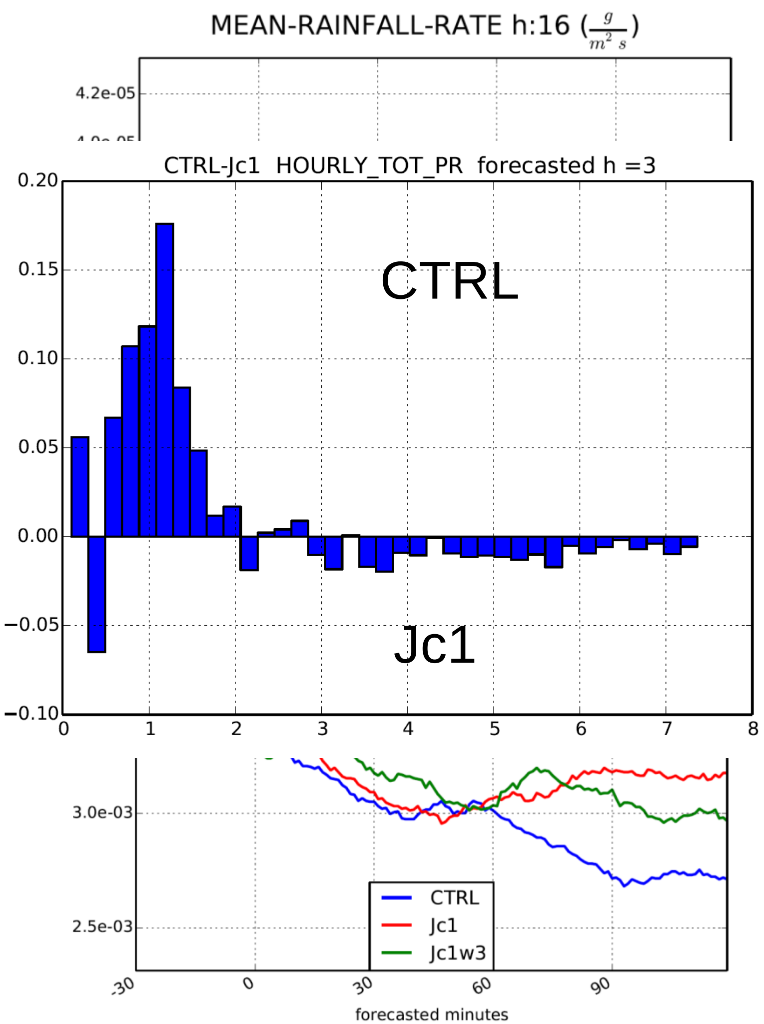
- The impact of the reduction in the spurious gravity waves should not degrade the observation penalty Jo.
- In the CTRL, the AG reduces the value of the Jo term, for every cycle.
- The Jo term for CTRL with AG and Jc1 is very similar.
- No differences are visible in the Jo term using different weights of the Jc term.

Number of iterations in the minimization



- The use of a Jc term can have an impact on the number of iterations during the minimization in 4D-VAR.
- Computational time is important. The parameters for the Jc term should take it into account.
- Comparing CTRL and CTRLnoAG, there are some differences only in the first few cycles.
- The introduction of Jc1 tends to increase the number of iterations by 50% in the first cycle, to around 0% after 8 cycles.
- A stronger Jc term (Jc3) leads to a higher amount of iterations.

One week test-run



- Hourly 4D-VAR from 25th to 29th June 2016. Rain rate.
- Mean: very similar, CTRL a larger spin down at the beginning of the assimilation window.
- 99th percentile: similar until T+60min, CTRL lower maximums after.
- CTRL: strong decrease in precipitation for higher values during the forecast.
- Distribution between T+2h and T+3h:
  - Subdivide precipitation values in bins.
  - Compute difference of frequency for every bin
  - Normalization with total frequency of difference
- Jc1 shows higher frequency in medium high precipitation, CTRL more for lower values.

Conclusions

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- 4D-VAR creates spurious gravity waves at the beginning of the assimilation window.
- The imbalances in the initial state are mainly due to the use of the vertically adaptive grid in 4D-VAR.
- A Jc term in the cost function is introduced.
- Jc aims to reduce gravity waves without increasing the observation penalty and with affordable computational costs.
- During a rainy summer week experiment, CTRL shows a spin down at the beginning of the assimilation window. Jc reduces it.

QUESTIONS?

Questions

- Incremental 4DVAR
- PF model and AVG
- Internal gravity waves
- Metric for internal gravity waves
- Case with real gravity waves
- Skills during the one-week test run
- qqplot for precipitation in one week test
- UKV extra observations assimilated
- New LBC

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Incremental 4D-VAR

- Based on the formulation of Rawlins et al. 2006

$$\begin{aligned} \mathbf{x}^a &= \mathbf{x}^g + \delta\mathbf{x} \\ \delta\mathbf{w} &= S(\mathbf{x}^g + \delta\mathbf{x}) - S(\mathbf{x}^g) \simeq \mathbf{S}\delta\mathbf{x} \\ \delta\mathbf{w}^b &= S(\mathbf{x}^b) - S(\mathbf{x}^g) \end{aligned}$$

- g, first guess; a, analysis; b, background; o observations
- *S* is a non-linear simplification operator with tangent linear approximation **S**
- 4DVAR Cost function, using the simplified increments (notation avoid sums)

$$J(\delta\mathbf{w}) = \frac{1}{2}(\delta\mathbf{w} - \delta\mathbf{w}^b)^T\mathbf{B}^{-1}(\delta\mathbf{w} - \delta\mathbf{w}^b) + \frac{1}{2}(\underline{\mathbf{y}} - \underline{\mathbf{y}^o})^T\underline{\mathbf{R}^{-1}}(\underline{\mathbf{y}} - \underline{\mathbf{y}^o})$$

Incremental 4D-VAR

- In the minimization a CVT (Control Variable Transform) is used.
- The **B** become an Identity
- New variable using CVT

$$\delta \mathbf{w} = \mathbf{U}\mathbf{v} = \mathbf{U}_p U_a U_v \mathbf{U}_h \mathbf{v}$$

$$J_b = \frac{1}{2}(\mathbf{v} - \mathbf{v}^b)^T(\mathbf{v} - \mathbf{v}^b)$$

$$J_o = \frac{1}{2}(\underline{\mathbf{y}} - H(\mathbf{x}^g) - \mathbf{H}\mathbf{U}\underline{\mathbf{v}})^T \underline{\mathbf{R}^{-1}}(\underline{\mathbf{y}} - H(\mathbf{x}^g) - \mathbf{H}\mathbf{U}\underline{\mathbf{v}})$$

- $\mathbf{U}_a$, is the vertically adaptive grid transform (AG; Piccolo and Cullen, 2011)

PF model and vertically adaptive grid

- The PF model is hydrostatic:

$$\frac{-\partial p}{\partial z} = g\rho$$

- Given the geostrophic balance. The geostrophic wind is normal to pressure gradient and proportional to it

$$f \cdot \vec{e}_z \times \vec{u} + \frac{1}{\rho}\vec{\nabla} p = 0 \Rightarrow \vec{u}_g = \frac{1}{\rho f}\vec{e}_z \times \vec{\nabla} p \Rightarrow \left(\frac{-1}{\rho f}\frac{\partial p}{\partial y}; \frac{1}{\rho f}\frac{\partial p}{\partial x} \right)$$

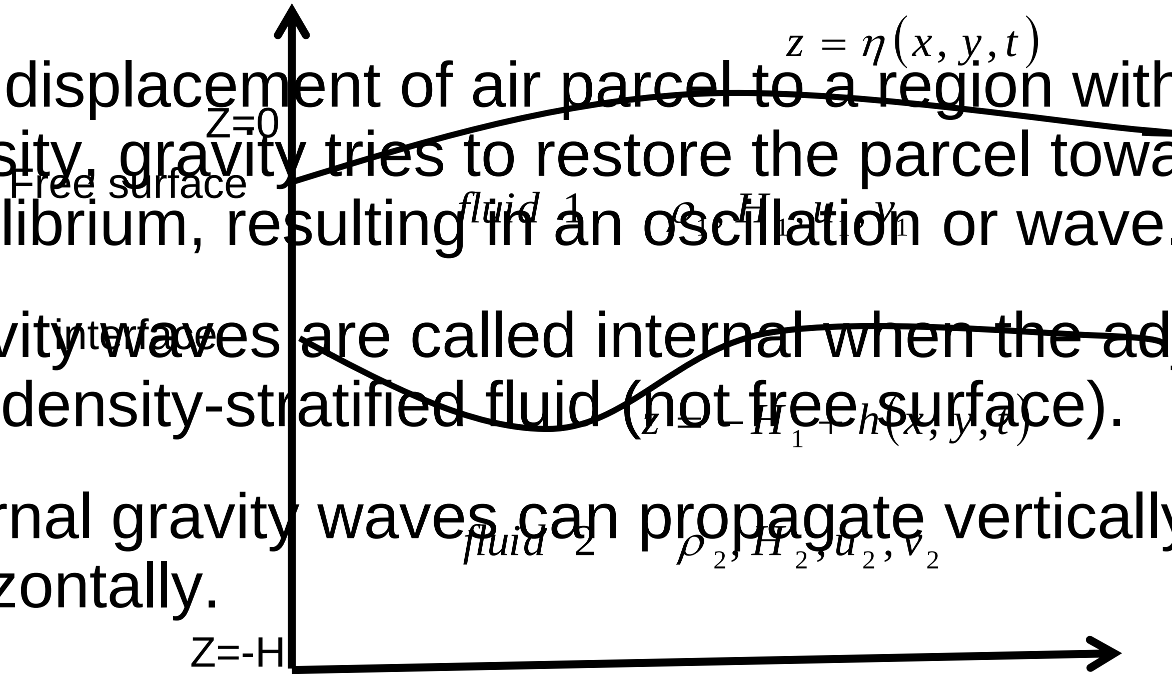
- Using the hydrostatic balance

$$\left.\frac{\partial p}{\partial x}\right|_z = g\rho \left.\frac{\partial z}{\partial x}\right|_p \qquad \left.\frac{\partial p}{\partial y}\right|_z = g\rho \left.\frac{\partial z}{\partial y}\right|_p$$

- Changing in the z planes can lead to unbalancing in the wind and then to gravity waves

Internal gravity waves

- In a displacement of air parcel to a region with different density, gravity tries to restore the parcel toward equilibrium, resulting in an oscillation or wave.
- Gravity waves are called internal when the adjustment are in a density-stratified fluid (not free surface).
- Internal gravity waves can propagate vertically as well as horizontally.



Internal gravity waves

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- A parcel with density ρ_0 moving in an environment of density ρ_z:

$$\rho_0 \frac{\partial^2 z'}{\partial t^2} = -g\left(\rho_0 - \rho\left(z'\right)\right)$$

$$\rho\left(z\right) - \rho_0 = \frac{\partial \rho\left(z\right)}{\partial z} z'$$

$$\Rightarrow \frac{\partial^2 z'}{\partial t^2} = \frac{g}{\rho_0} \frac{\partial \rho\left(z\right)}{\partial z} z' \Rightarrow z' = z'_0 e^{\sqrt{-N^2}t} \qquad N = \frac{g}{\theta} \sqrt{\frac{d\theta}{dz}}$$

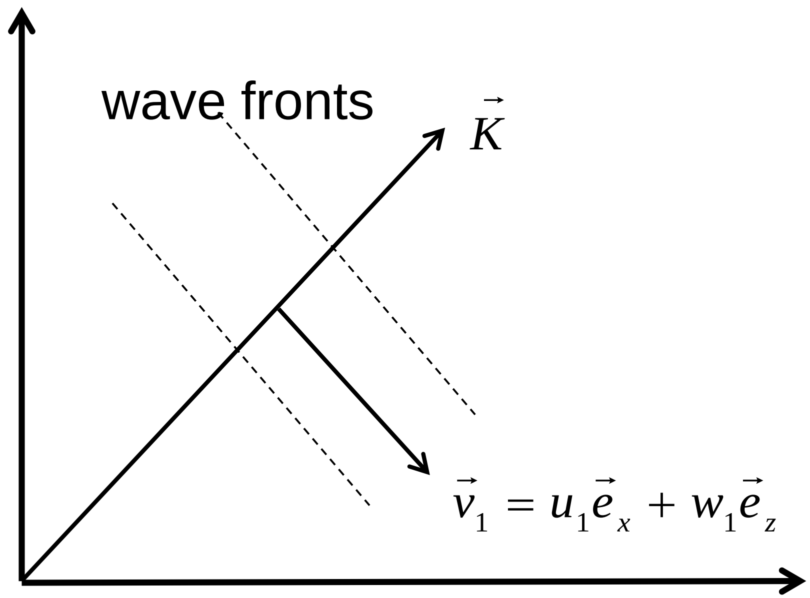
z’ vertical displacement from equilibrium
N Brunt-Vaisala frequency, angular frequency of oscillation of the air parcel

A measure for internal gravity waves

- Mean absolute pressure tendency at level 1, relates the change in surface pressure to the vertically integrated mass divergence of the overlying air column (Trexler and Koch, 2000).

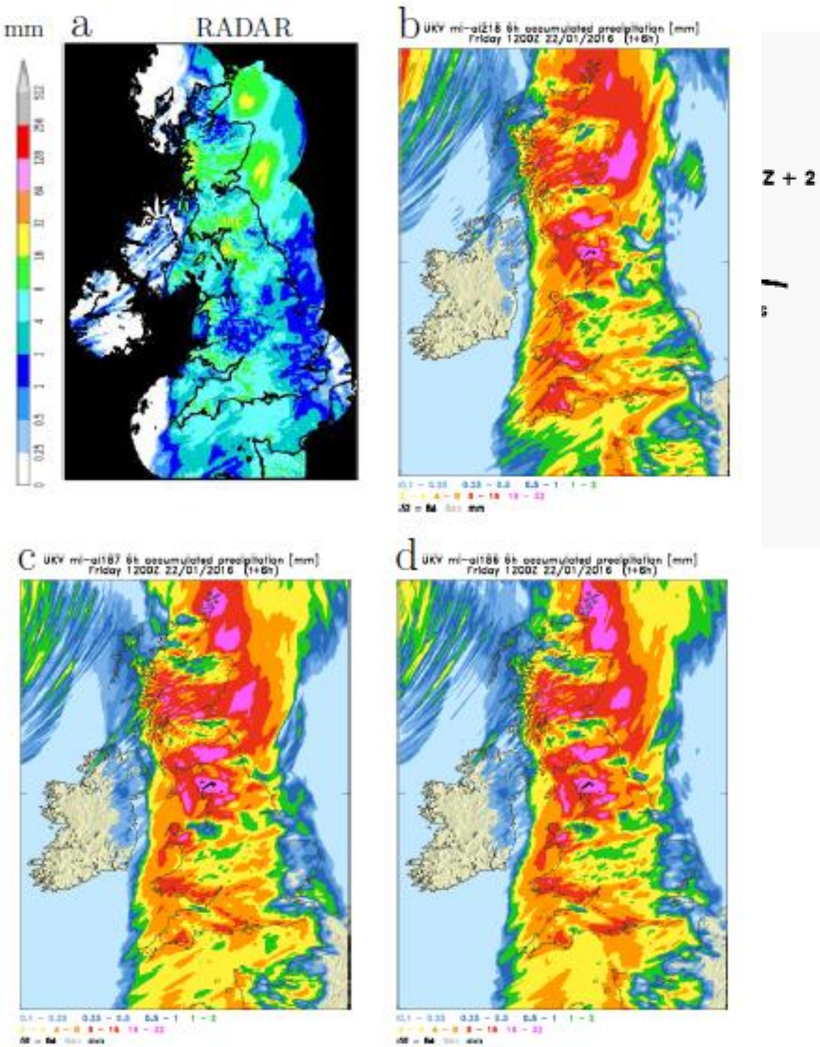
$$\frac{\partial p_{sfc}}{\partial t} \approx -\int_0^{ptop} (\nabla \cdot \vec{v})\,dp$$

$$w_{sfc} = 0 \qquad w_{top} = 0$$



- Using the horizontal velocity vector, $\vec{K}$ the wave vector

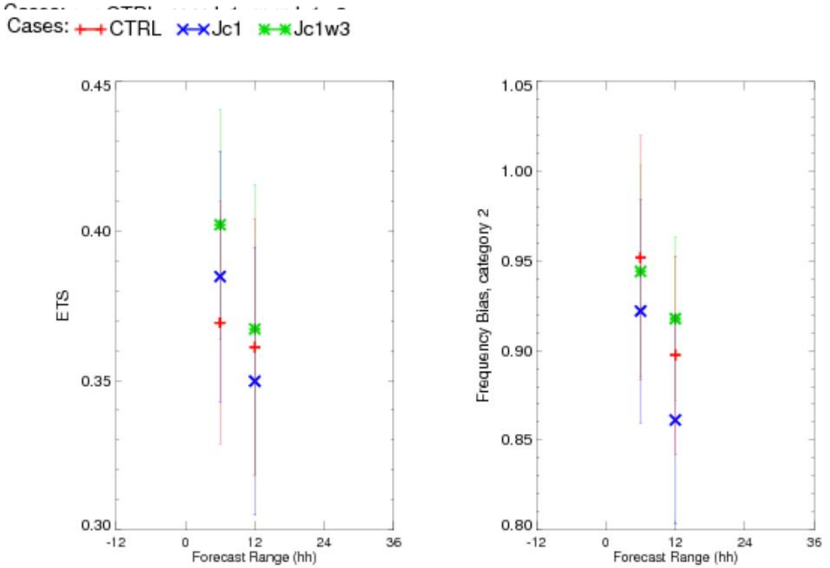
Real gravity waves



- Front system: area favourable for gravity waves in the exit region of the upper level jet streak
- Flow strongly diffluent and unbalanced
- 6hour accumulated precipitation
- All assimilation method similar results
- a) 3h 3D-VAR, b) CTRL c) Jc1; all forecast local maximum of precipitation.

Skills during the one-week test run

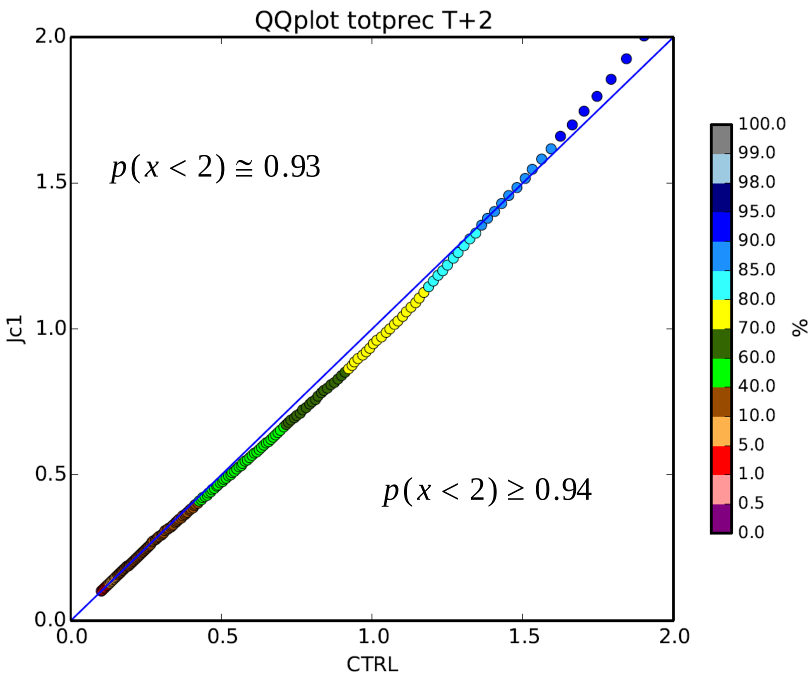
4mm
0.2mm



- Accumulated precipitation over 6 hours:
  - Low threshold: similar results for ETS and FBIAS after 6 and 12 hours.
  - Higher threshold: CTRL some better values after 12 hours, but inside confidence intervals
- Surface temperature and surface wind give very similar results
- Locally some positive impacts, not considering the whole area and averaging over a week.

Courtesy of Bruce Macpherson

One week test-run qq plot



- qq plot instead of relative distribution.
- Points with precipitation greater than 0.1mm/h, accumulated between T+2h and T+3h.
- Jc1 shows higher probabilities for values of high precipitation
- CTRL higher probabilities for values between 1.5mm/h and 2mm/h, but lower for precipitation between 0.5mm/h and 1.5mm/h

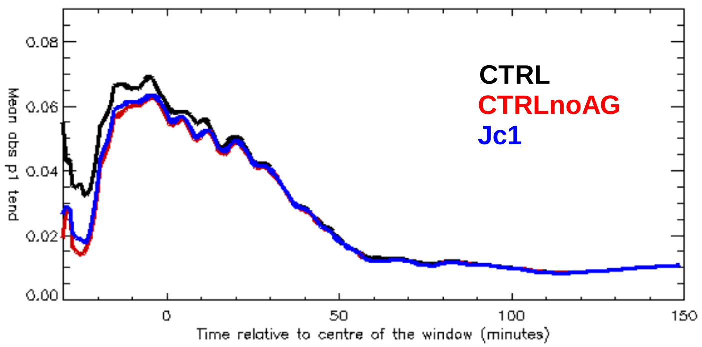
Courtesy of Bruce Macpherson

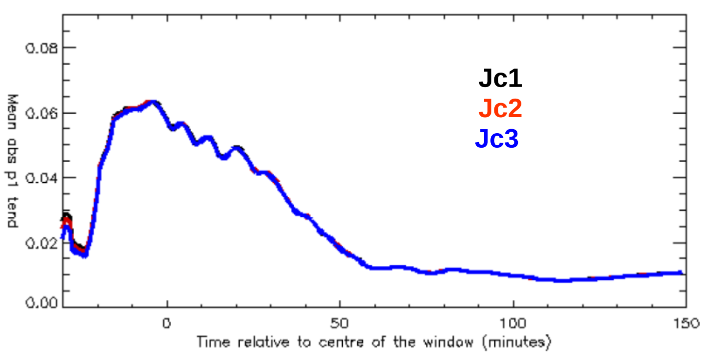
UK 1.5km – extra observations *not* assimilated in global model

- Radar-derived surface rain rate (hourly, 5km resolution)
- Visibility from SYNOPs (hourly)
- T_{2m} & RH_{2m} from roadside sensors (hourly)
- Doppler radial winds (3-hourly)
- SEVIRI Channel 5 radiances above low cloud
- High-resolution AMVs from MSG
- GeoCloud cloud fraction profiles (3-hourly, 5km resolution)
  - zero cloud down to cloud top, missing data below
- Cloud fraction profiles from SYNOPs (3-hourly)
  - zero cloud up to cloud base, missing data above

Courtesy of Bruce Macpherson

Introduction of new LBC





DA window

- Pressure tendency at level 1.
- Run at 15Z using the LBC at 12UTC: problems in the gravity waves after 10 minutes of run. Solution in other tests using smooth BC.
- The use of the vertically adaptive grid (AG) introduces instabilities at the beginning of the assimilation window
- CTRL: run with AG and without Jc term.
- CTRLnoAG: run without AG and without Jc term.
- Jc1: run with AG and with Jc1.
- CTRLnoAG and Jc1 have less gravity waves than CTRL for the first 50 minutes.
- No differences are visible using different Jc approaches