Modelling spatial correlations for observation errors with Lanczos : application to SEVIRI and RADAR data

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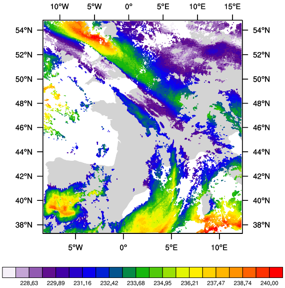
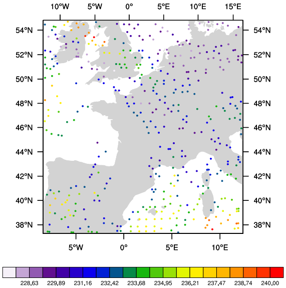
Introduction

- Horizontal correlations are not represented into the obs. error covariance matrix **R**.
- This certainly contributes to the undersampling of observations in our assimilation schemes.

Example with MSG/SEVIRI data in AROME



(a) Non-cloudy obs. (3km)



(b) Assimilated obs. (70km)

Introduction

Context

- We need $\mathbf{R}^{-1}$ when writting the gradient of the cost-function.
- $\mathbf{R}$ (and $\mathbf{R}^{-1}$) can probably be represented in block-diagonal form ; each block corresponding to an independent instrument.
- Even though, dimensions are big :

SEVIRI one channel is $p \sim 3712^2 \sim 10^7$ over the globe ; $p \sim 4 \cdot 10^5$ for AROME.

RADAR One elevation from a single radar gives $p \sim 512^2 = 2 \cdot 10^5$.

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Possible approaches

- Finite elements version of the diffusion equation [Lindgren et. al 2011].
- Modelling of $\mathbf{R}$ with interpolations and estimating a truncated inverse with Lanczos [Fisher 2014].

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Context – II

- Modelling interchannel correlations (with direct methods) for infrared sounders has proven useful [Weston et.al 2014];
- Estimating **R** may be possible with the diagnostic from [Desroziers et. al 2005].
- Progress has been made for SEVIRI radiances and radial wind from Doppler RADAR [Waller et. al 2016].

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Goal of this study

Evaluate the approach proposed by [Fisher 2014]:

- in a limited area context (for AROME) ;
- for SEVIRI and RADAR data.

Outline

1. Estimating spatial correlations in **R**
2. Modelling **R** with interpolations
3. Lanczos-based truncated inverse

Estimating spatial correlations in **R** : SEVIRI

Following [Desroziers et. al 2005]:

$$\mathbf{R} \approx \mathbb{E}(\mathbf{d}_a \mathbf{d}_b^\top)$$

Correlations in **R** are estimated through a time average/median.

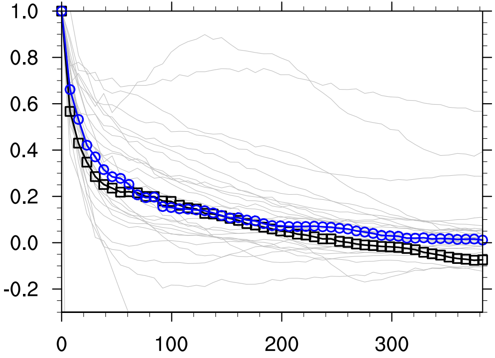

Figure : Application of the Desroziers diagnostic to the 6.2μm WV channel from SEVIRI, as in [Guedj et.al 2014].

Estimating spatial correlations in **R** : RADAR

[Waller et. al 2016]

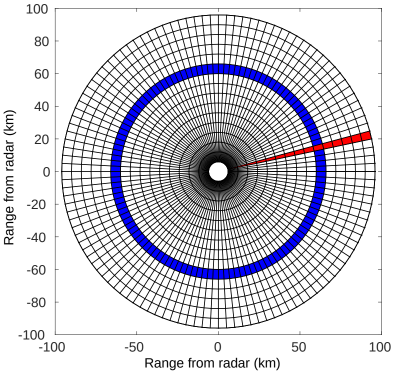

Figure : Geometry

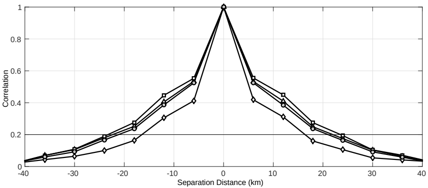

Figure : Azimuthal direction

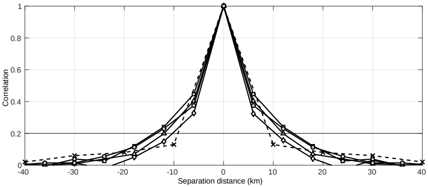

Figure : Radial direction

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Modelling $\mathbf{R}$ with interpolations

[Fisher 2014] has proposed the following square-root model :

$$\mathbf{R} = \mathbf{U}\mathbf{U}^{\mathsf{T}}$$

where $\mathbf{U}$ is a sequence of operators :

$$\mathbf{U} = \mathbf{\Sigma_o}\mathbf{P}\mathbf{S}^{-1}\mathbf{D}\mathbf{S}$$

where :

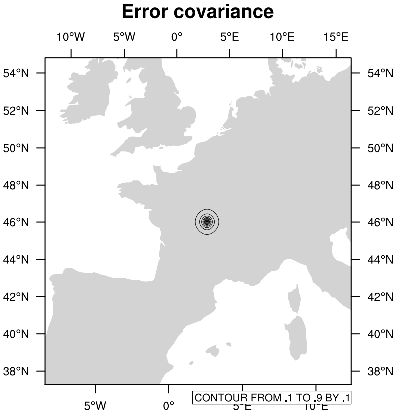
- $\mathbf{S}^{-1}\mathbf{D}\mathbf{S}$ is a spectral based correlation model ;
- $\mathbf{P}$ is the interpolation from a regular grid to the observation space ;
- $\mathbf{\Sigma_o}$ is the multiplication by the standard deviations.

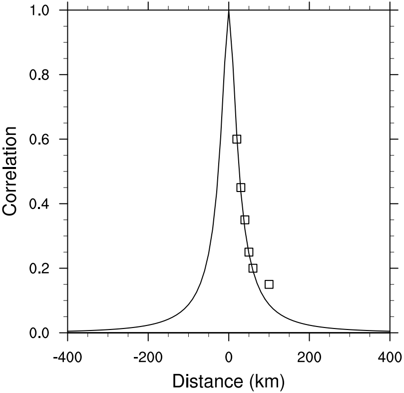
Note :

- the regular grid may be of coarse resolution ;
- we do not need to cover the whole analysis domain ;
- we can use alternative (gridpoint) correlation models.

Modelling **R** with interpolations : SEVIRI

- Define a grid covering the AROME domain, at 10 km spatial resolution (240 $\times$ 256 points).
- Use non-periodic hyperGaussians recursive filters ($\sigma = 60$ km, $\gamma = 5$, Purser et. al 2003b).





Modelling **R** with interpolations : SEVIRI

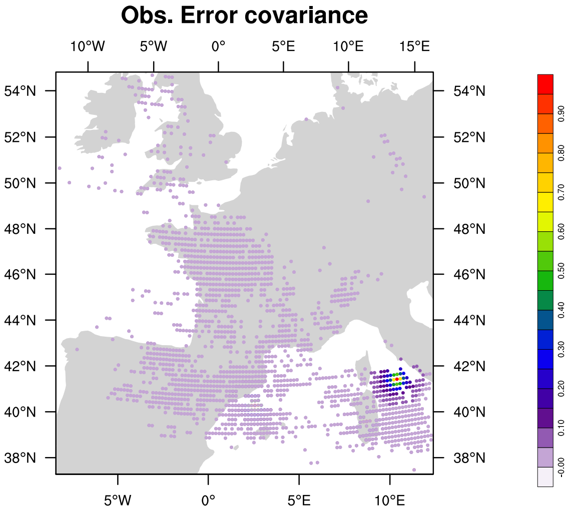
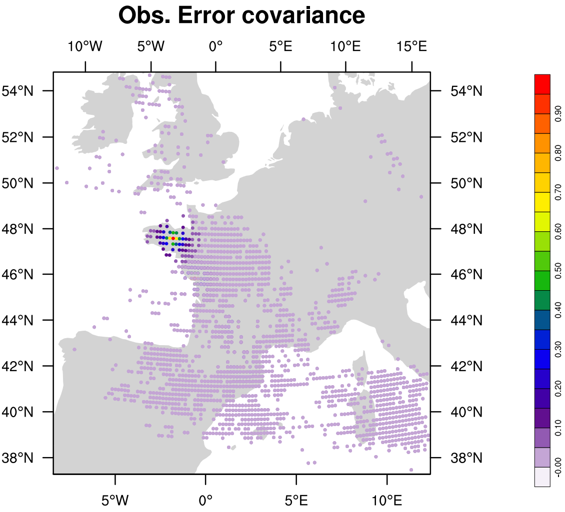


Fig: Spatial correlations modelled in **R**.

Modelling **R** with interpolations : SEVIRI



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Modelling **R** with interpolations : SEVIRI

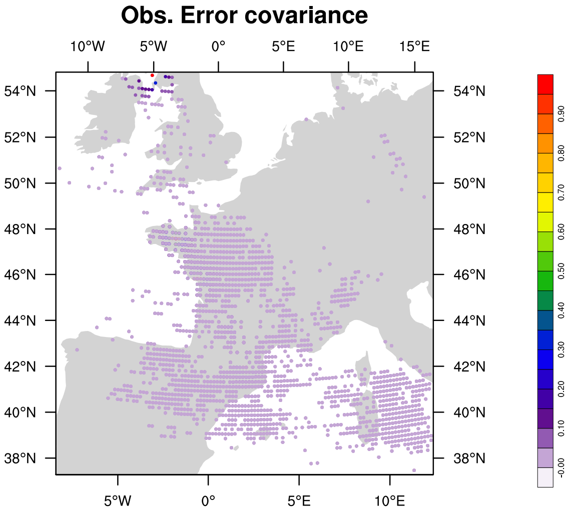
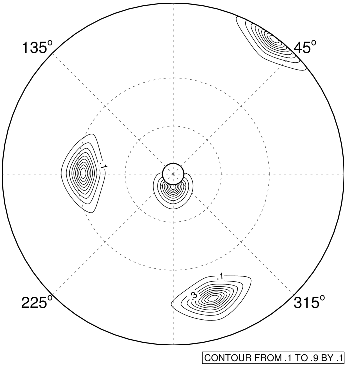


Fig: Spatial correlations modelled in **R**.

Modelling **R** with interpolations : RADAR

- Grid covering the radar only.
- Convolution with 1D recursive filters ;
- Periodic in the azimuth, non-periodic in the radial direction.



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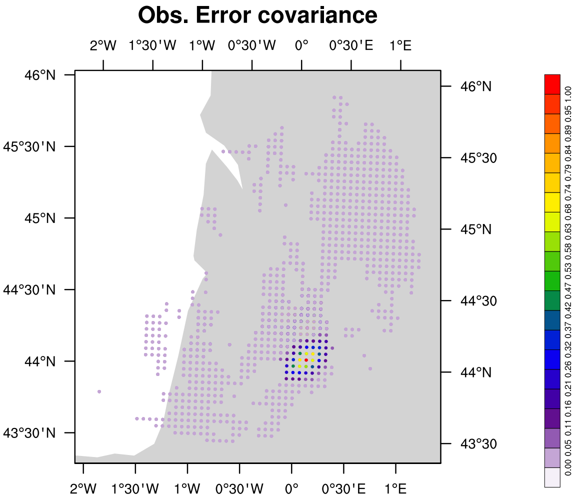

Fig: Spatial correlations modelled in **R**.

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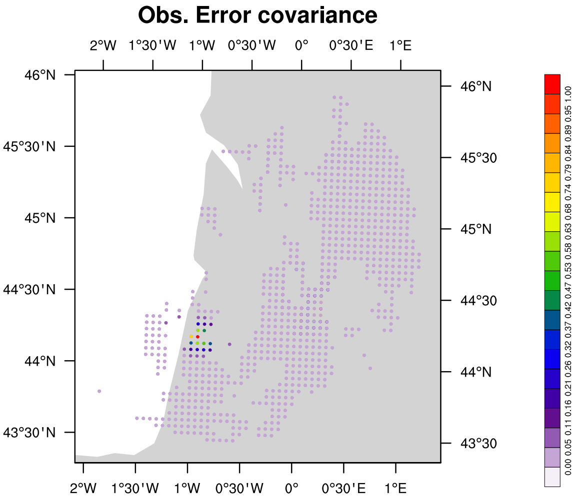

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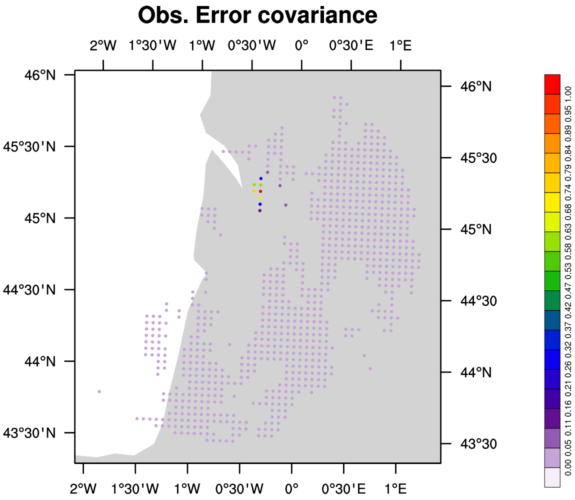

Fig: Spatial correlations modelled in **R**.

Outline

Lanczos-based truncated inverse

A reduced rank approximation of **R** may be obtained from the Lanczos algorithm :

$$\mathbf{R} \approx \mathbf{\Sigma}_o \left(\sum_{k=1}^{K} \lambda_k \mathbf{v}_k \mathbf{v}_k^{\mathrm{T}} \right) \mathbf{\Sigma}_o$$

When $K < p$, this approximation can be regularized :

$$\mathbf{R} \approx \widehat{\mathbf{R}} \equiv \mathbf{\Sigma}_o \left(\alpha \mathbf{I} + \sum_{k=1}^{K} (\lambda_k - \alpha) \mathbf{v}_k \mathbf{v}_k^{\mathrm{T}} \right) \mathbf{\Sigma}_o$$

allowing the explicit inversion formula :

$$\widehat{\mathbf{R}}^{-1} = \mathbf{\Sigma}_o^{-1} \left(\alpha^{-1} \mathbf{I} + \sum_{k=1}^{K} (\lambda_k^{-1} - \alpha^{-1}) \mathbf{v}_k \mathbf{v}_k^{\mathrm{T}} \right) \mathbf{\Sigma}_o^{-1}$$

Lanczos-based truncated inverse

The regularisation parameter α is chosen to conserve the total trace :

$$\mathrm{Tr}[\mathbf{R}] = \mathrm{Tr}[\widehat{\mathbf{R}}] \Longrightarrow \alpha = \frac{p - \sum_{k=1}^{K} \lambda_k}{p - K}$$

α is one minus the fraction of variance explained by the first K eigenvectors.

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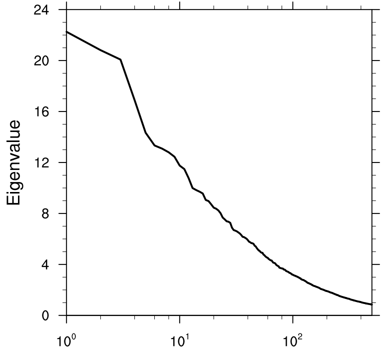
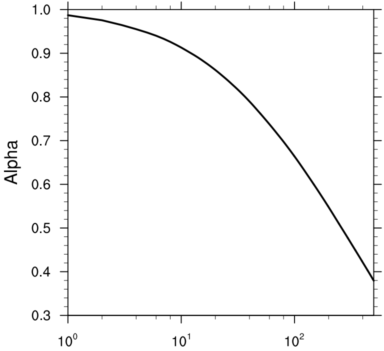


Eigenspectrum λ_k – SEVIRI



$\alpha(K)$ – SEVIRI

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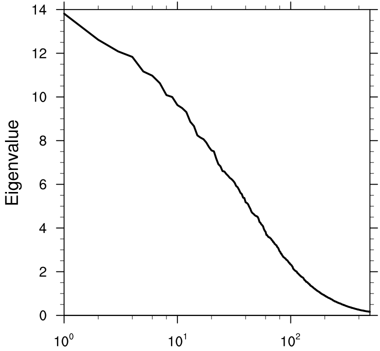
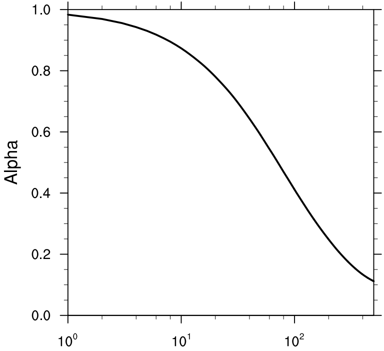


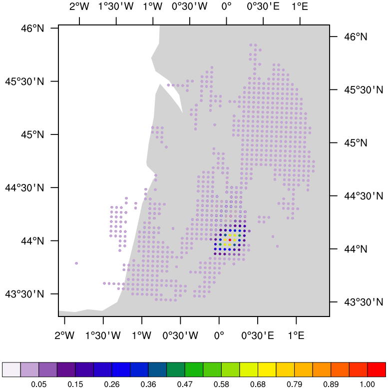
Eigenspectrum λ_k – RADAR



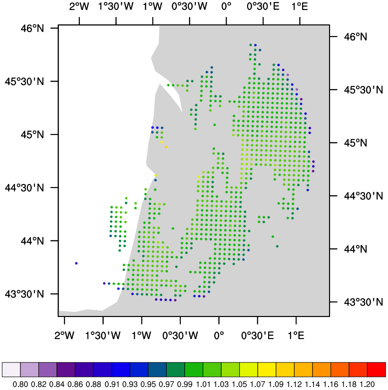
$\alpha(K)$ – RADAR

Effects of the truncation – RADAR

Truncation $K = 500$



Spatial correlations



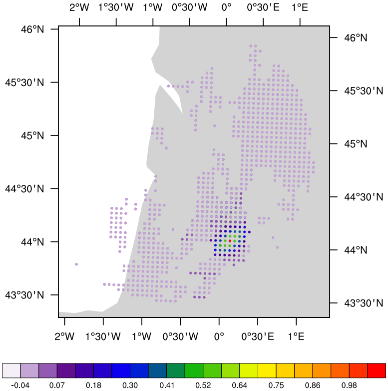
Variances

Truncating the spectrum introduces :

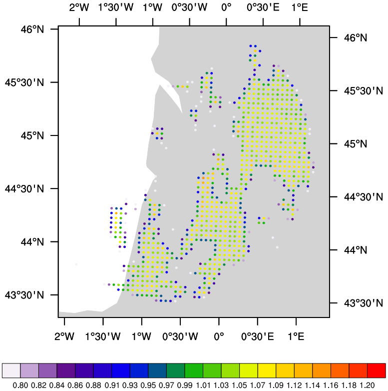
- erroneous long range correlations ;
- errors in the variances.

Effects of the truncation – RADAR

Truncation $K = 100$



Spatial correlations



Variances

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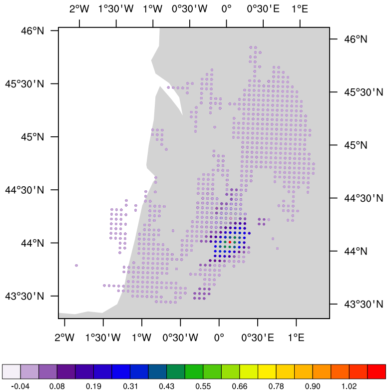
Effects of the truncation – RADAR

Truncation $K = 50$



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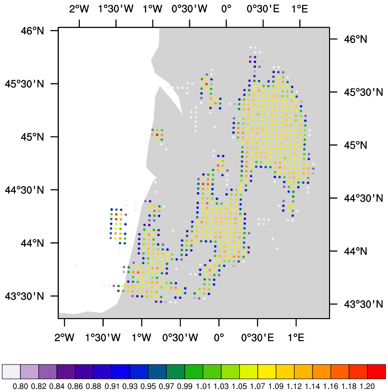

Variances

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Conclusions

Estimation : based on Desroziers' diagnostic, both SEVIRI and RADAR observations are significantly correlated.

Modelling : we can build a spatial correlation model to represent those correlations in observation space.

Inverse : use of the Lanczos algorithm [Fisher 2014]...

- may require a large number of eigenvectors (*e.g.*, 500).
- may introduce erroneous long range correlations / wrong variances if truncation is too severe.

but otherwise works !

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