Vertical Localization for EnKF radiance assimilation

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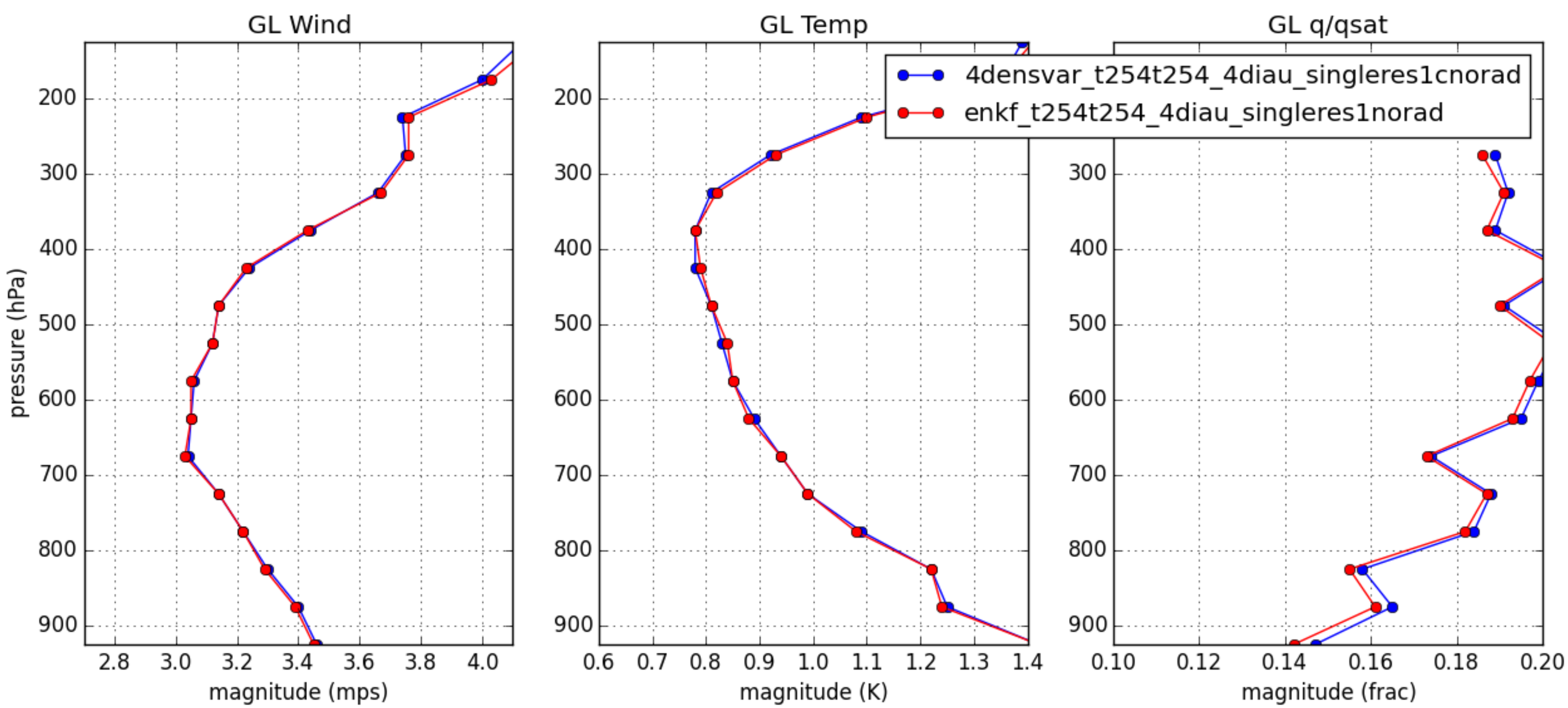
Motivation (1)

- ***Hypothesis***: 4DEnVar (non-hybrid) and EnKF should perform similarly if all 'extra' constraints turned off in Var solver.
- ***Experiment***: T254 single resolution 4DEnVar (no static **B**, balance constraint) vs 'pure' EnKF (80 members, operational localization settings).

Motivation (2)

4DEnVar vs EnKF, no radiances

RMS innovations for background

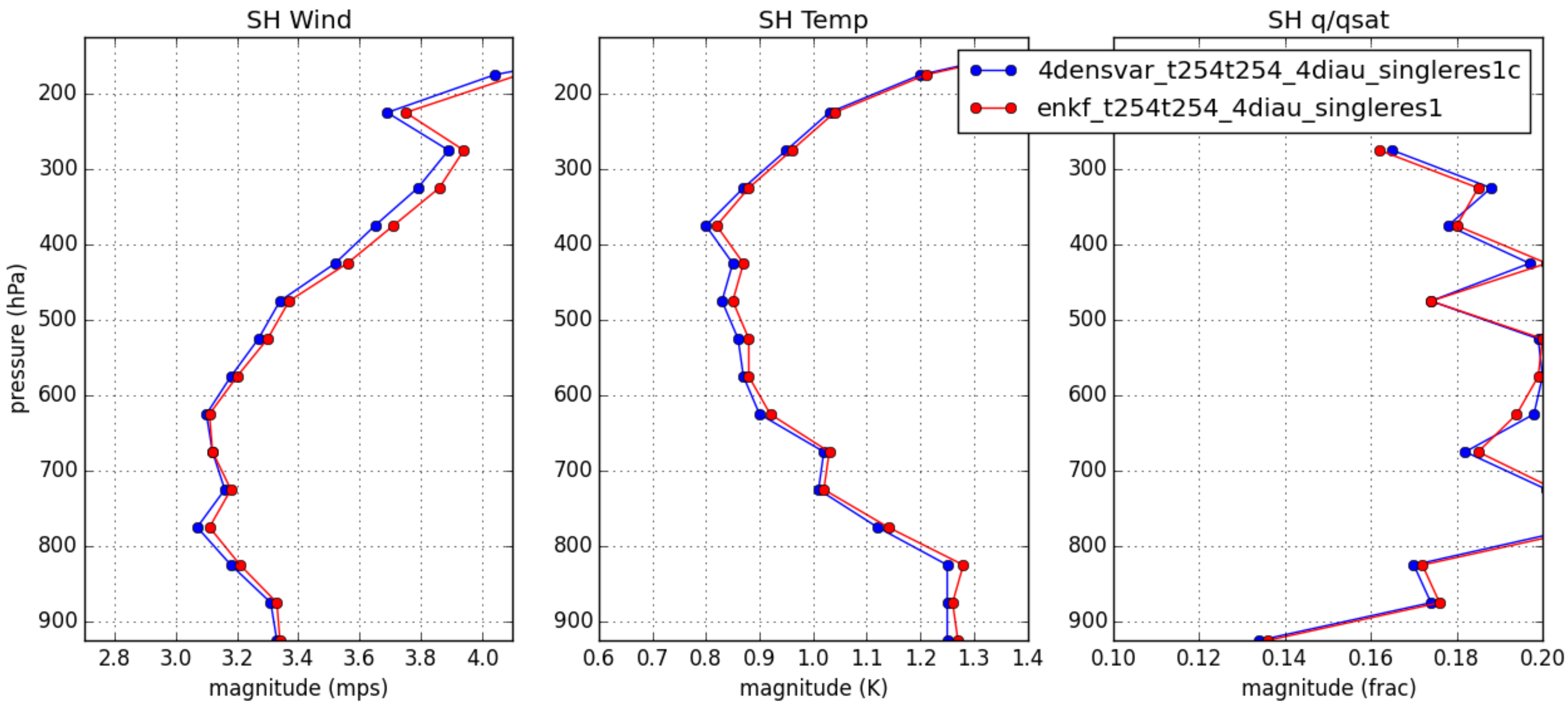


Virtually no difference, EnKF (red) perhaps slightly better for humidity

Motivation (2)

4DEnVar vs EnKF, including radiances

RMS innovations for background



4DEnVar (blue) slightly better, esp in SH. Why?
Hypothesis: Difference in vertical localization

Model and Observation-Space Localization

Model space:

- state-space covariances are tapered.
- Involves distances between state variables only.
- **H** applied after.
- EnVar algorithms use this form.

$$\mathbf{K} = (\rho_m \circ \mathbf{P}^f)\mathbf{H}^{\mathrm{T}}[\mathbf{H}(\rho_m \circ \mathbf{P}^f)\mathbf{H}^{\mathrm{T}} + \mathbf{R}]^{-1}$$

Observation space:

- Computed after **H** applied.
- Involves distances between state and observation space quantities.
- Much simpler to implement in EnKF systems.

$$\mathbf{K} = [\rho_{o1} \circ (\mathbf{P}^f\mathbf{H}^{\mathrm{T}})][\rho_{o2} \circ (\mathbf{H}\mathbf{P}^f\mathbf{H}^{\mathrm{T}}) + \mathbf{R}]^{-1}$$

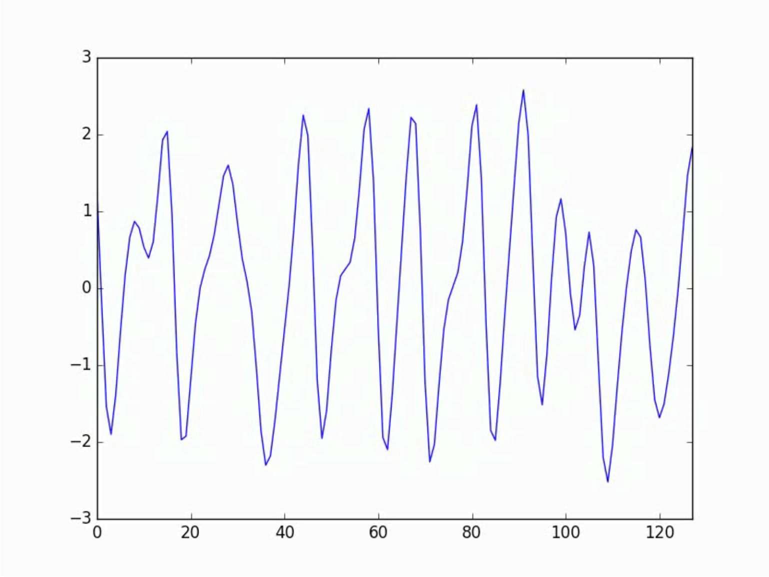
Experiments with a simple model

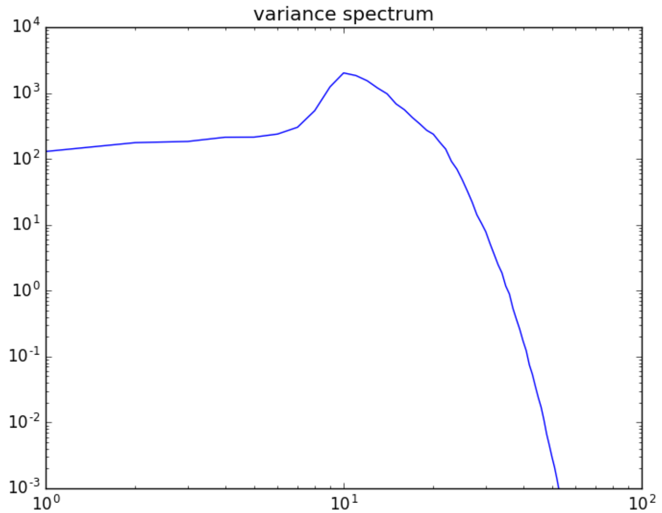
- 1-d Kuramoto-Sivashinsky equation (https://www.encyclopediaofmath.org/index.php/Kuramoto-Sivashinsky_equation), one of the simplest PDEs that exhibits spatio-temporal chaos.

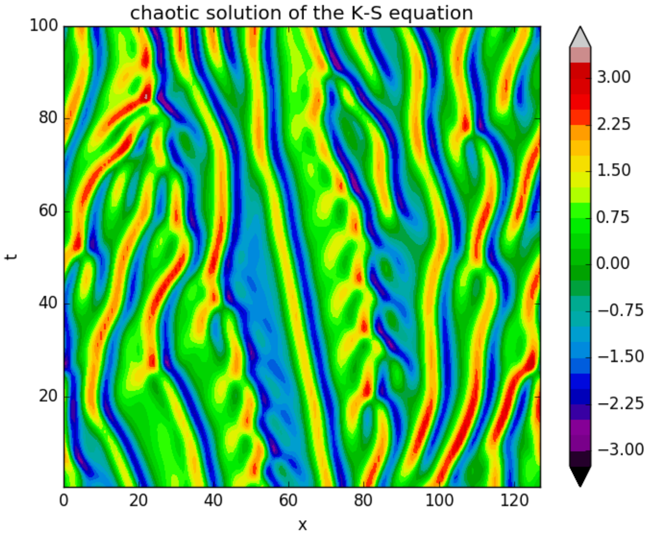
- u_t + u*u_x + u_xx + d*u_xxxx = 0, periodic BCs on [0,2*pi*L].
  - Energy enters the system at long wavelengths via u_xx (an unstable diffusion term)
  - cascades to short wavelengths due to the nonlinearity u*u_x
  - dissipates via d*u_xxxx
  - Solved with spectral method, using N Fourier collocation points.

- Python code available at https://github.com/jswhit/pyks.git

Nature run: L=16, N=128, dt=0.5, d=1 (semi-implicit RK3 scheme)







Data assimilation experiments

- 10 members, assimilation every 4 time steps (2 time units).
- Forward observation operator (**H**) includes an averaging kernel, either Gaussian or boxcar (running average). **R**=0.1.
- Serial EnSRF with either ***observation space*** or ***model space*** localization. Observation 'location' assumed to be center of averaging kernel.
- Multiplicative inflation using Hodyss et al 2016 (*dx.doi.org/10.1175/MWR-D-15-0329.1*) algorithm.

‘Modulated ensemble’ approach to model-space localization in the EnKF

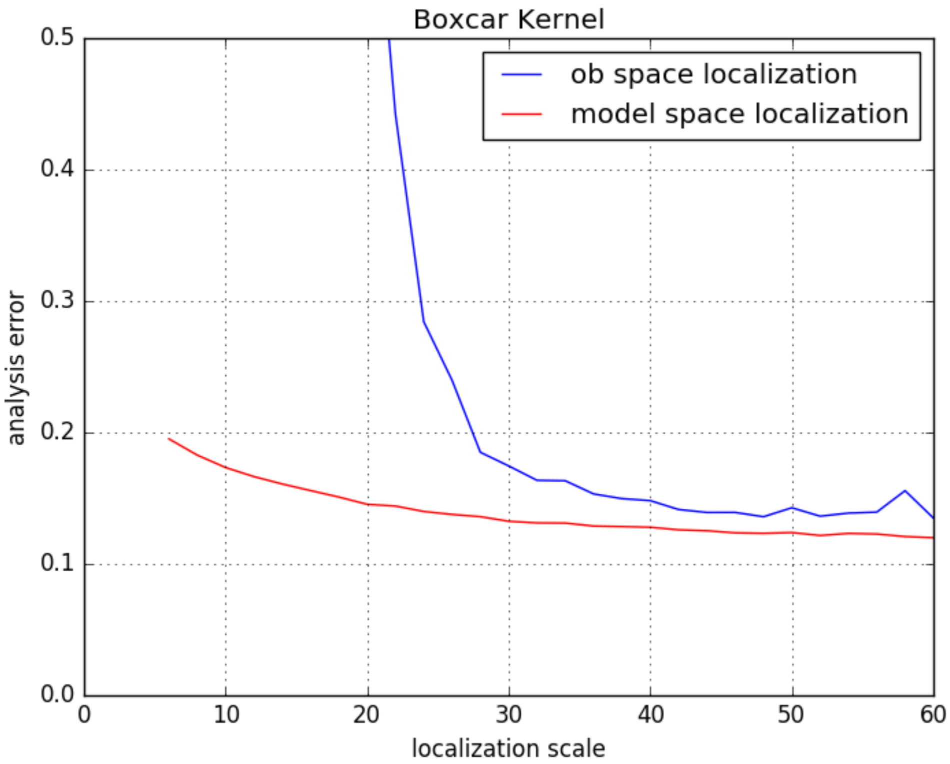
Model space localization is $\mathbf{P}_{loc} = \boldsymbol{\rho} \circ \mathbf{P}_{sample}$ ($\circ$ denotes element-wise product)

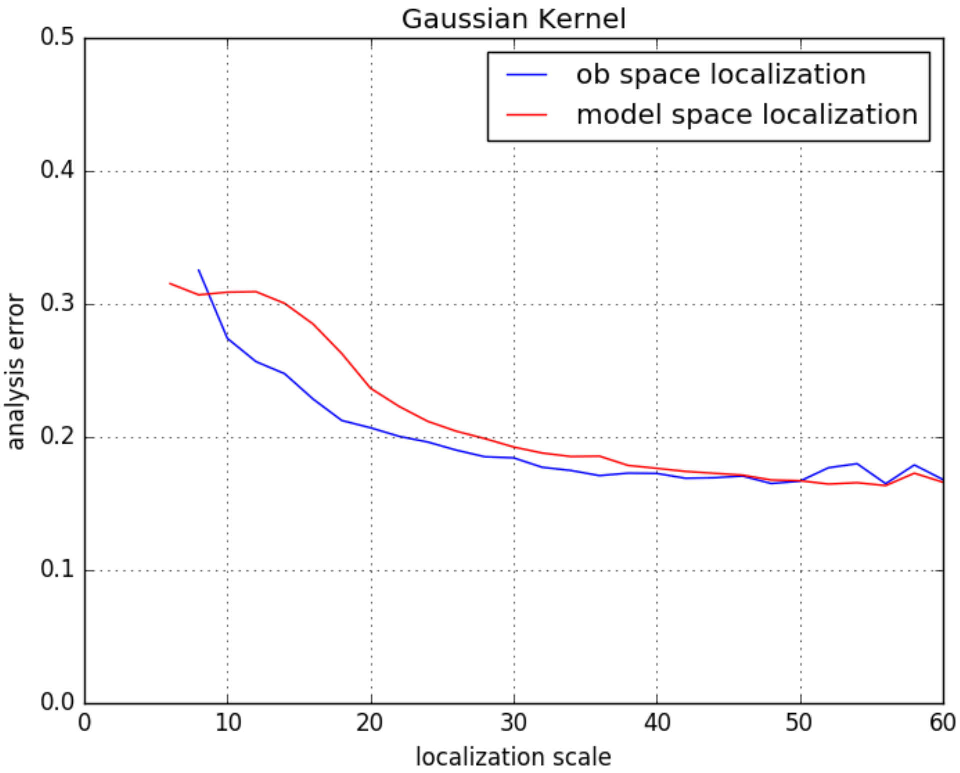
Let $\mathbf{P}_{sample} = \mathbf{X}\mathbf{X}^T$, $\boldsymbol{\rho} = \mathbf{L}\mathbf{L}^T$, then $\mathbf{P}_{loc} = \mathbf{L}\mathbf{L}^T \circ \mathbf{X}\mathbf{X}^T = \mathbf{Z}\mathbf{Z}^T$, where $\mathbf{Z} = \mathbf{L} \, \Delta \, \mathbf{X}$ and Δ denotes ‘modulation product’ (Bishop and Hodyss, 2008[1])

If **X** has *N* columns (ens. members) and **L** has *M* columns (eigenvectors), then $\mathbf{Z} = [[X_1 \circ L_1, X_2 \circ L_1, \ldots, X_N \circ L_1], [X_1 \circ L_2, X_2 \circ L_2, \ldots, X_N \circ L_2], \ldots, [X_1 \circ L_M, X_2 \circ L_M, \ldots, X_N \circ L_M]]$

[1] *doi: 10.1111/j.1600-0870.2008.00372.x*

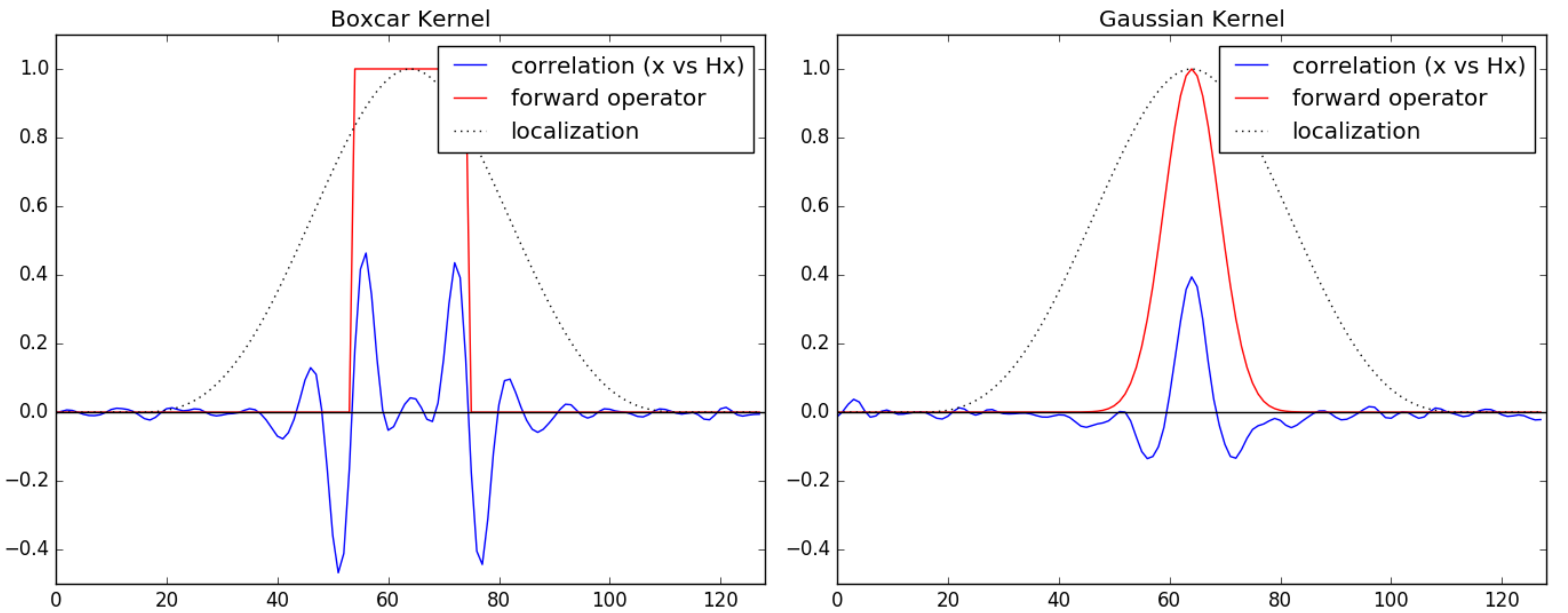
RMS analysis error as a function of localization scale





- *model space localization works better for Boxcar kernel.*
- *ob space localization works better (for short localization scales) for Gaussian kernel.*
- Why?

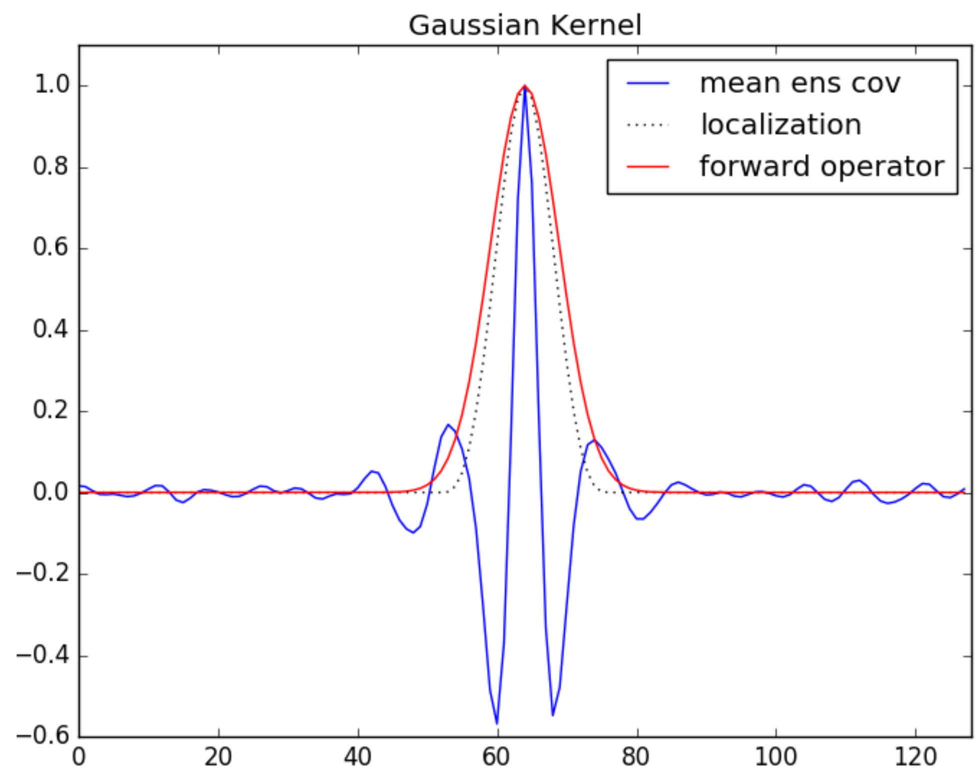
Mean correlation between ob priors and state priors (using localization=50)



- *correlation between state and ob space is maximum at edge of Boxcar kernel (not center).*

Why is ob space localization better in some circumstances?

- Localization applied to $\mathbf{P}^b$ removes some of negative side lobes.
- **H** operator applied to localized covariance then produces too large a value for **K**..
- See Lei and Whitaker 2015: DOI: dx.doi.org/10.1175/MWR-D-14-00413.1
- Symptom (seen with some satellite obs): single-ob experiment with/without localization, increment is larger *with* localization than without.



Conclusions (simple model expts)

- Model space localization performs better when correlation between ob prior and model priors is not maximum at center of averaging kernel (nominal 'ob location'). Advantage is even larger than shown by Campell et al 2009 (dx.doi.org/10.1175/2009MWR3017.1).
  - Ob space localization with 'empirical localization functions' (ELFs) can work in this circumstance …

Example of Empirical Localization Function (ELF) with Boxcar Kernel **H** (Lorenz 40 variable model) from Anderson and Lei 2013: http://dx.doi.org/10.1175/MWR-D-12-00330.1

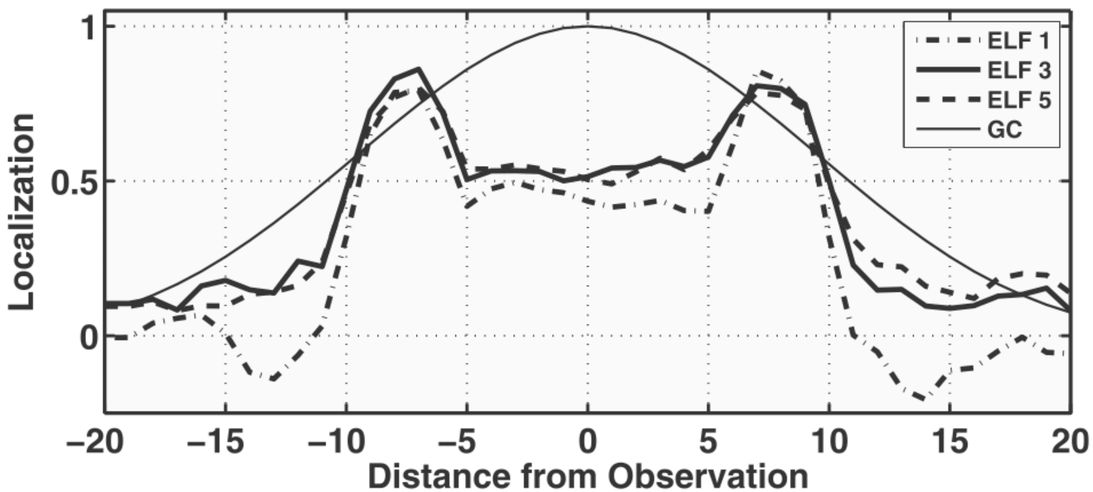


FIG. 9. The first, third, and fifth empirical localization functions and the Gaspari–Cohn function with the smallest associated RMSE for 320 observations of the sum of 17 state variables with observation error variance 1 assimilated every time step with 20-member ensembles. The units for the horizontal axis are the number of grid intervals separating the state variable from the observation.

Conclusions (simple model expts)

- Model space localization performs better when correlation between ob prior and model priors is not maximum at center of averaging kernel (nominal 'ob location').
- If correlation is maximum at center of averaging kernel, ob space localization can perform better if there are negative side lobes in correlation and localization scale is short (Lei and Whitaker 2015: DOI: dx.doi.org/10.1175/MWR-D-14-00413.1).

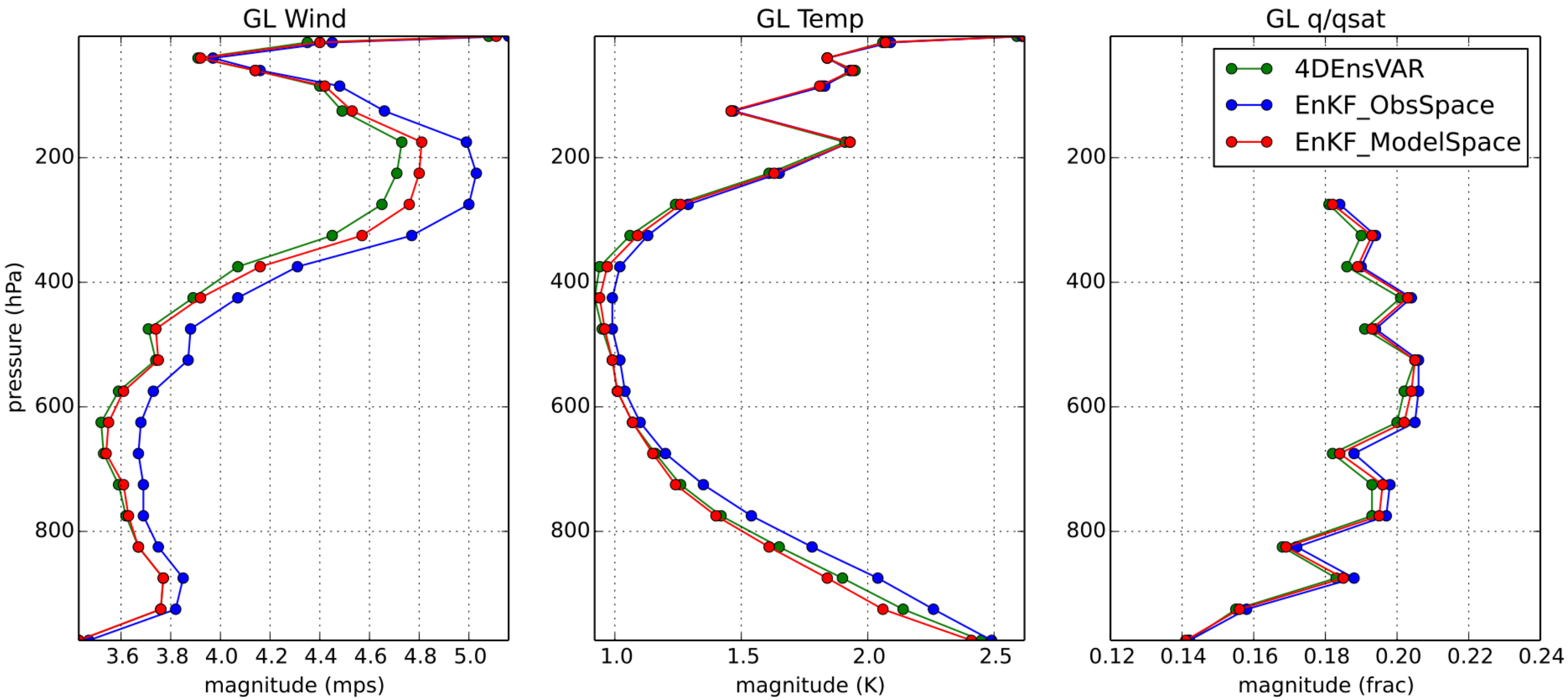
EnKF radiance assimilation with model-space localization

- As before, with GFS T254 80 member ensemble, but using model space localization
- ‘Modulated ensemble’ approach used to implement model-space localization *in the vertical only.*

‘Modulated ensemble’ approach to model-space vertical localization

- Assume localization is separable. Perform horizontal localization in ob space, vertical in model space.
- Truncate the vertical localization matrix, retaining the *M* eigenvectors that explain 90-95% of the variance (*M* is O(10) for the current operational configuration, *N*=80). The ‘modulated ensemble’ then contains *MN* members.
  - ***Horizontal localization*** still computed in observation space.
  - ***The EnKF algorithm is unchanged***, except that vertical localization is turned off, and ‘modulated’ ensemble (in model and observation space) is ingested instead of ‘raw’ ensemble.
  - ***Only the original N ensemble members are updated*** in the DA, using covariances derived from the full *MN* member modulated ensemble.
  - Not straightforward to implement in LETKF, since analysis weights apply to full MN member ensemble (not original N member ensemble).

Results (radiance-only assimilation)

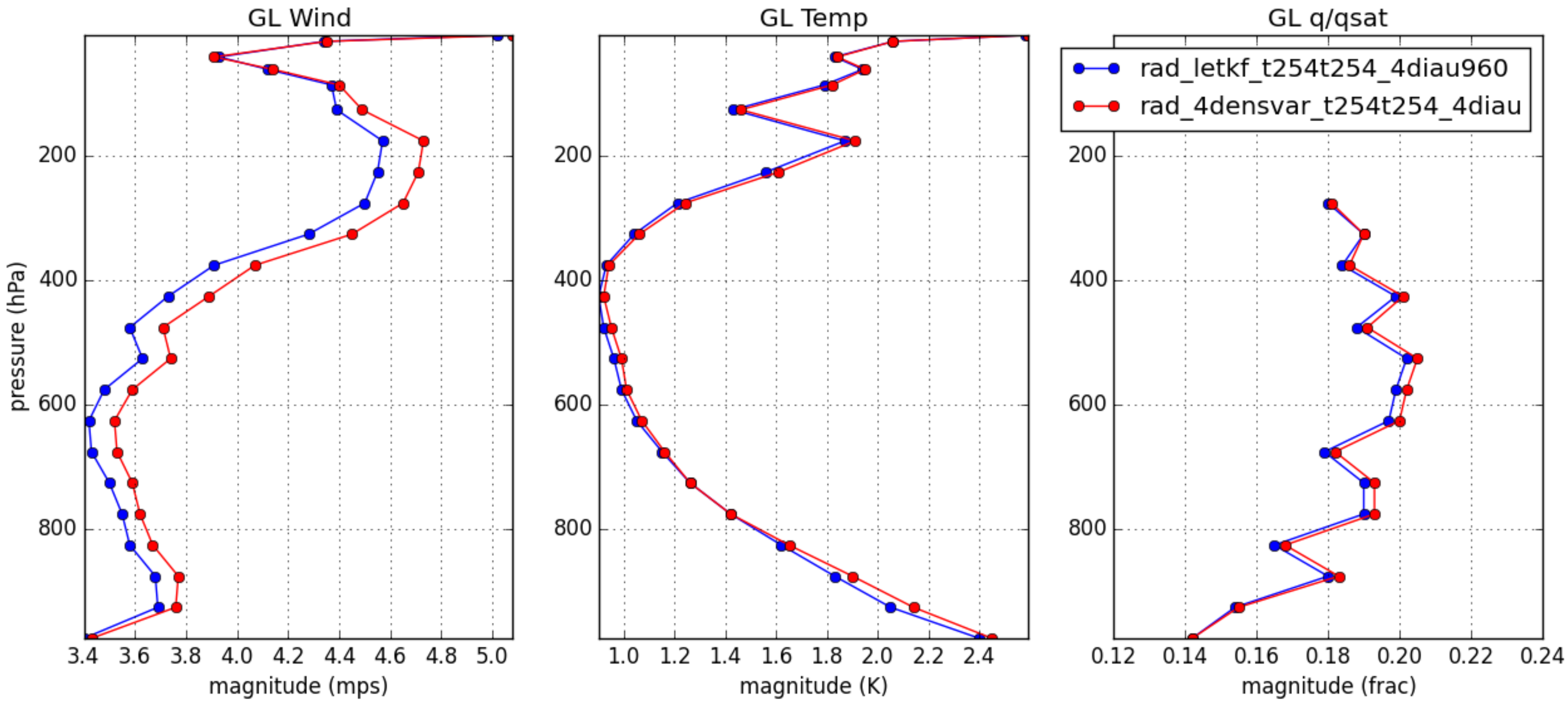


- *Using model space localization in the EnKF improves the use of radiance data.*
- *Performance similar to 4DEnVar.*

How many members are needed to turn off vertical localization?

- Since 12 eigenvectors of vertical localization matrix explain 99% of variance, this suggests that 80*12=960 members should be sufficient.
- We have run a 960 member LETKF ensemble without vertical localization (very efficient in LETKF, since analysis weights can be computed for entire column at once).

4DEnVar (80 members) vs LETKF (960 members): radiances only



- *Significant improvement from increase ensemble size/elimination of vertical localization.*
- *Performance superior to 4DEnVar with 80 members.*

Conclusions

- Care must be taken when assimilating radiance observations in the EnKF with observation-space localization.
- O(1000) members should be enough to obviate the need for vertical localization.
- Model space localization improves the assimilation of radiance observations. Can be implemented in EnKF using 'modulated ensembles', but with a significant increase in cost.
  - Alternately, empirically derived localization functions (ELFs, Lei et al 2016: dx.doi.org/10.1002/2016MS000627) for each instrument/channel.

Example from Lei et al 2016: AMSU-A Channel 9