Conservation laws and the local ensemble transform Kalman filter

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Hans-Ertel-Zentrum
für Wetterforschung

Motivation

Numerical discretization schemes have a long history of incorporating the most important conservation properties of the continuous system in order to improve the prediction of the nonlinear flow.

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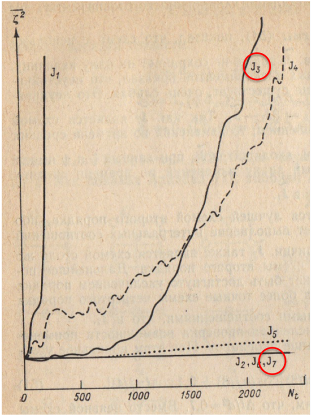

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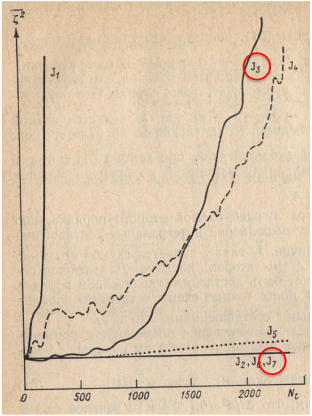

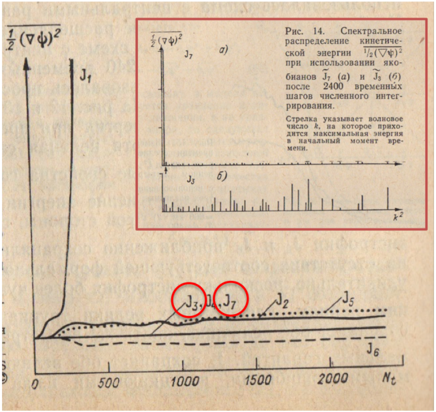

Arakawa and Lamb 1977

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- Goal: Explore which conservation properties are well recovered when using LETKF
- Explore which constraints should be included in LETKF in future

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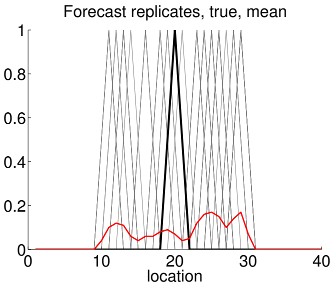

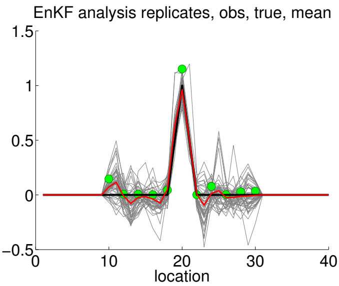

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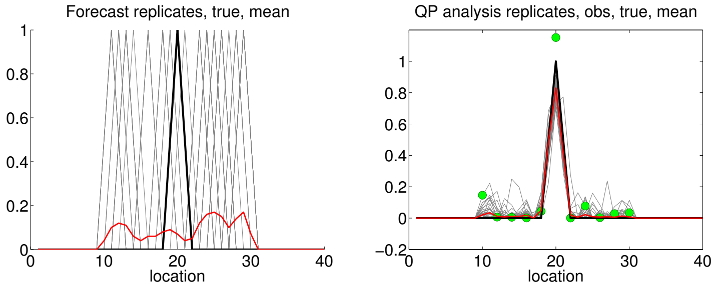

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- Study conservation of mass, energy and enstrophy with LETKF
- including dependence of the results on the observational type
- Show implication on the prediction

Experimental set-up

- Non-linear dynamics with nonlinear shallow water model
- Model settings:
  1. Mirror boundaries,
  2. constant $f = 0.0001$,
  3. 259×259 grid points with spacing 50km
  4. leapfrog scheme with time step 125s
  5. Asselin filter with 0.01.

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- Numerical discretization of the dynamics is such that mass, energy and momentum are conserved (c.f. Z. Janjic 1984), and enstrophy for non divergent flow .
- Rossby radius of deformation $\sqrt{gh_0}/f \approx 2300$ km

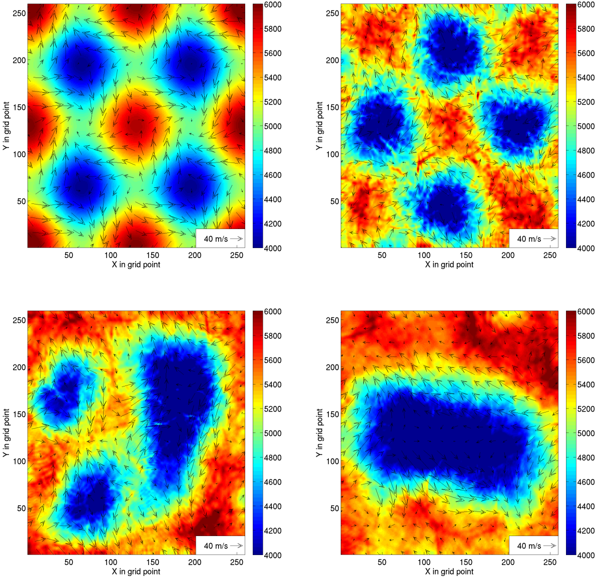
Nonlinear shallow water model

Initial

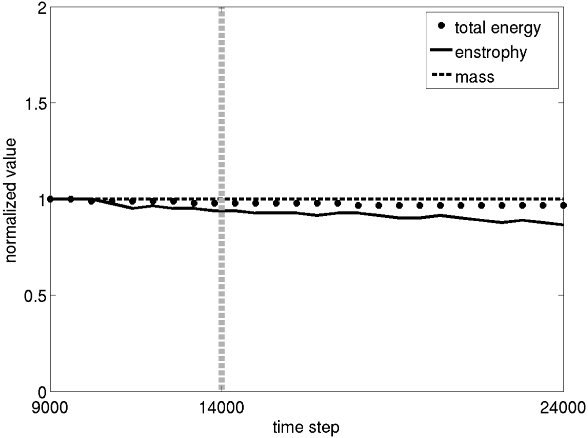
Day 13

Day 20

Day 35



Nonlinear shallow water model



Time evolution of mass, total energy and enstrophy, normalized with respective initial values, in a nature run.

LETKF experiments

- different localization and the observational coverage
- 32 members + 1 deterministic run, constant inflation = 1.05
- 50 assimilation cycles
- Observations, u, v and h, or u and v, or h only from nature run
- Linear observation operator
- Gaussian observation error with standard deviations of 1.5m/s and 150 m.
- 1h updates

Diagnostics for analysis (ensemble mean)

1. RMSE
2. Normalized energy, enstrophy, mass and divergence.
3. Noise (e.g. Janjic et al. 2011)

$$\mathcal{N} = \frac{\sum_{i,j=1}^{N_x,N_y} [\nabla^2 u(i,j)]^2 + [\nabla^2 v(i,j)]^2}{\sum_{i,j=1}^{N_x,N_y} [u(i,j)^2 + v(i,j)^2]}$$

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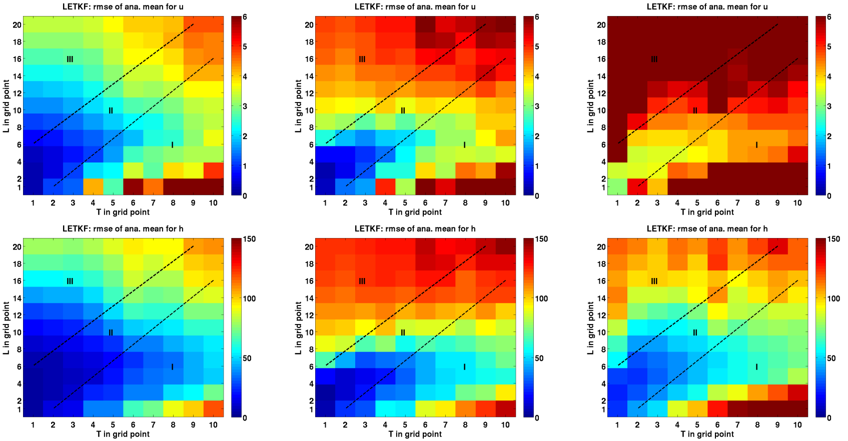
Relative to:

- nature run
- the initial state

RMSE

u

h

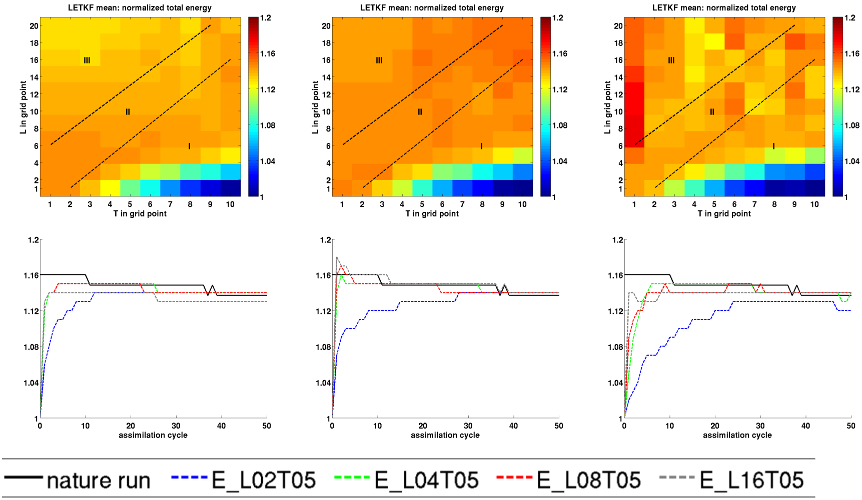


Obs u,v and h

Obs u and v

Obs h

Energy


LETKF mean: normalized total energy
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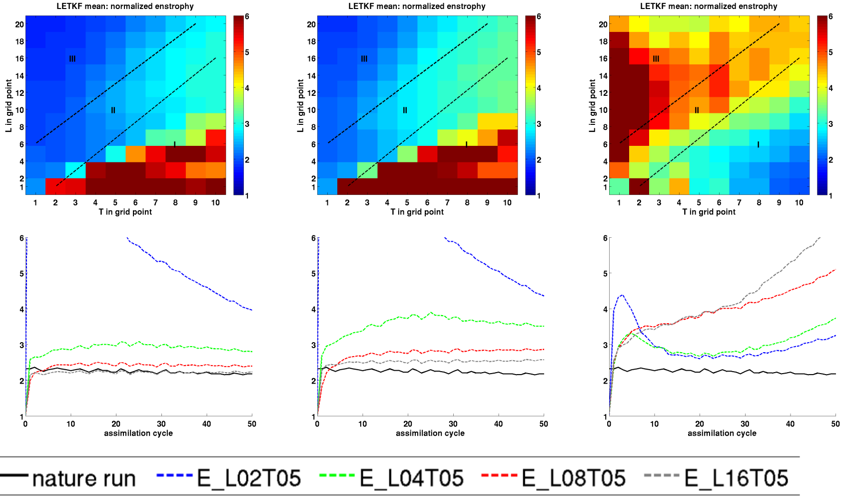
L in grid point
T in grid point
III
II
I
assimilation cycle
nature run
E_L02T05
E_L04T05
E_L08T05
E_L16T05

Enstrophy



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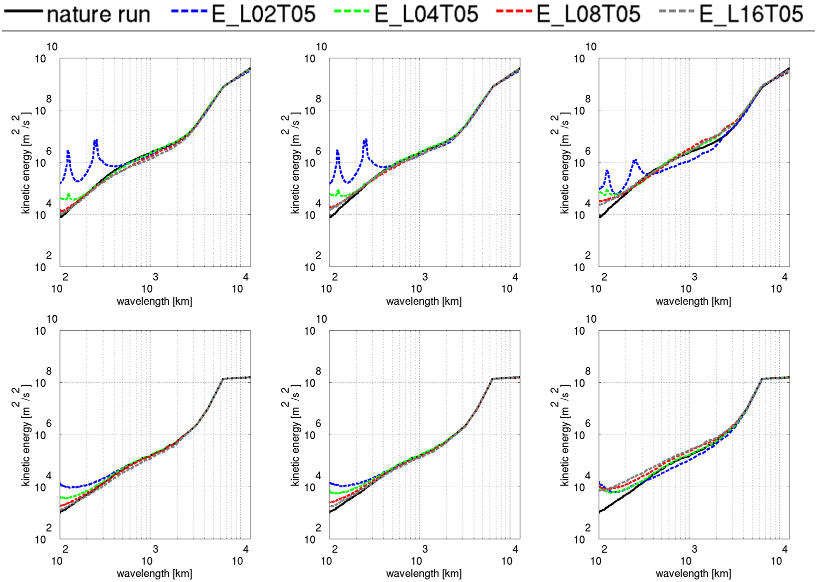
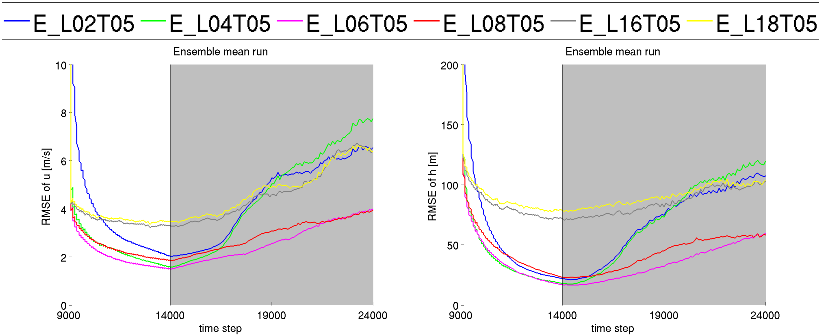
Kinetic energy spectra



Averaged over the first (upper) and last five assimilation cycles (lower).

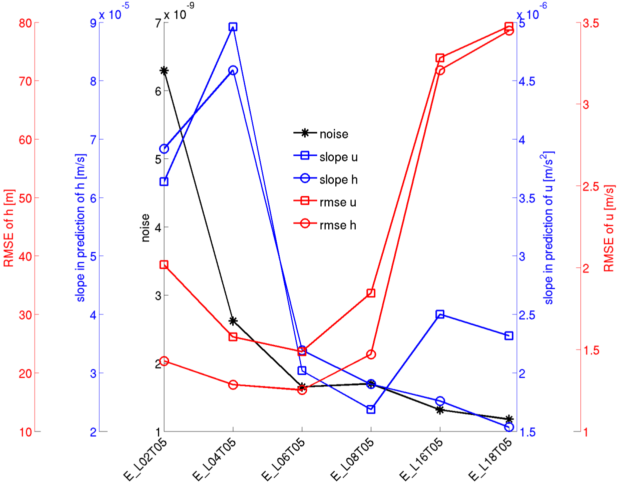
Prediction



RMSE for u

RMSE for h

Prediction



Noise during assimilation and the analysis RMSE are good indicators of the quality of the prediction.

Conclusion

- Although total energy of the analysis ensemble mean converges towards the nature run value with time, enstrophy does not.
- LETKF effects energy spectrum, enstrophy, divergence and noise.
- Assimilation of velocity observations bounds enstrophy.
- Observations of height improve divergence but cannot control enstrophy.
- Multiplicative inflation increases enstrophy and energy at small scales.
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More in:

Zeng,Y. and T. Janjic: Study of conservation laws with the Local Ensemble Transform Kalman Filter. Q.J.R. Meteorol. Soc.. doi: 10.1002/qj.2829