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Scale-Dependent Spatial Covariance Localization in Ensemble-Variational Data Assimilation

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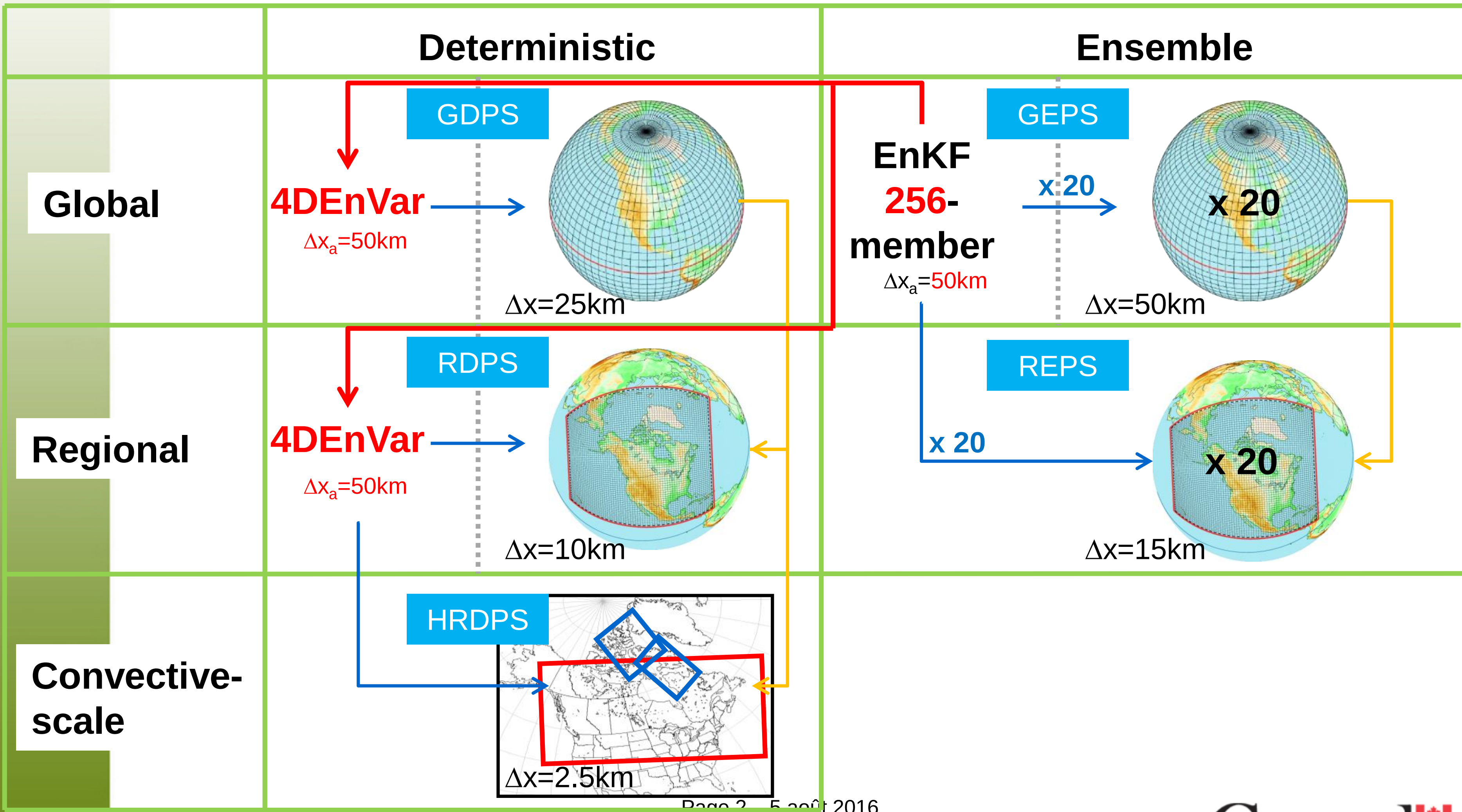
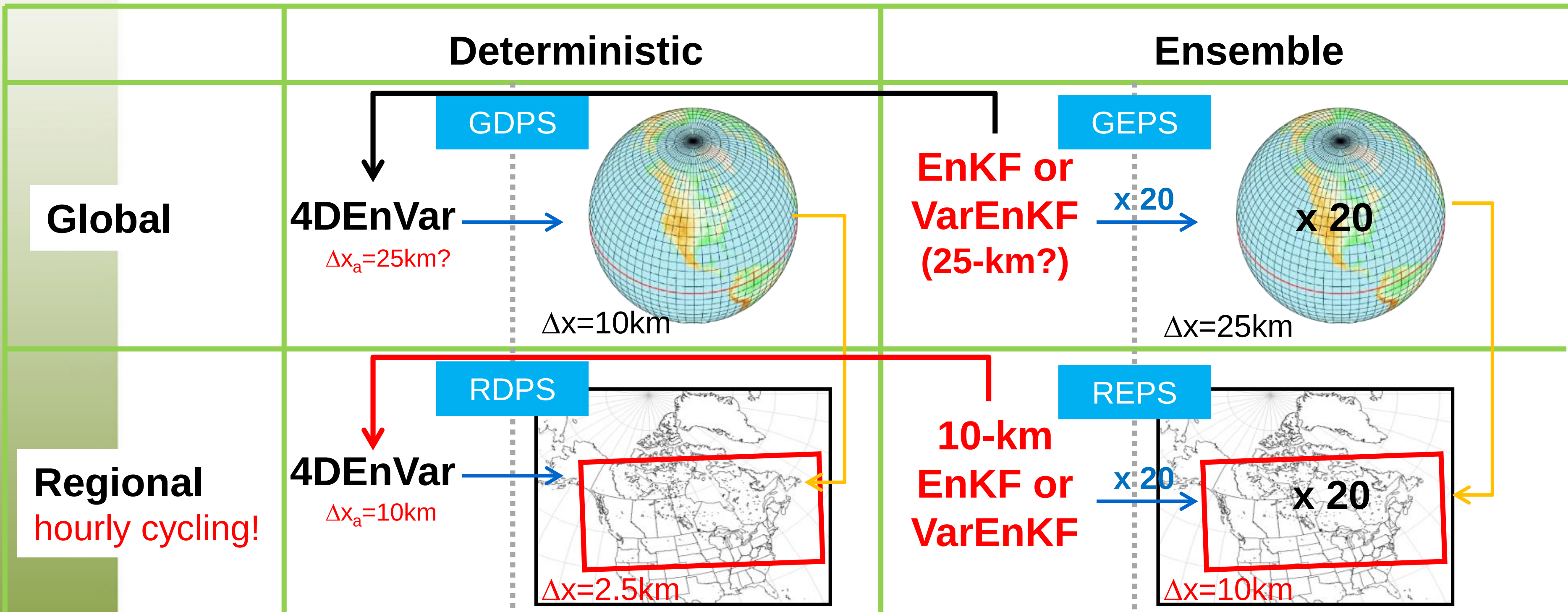
ECCC's NWP systems since 2016



ECCC's NWP systems in ~2020



The range of analysed scales will increase with time <u>in both</u> global and limited-area NWP. DA methods that can cope with this challenge are needed.

Spatial Covariance Localization

- Spatial covariance localization is essential to obtain useful analyses with “small” ensembles (a 256-member ensemble is still "small"!).

- Currently, ECCC's EnVar uses simple localization of ensemble covariances, similar to EnKF: single length scale in both horizontal and vertical localizations based on Gaspari and Cohn (1999) 5th order piecewise rational function.

- Comparing various NWP studies, seems that the best amount of horizontal localization depends on application/resolution:
    - convective-scale assimilation: ~10km
    - mesoscale assimilation: ~100km
    - global-scale assimilation: ~1000km – 3000km (2800km at ECCC)

A one-size-fits-all approach for localization does not seem appropriated for analysing a wide range of scales.

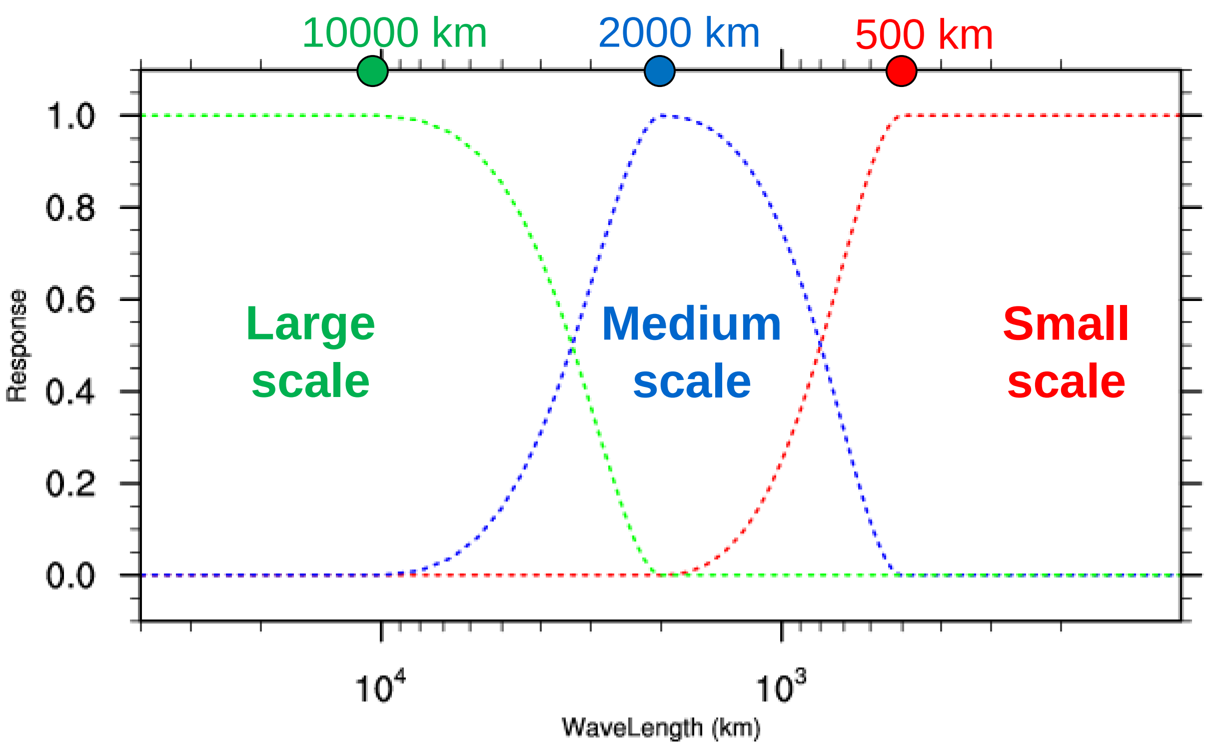
Scale-dependent covariance localization
Introduction

Definition: Simultaneously apply appropriate (i.e. different) localization to different range of scales.

- The approach can be applied to both horizontal and vertical localization but this presentation will only focus on horizontal-scale-dependent horizontal localization.
- **Pros**:
  - Seems appropriated for multi-scale analysis.
  - In limited-area: Could avoid the need of multi-step or large-scale blending approaches.
- **Cons**:
  - Adds more parameters to tuned.
  - Increases the cost of the analysis step (at least in our formulation).

Horizontal Scale Decomposition

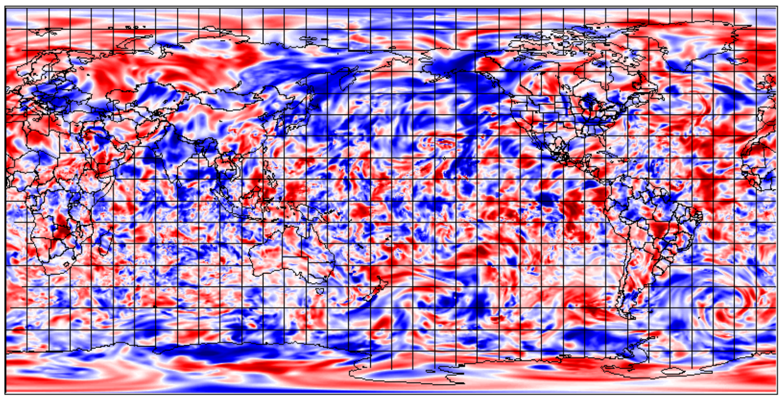
Filter response functions for decomposing with respect to 3 horizontal scale ranges



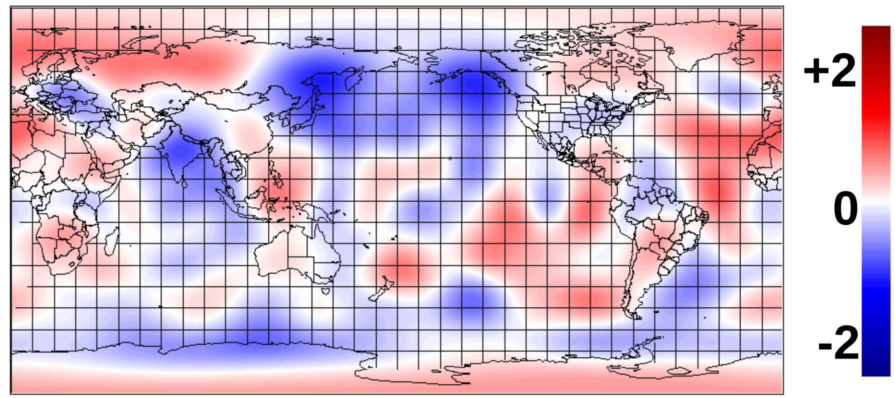
Horizontal Scale Decomposition

Perturbations for ensemble member #001 – Temperature at ~700hPa

Full

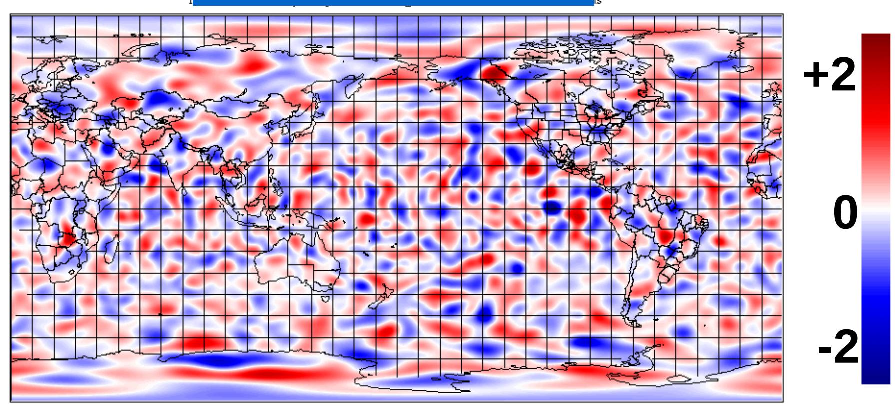


Large scale



Small scale

Medium scale



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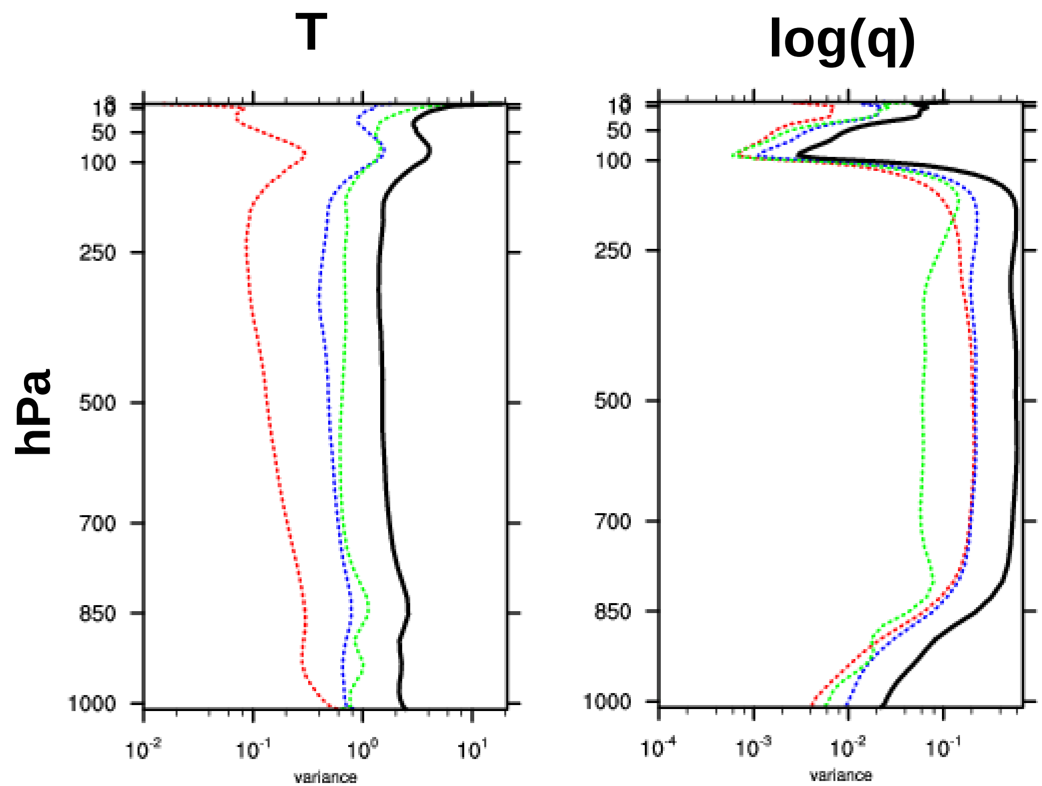
Pa

Horizontal Scale Decomposition

Waveband variances

Large scale
Medium scale
Small scale
All the scales

Horizontal scale-dependent localization leads to (implicit)…
<u>variable-dependent</u> horizontal localization



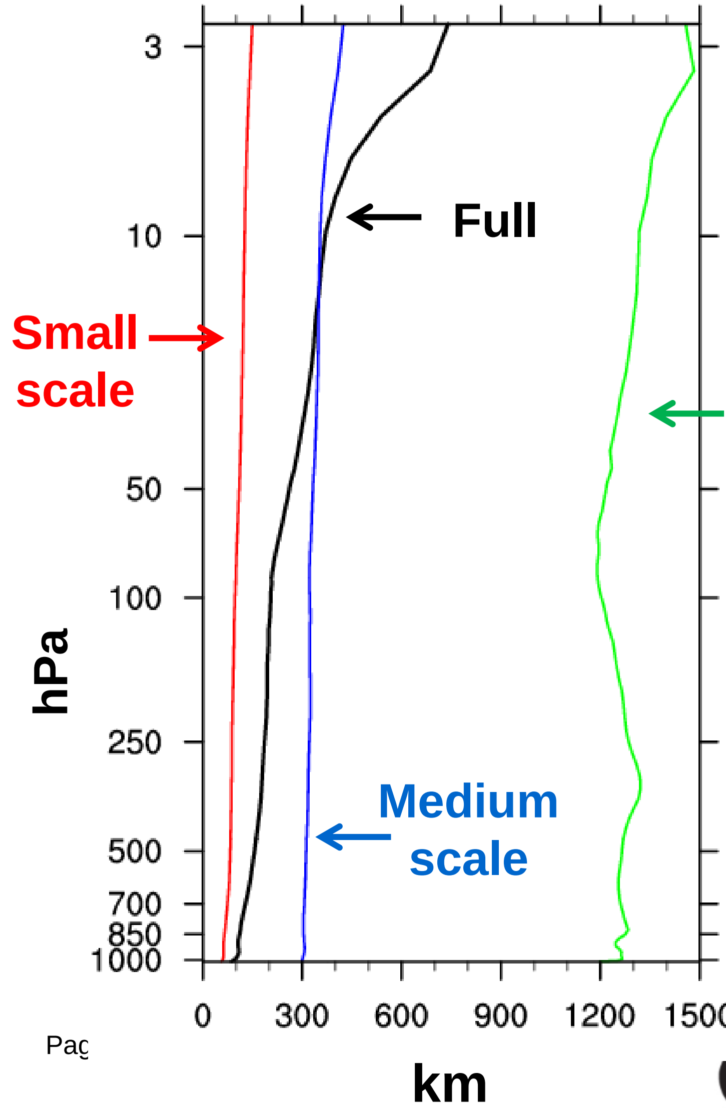
6-h perturbation from 256-member EnKF

Horizontal Scale Decomposition

Homogeneous horizontal correlation length scales

6-h temperature perturbation from 256-member EnKF

Horizontal scale-dependent localization leads to (implicit)…
vertical-level-dependent horizontal localization



Scale-dependent covariance localization

Implementation in EnVar

Current (one-size-fits-all) Approach

- **Analysis increment computed from control vector ($B^{1/2}$ preconditioning) using:**

$$\Delta \mathbf{x} = \sum_k \mathbf{e}_k \circ \left(\mathbf{L}^{1/2} \boldsymbol{\xi}_k \right)$$

k: member index

Scale-dependent Approach (Buehner and Shlyaeva, 2015, *Tellus*)

- **Varying amounts of smoothing applied to same set of amplitudes for a given member**

$$\Delta \mathbf{x} = \sum_k \sum_j \mathbf{e}_{k,j} \circ \left(\mathbf{L}_j^{1/2} \boldsymbol{\xi}_k \right)$$

k: member index
j: scale index

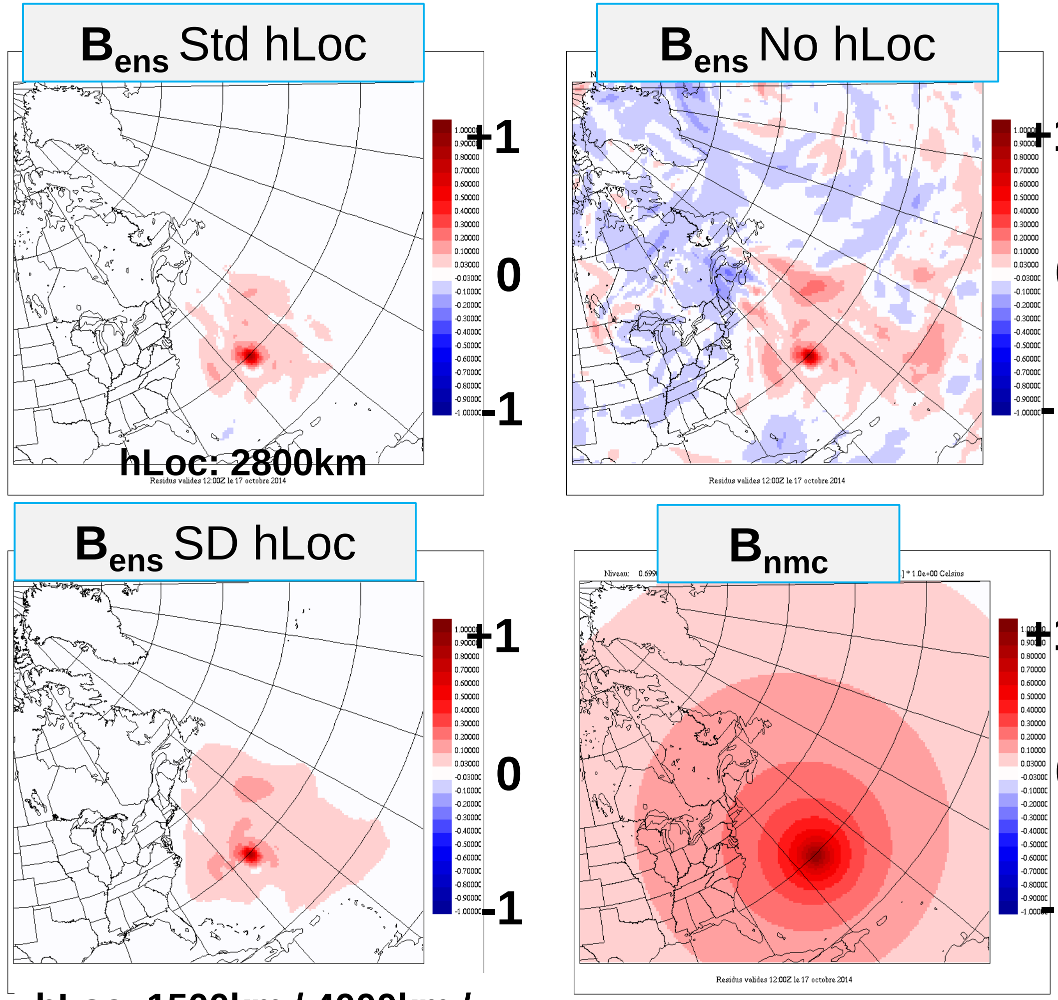
where $e_{k,j}$ is scale *j* of normalized member *k* perturbation

Scale-dependent covariance localization

Impact in single observation DA experiments

700 hPa T observation at the center of **Hurricane Gonzalo** (October 2014)

Normalized temperature increments (correlation-like) at 700 hPa resulting from various B matrices.



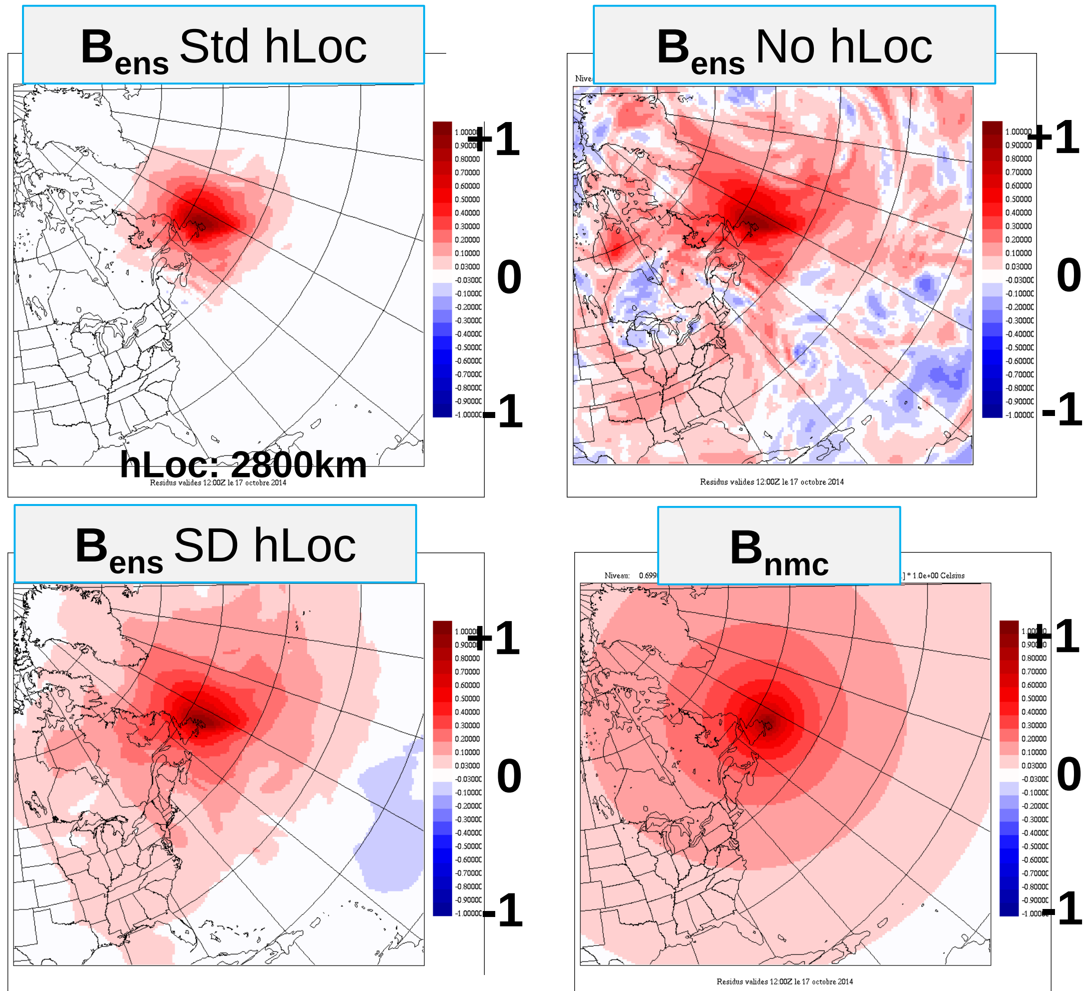
hLoc: 1500km / 4000km / 10000km

Scale-dependent covariance localization

Impact in single observation DA experiments

700 hPa T observation at the center of a **High Pressure**

Normalized temperature increments (correlation-like) at 700 hPa resulting from various B matrices.



hLoc: 1500km / 4000km / 10000km

Scale-dependent covariance localization

Forecast impact

- 2.5-month trialling (June-August 2014) in our global NWP system.
  - Why using the global system and not the regional system? Because the positive impact from the scale-dependent localization is likely to be greater in this system since…
    - An intermittent cycling strategy is used in the regional system
    - The global system has a wider range of horizontal scales

- 3DEnVar with 100% B_{ens} used in both experiments

  1) **Control experiment** with hLoc = 2800 km, vLoc = 2 units of ln(p)

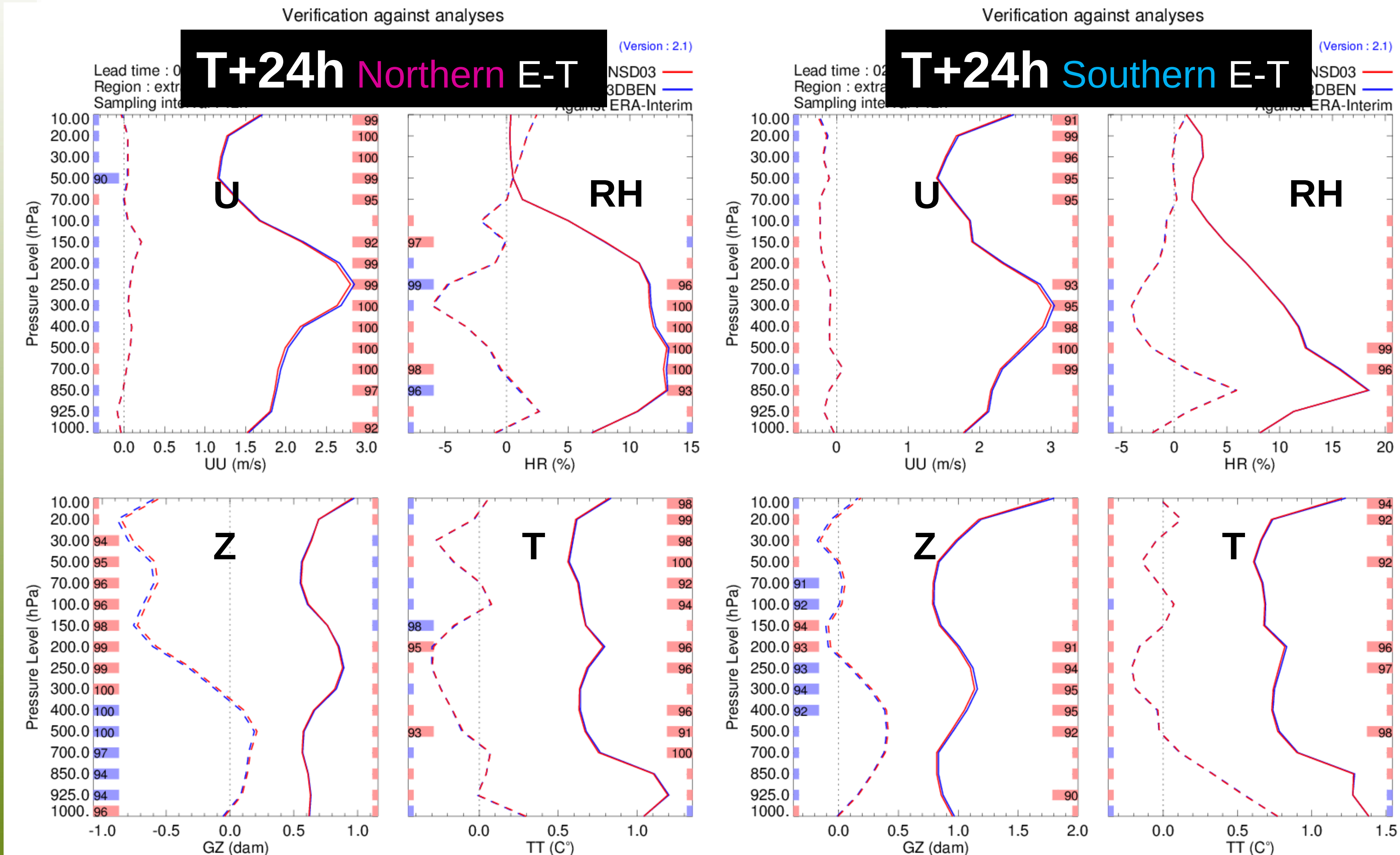
  2) **Scale-Dependent experiment** with a 3 horizontal-scale decomposition
     I. Small scale uses hLoc = 1500 km
     II. Medium scale uses hLoc = 2400 km
     III. Large scale with uses = 3300 km

     Ad hoc values!

     Same vLoc (2 units of ln(p)) for every horizontal-scale

Scale-dependent covariance localization

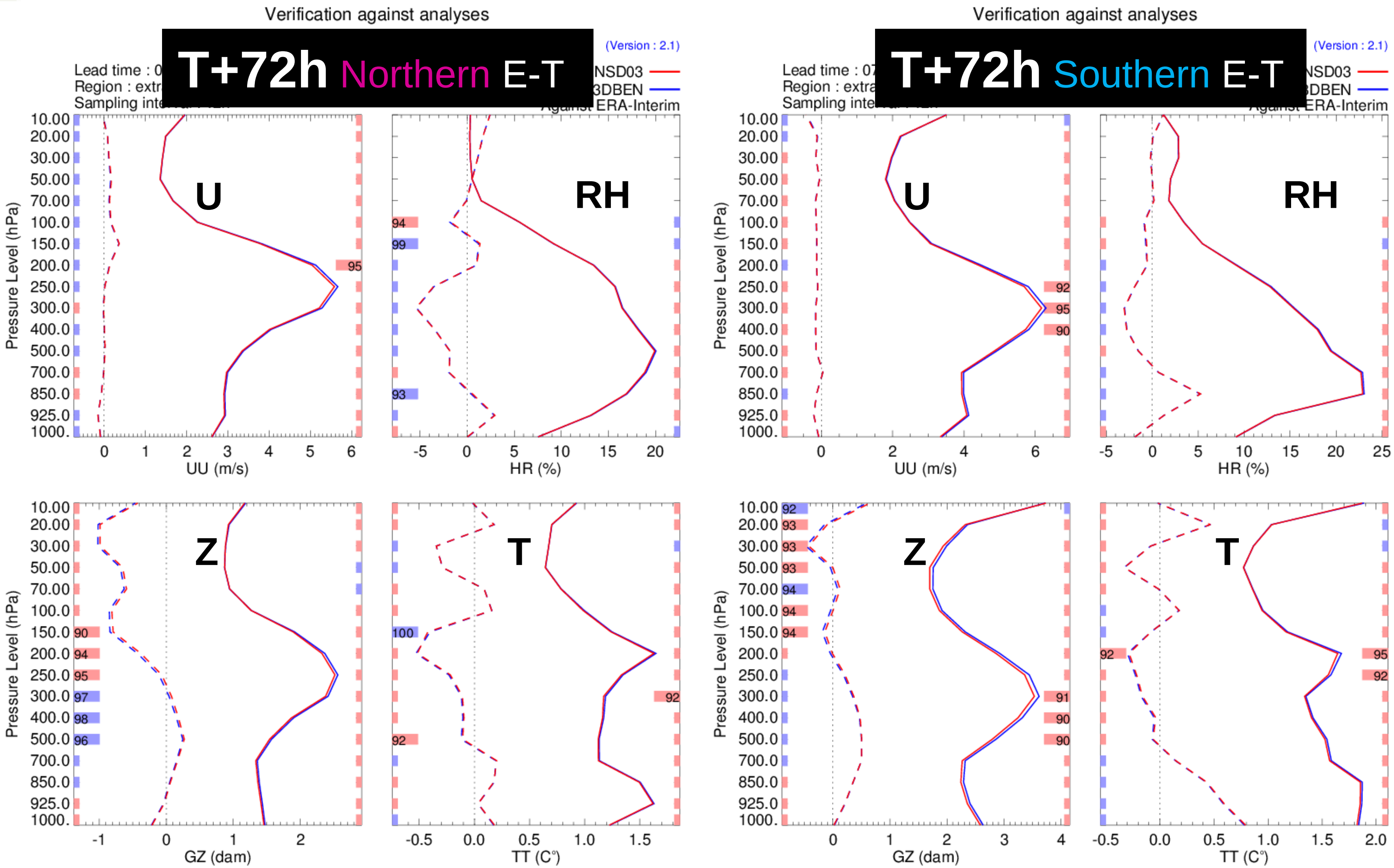
Forecast impact – Comparison against ERA-Interim



➢ Control ➢ Scale-Dependent

Scale-dependent covariance localization

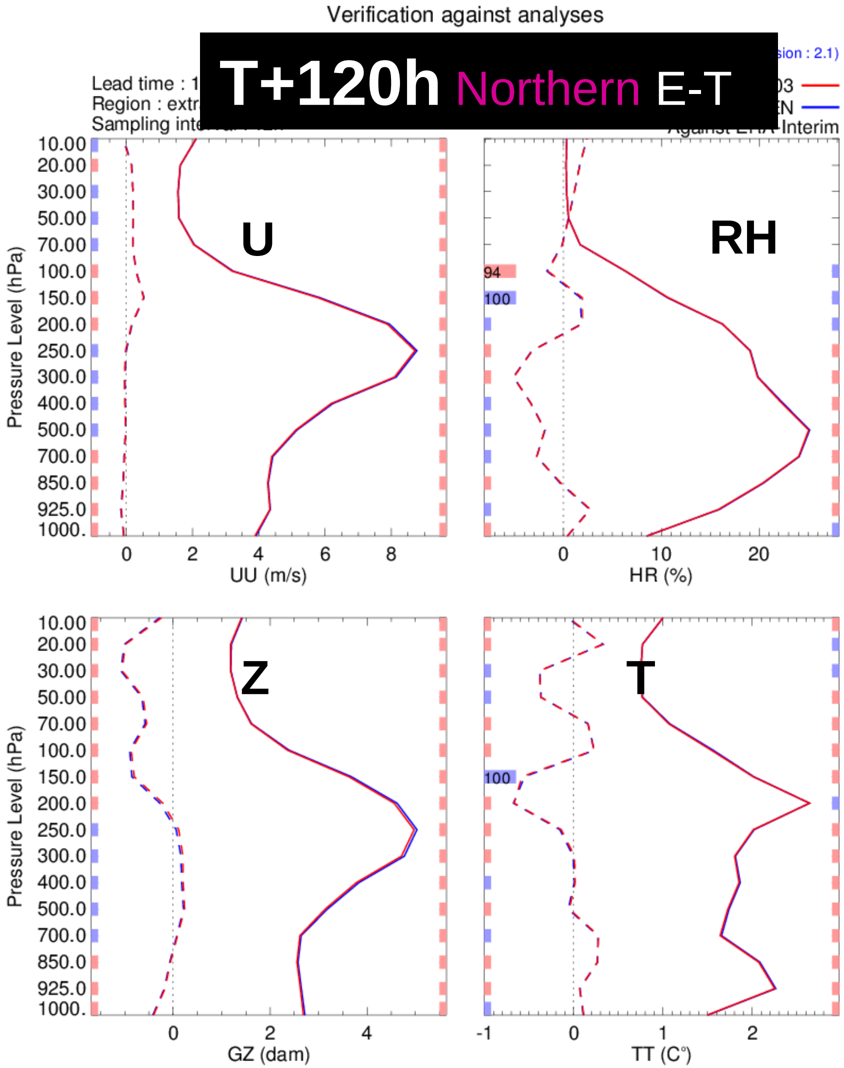
Forecast impact – Comparison against ERA-Interim

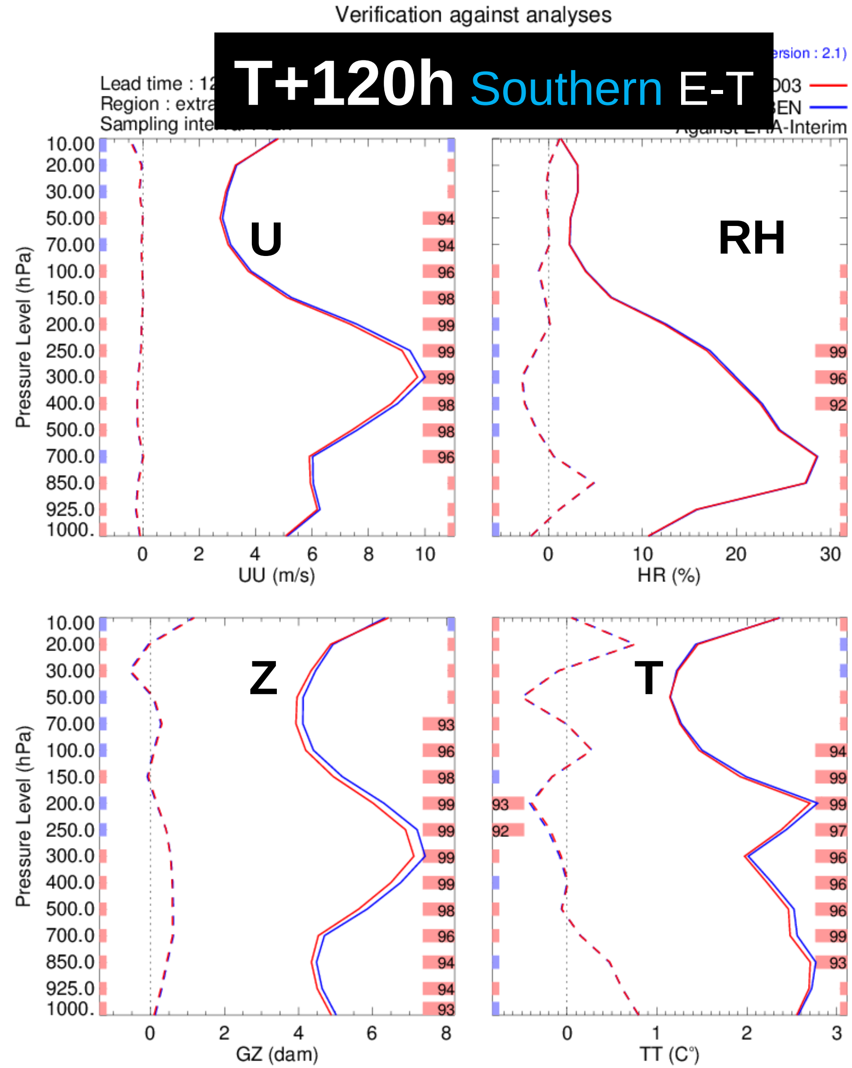


➤ Control ➤ Scale-Dependent

Scale-dependent covariance localization

Forecast impact – Comparison against ERA-Interim

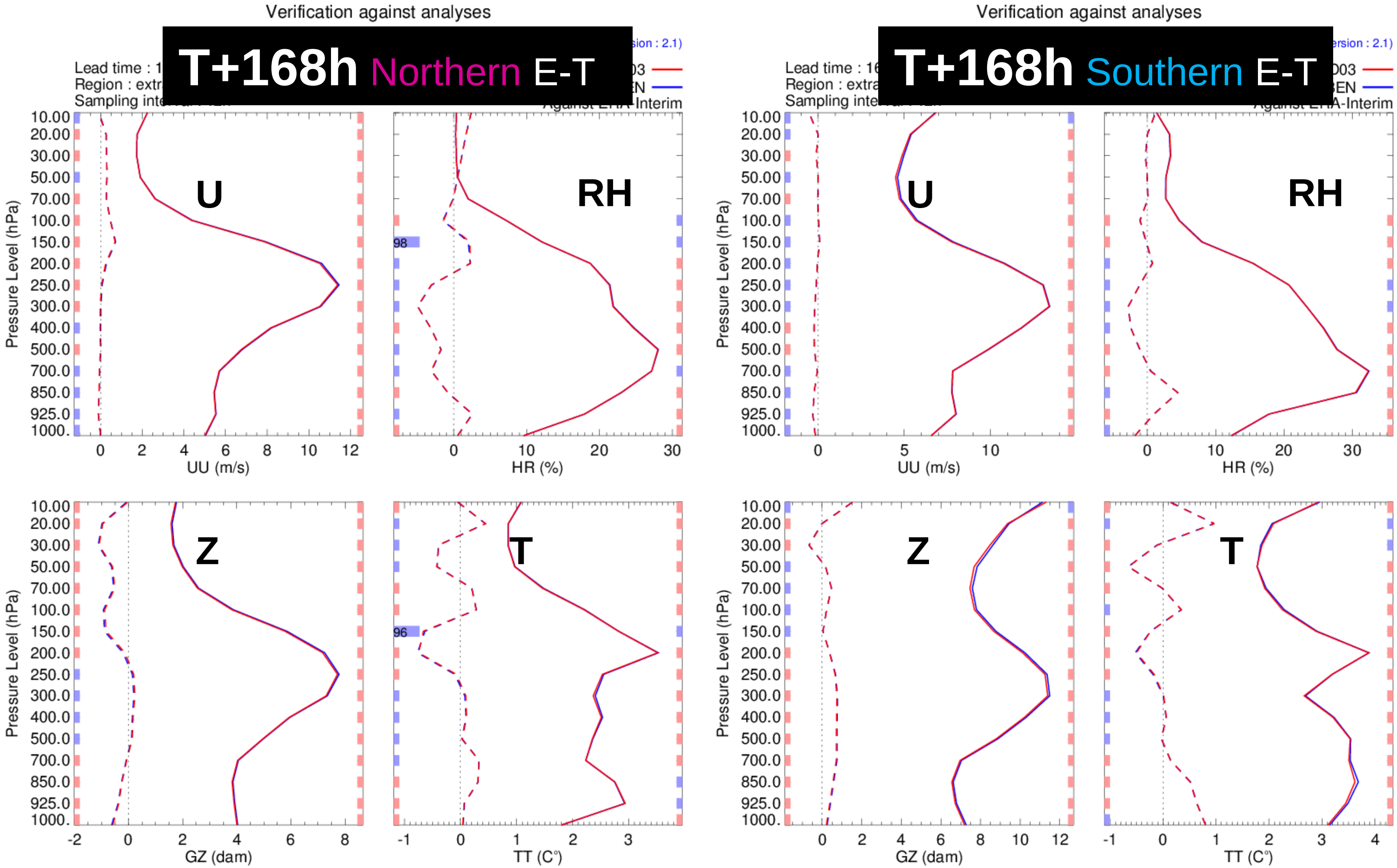




- **Control**
- **Scale-Dependent**

Scale-dependent covariance localization

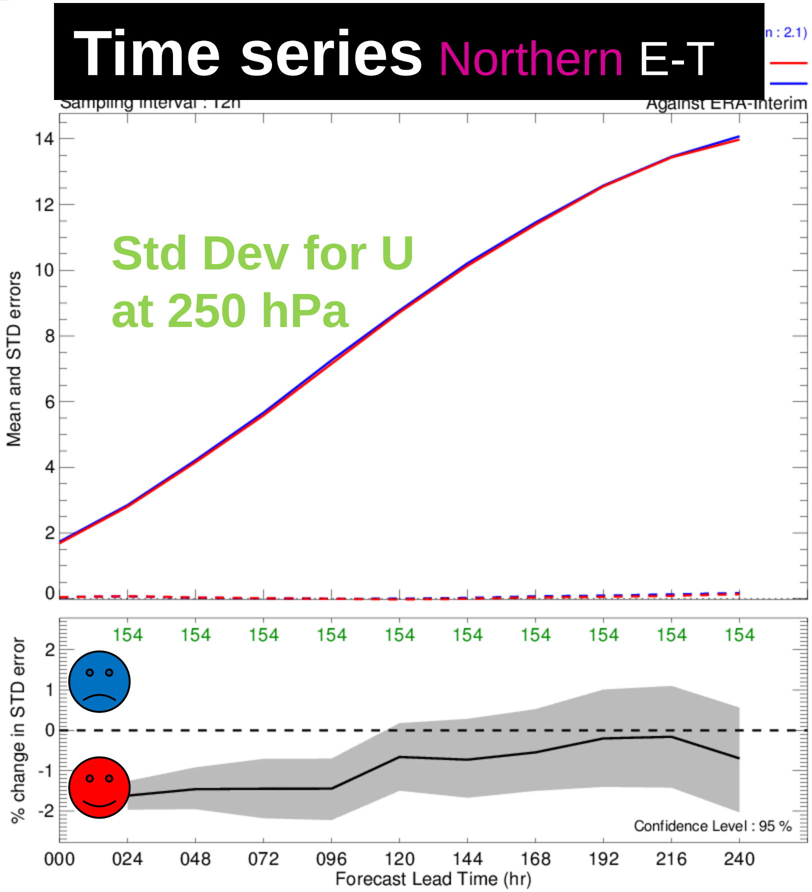
Forecast impact – Comparison against ERA-Interim

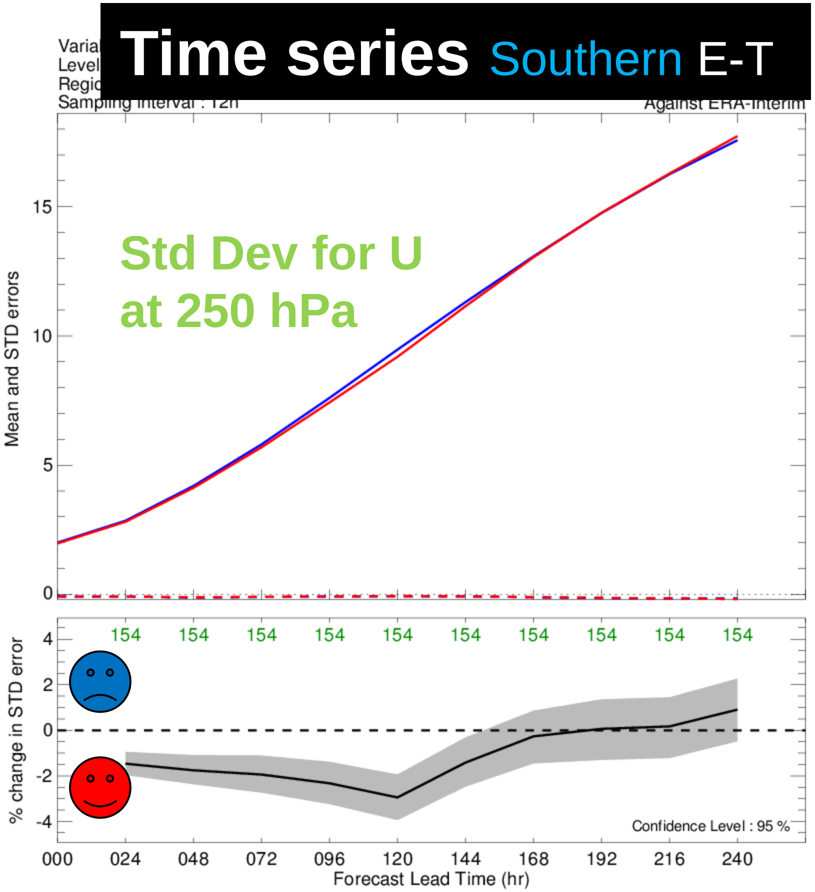


➢ Control ➢ Scale-Dependent

Scale-dependent covariance localization

Forecast impact – Comparison against ERA-Interim





- Control
- Scale-Dependent

Scale-dependent covariance localization

Forecast impact

Is it possible to do as good as S-D localization with a single localization approach?

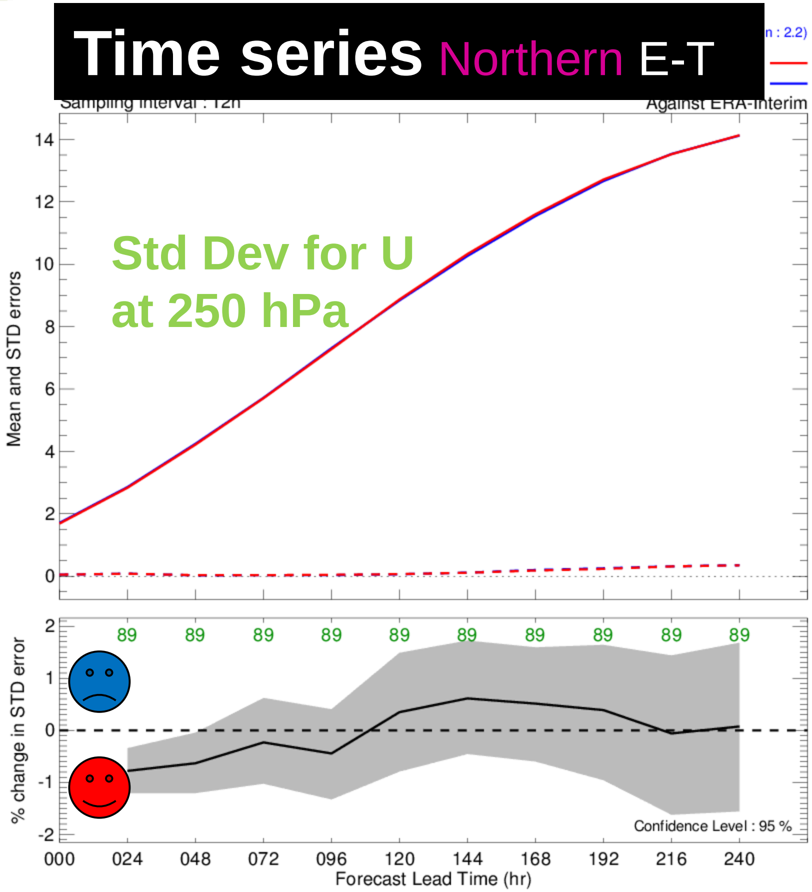
After all, perhaps our one-size-fits-all horizontal localization radius of 2800 km is not optimal...

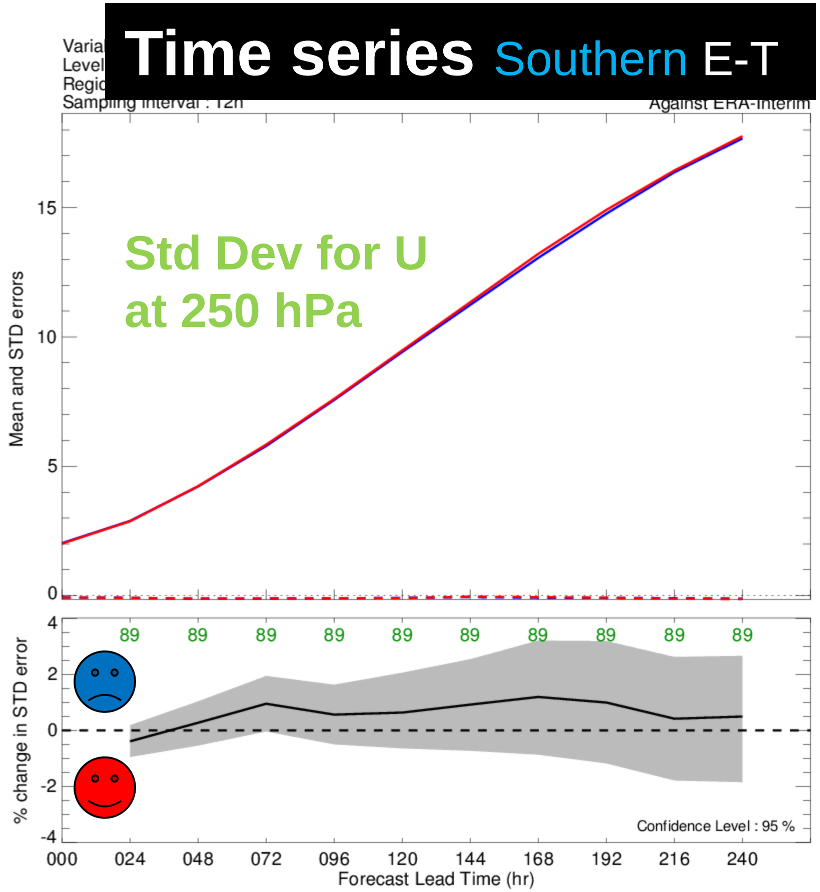
- **2 new 1.5-month trialling** (June-July 2014) with a single localization approach (still using 3DEnVar with 100% B_{ens})

  1) hLoc = 2400 km (the value used for **medium scale** in SD hLoc)
  2) hLoc = 3300 km (the value used for **large scale** in SD hLoc)

Scale-dependent covariance localization

Forecast impact – Comparison against ERA-Interim

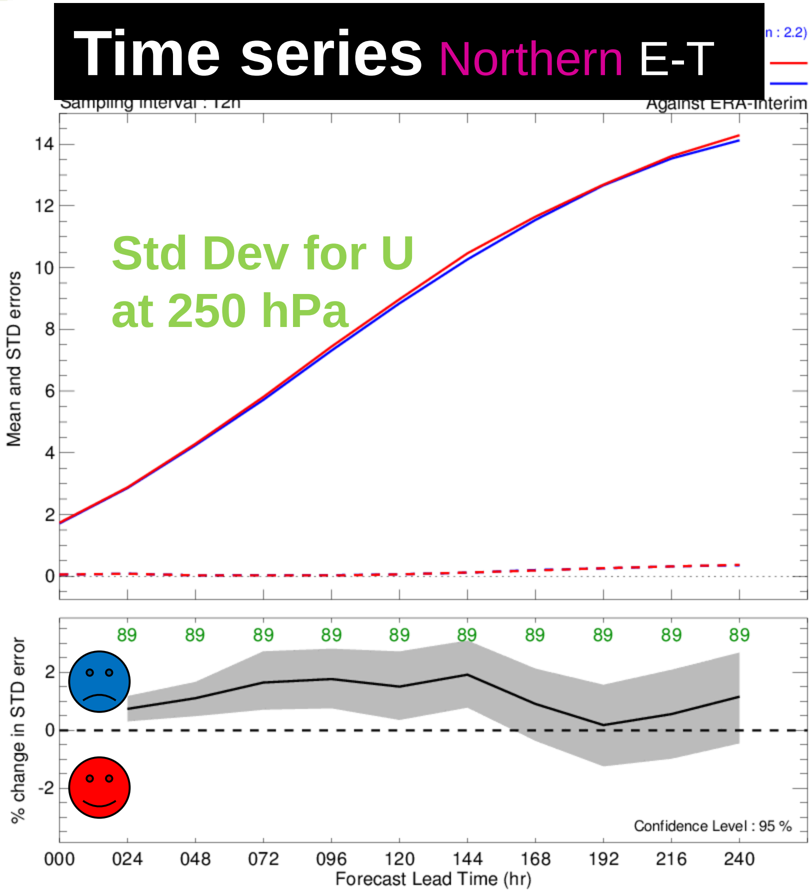


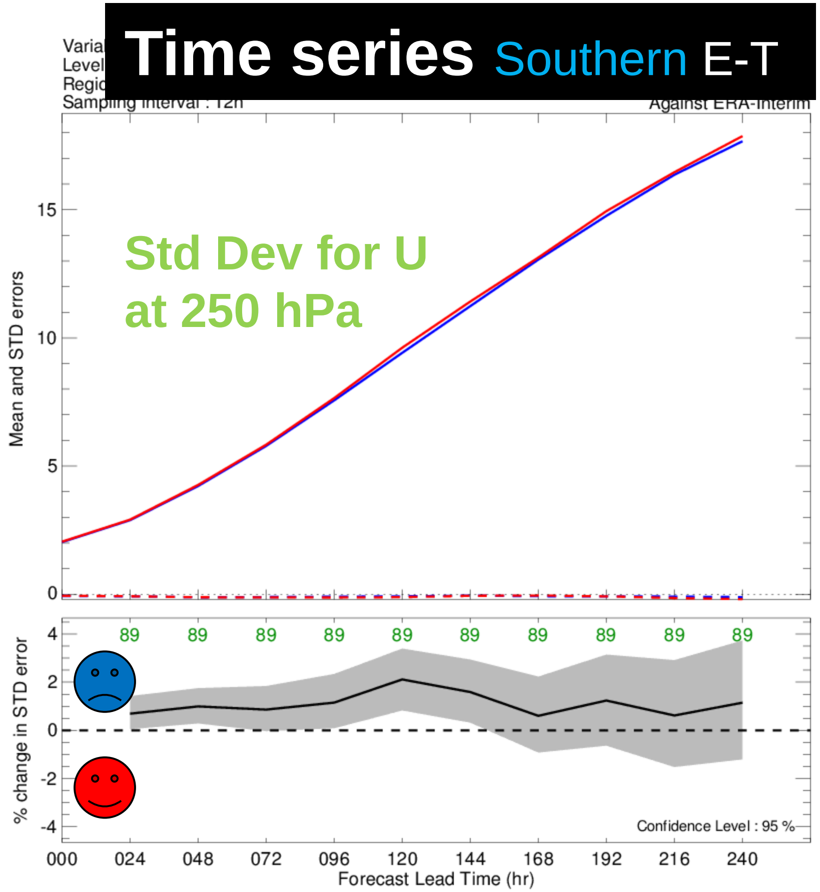


- Control (hLoc = 2800 km)
- hLoc = 2400 km

Scale-dependent covariance localization

Forecast impact – Comparison against ERA-Interim





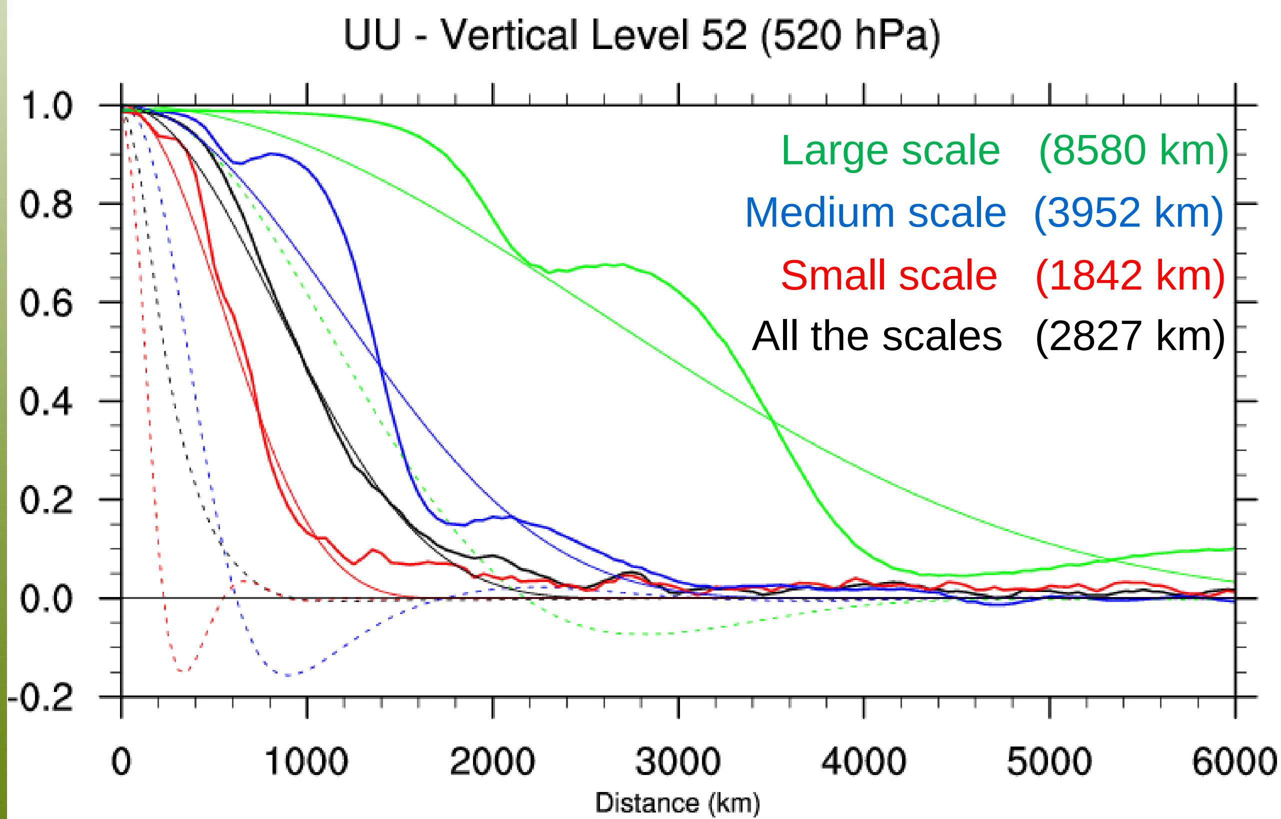
- **Control (hLoc = 2800 km)**
- **hLoc = 3300 km**

Scale-dependent covariance localization

Objective localization radii

What are the optimal horizontal localization radii for S-D localization?

Let's try Ménétrier et al. (2015, MWR) optimal linear filtering approach



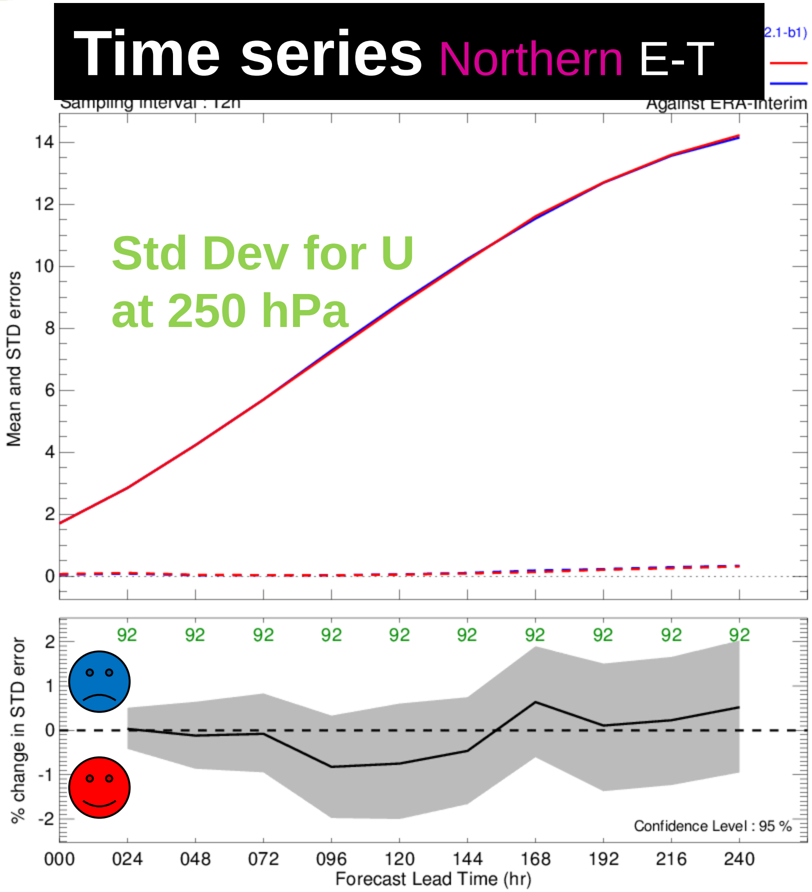
Correlation

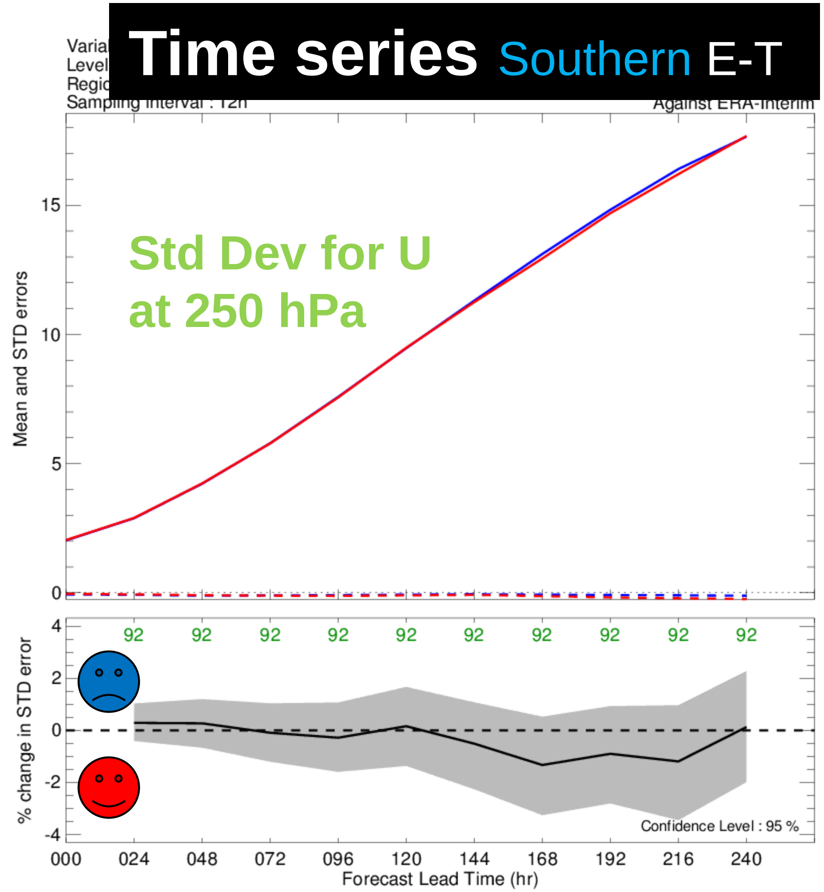
Optimal localization

Gaspari and Cohn function that best-fit the optimal localization

Scale-dependent covariance localization

Forecast impact – Comparison against ERA-Interim





- Control (hLoc = 2800 km)
- SD hLoc = 1500 / 4000 / 10000 km

Scale-dependent covariance localization

Objective localization radii

Why the localization radii based on Ménétrier et al. (2015, MWR) are not optimal for S-D localization?

Some possibilities…

1. I did not correctly coded this approach!
2. This approach can not fully identify the sampling errors in B_{ens}
3. Optimal localization does not only depends on sampling errors.
   e.g. Localization might also alleviate other important sources of errors. The observing network could also come into play as suggested by Flowerdew (2015, Tellus)

Scale-dependent covariance localization

Impact on dynamical balance

It is well know that localization can disrupt the dynamical balance of the analysis increments.

Does the scale-dependent approach increase or decrease this problem?

Balance diagnostics as in Caron and Fillion (2010; MWR)

- Rotational part: Charney's (1955) nonlinear balance equation

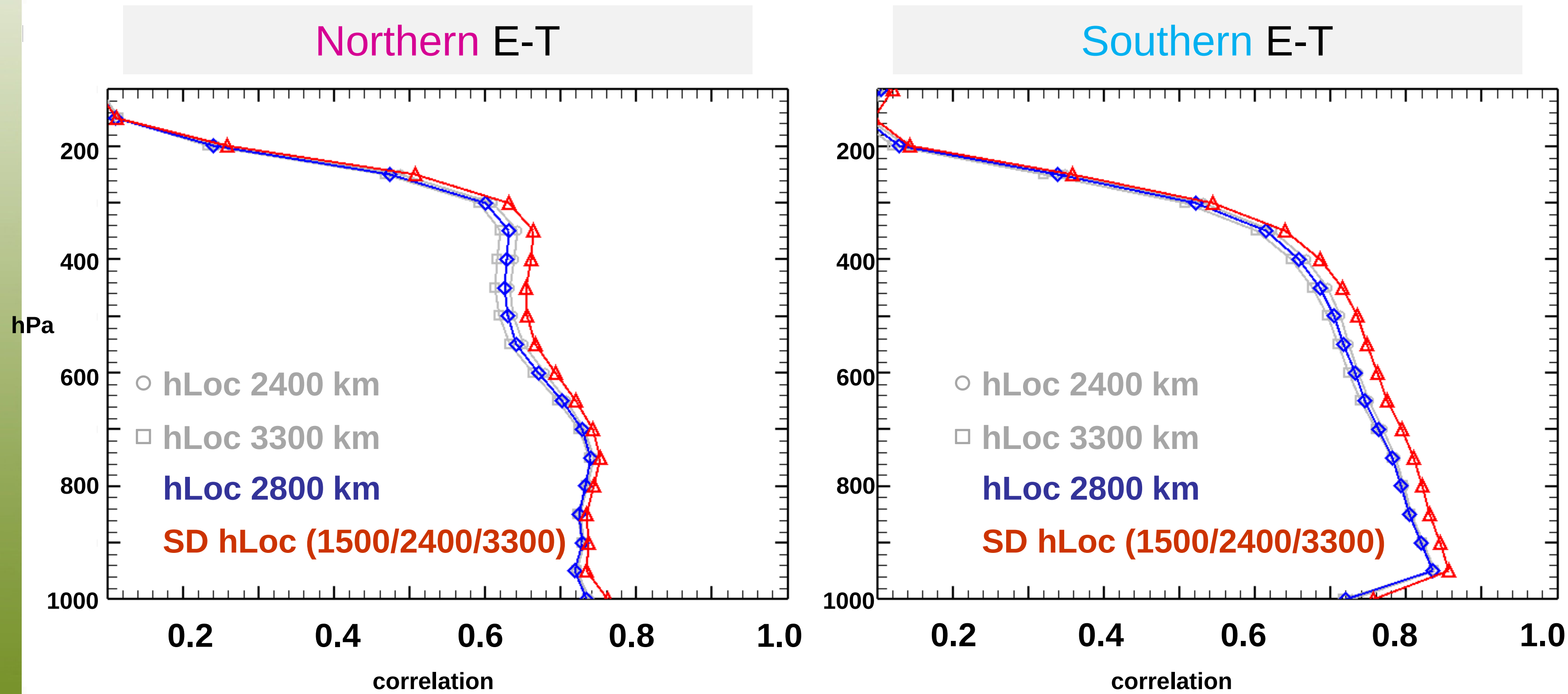
$$\nabla^2\Phi' = f\zeta' + 2\left[\left(\frac{\partial \bar{u}_r}{\partial x}\frac{\partial v'_r}{\partial y} - \frac{\partial \bar{u}_r}{\partial y}\frac{\partial v'_r}{\partial x}\right) + \left(\frac{\partial u'_r}{\partial x}\frac{\partial \bar{v}_r}{\partial y} - \frac{\partial u'_r}{\partial y}\frac{\partial \bar{v}_r}{\partial x}\right) + \left(\frac{\partial u'_r}{\partial x}\frac{\partial v'_r}{\partial y} - \frac{\partial u'_r}{\partial y}\frac{\partial v'_r}{\partial x}\right)\right] - \frac{\partial f}{\partial y}u'_r$$

Mass Wind

Scale-dependent covariance localization

Impact on dynamical balance – Rotational Part

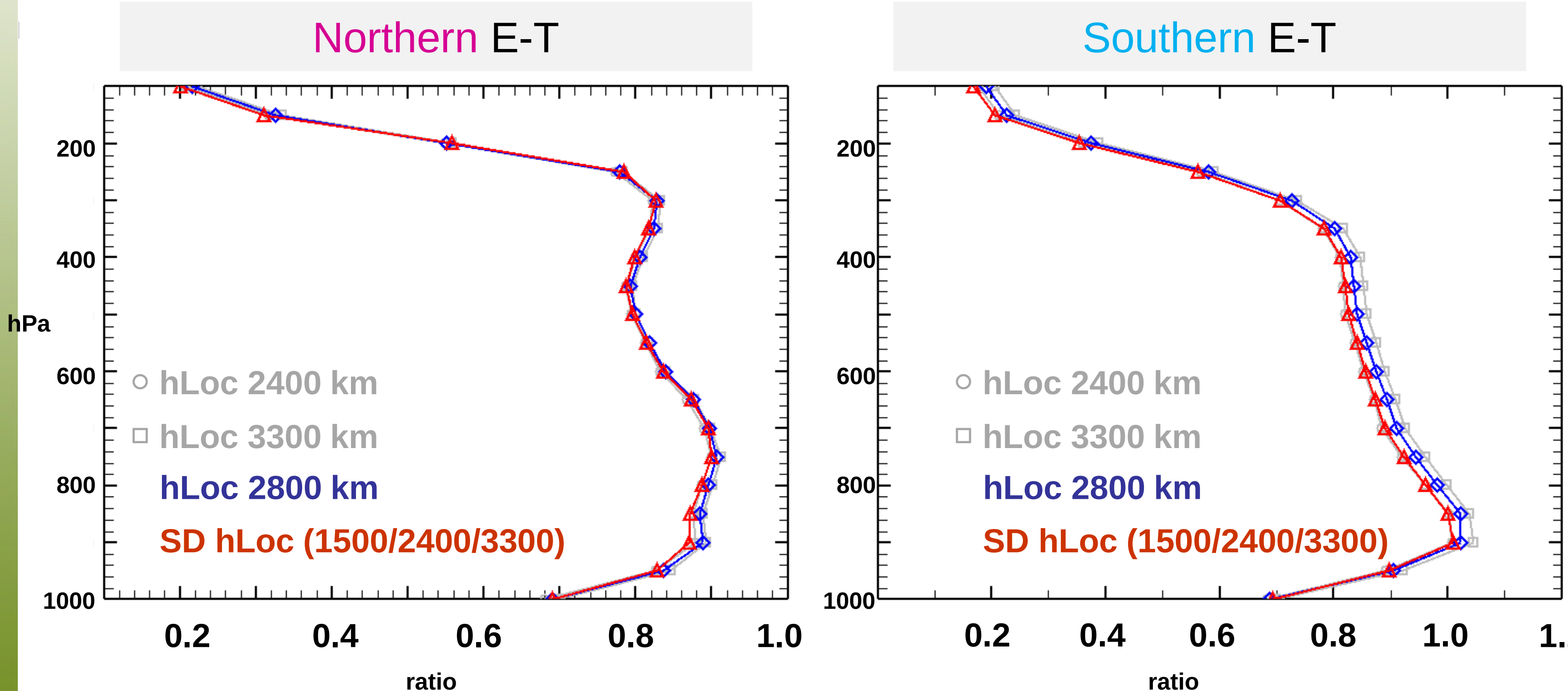
Vertical profile of average correlation between MASS and WIND



Scale-dependent covariance localization

Impact on dynamical balance – Rotational Part

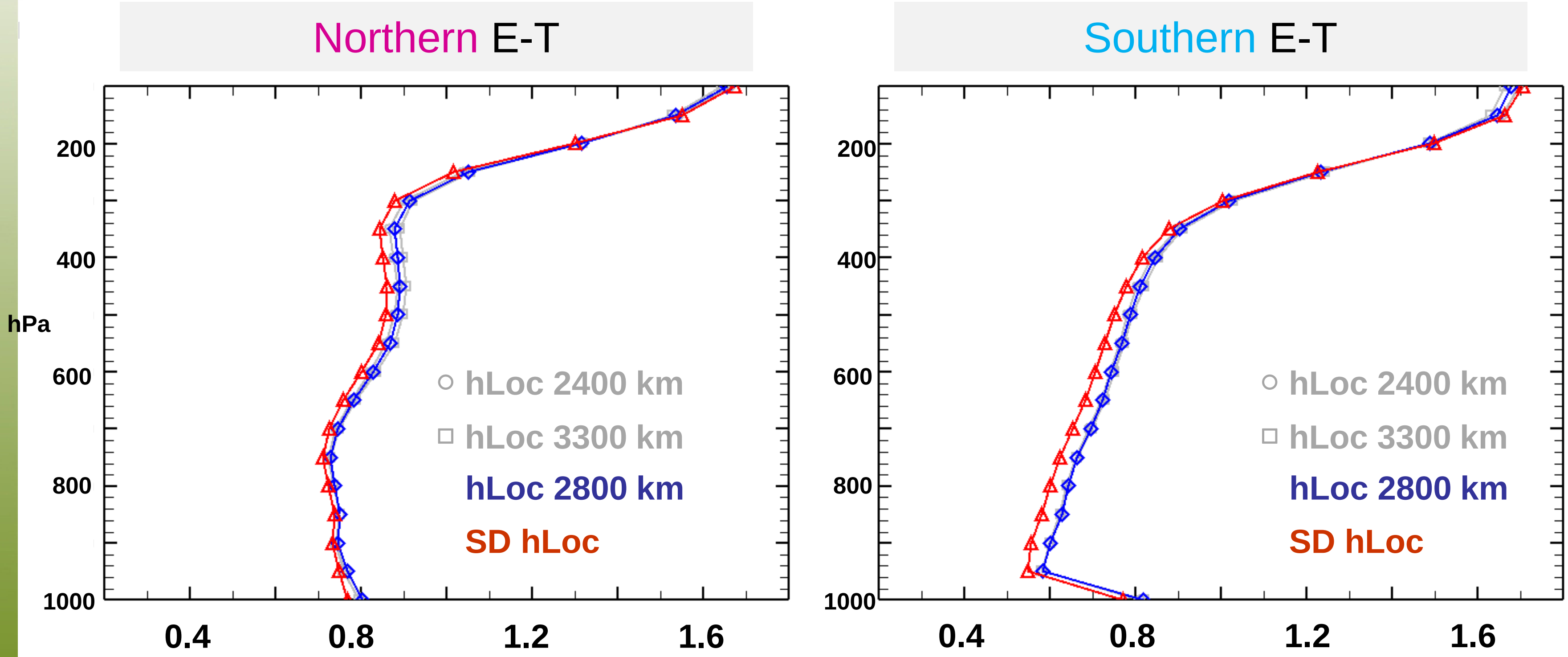
Vertical profile of **average rms(WIND) / average rms(MASS)**



Scale-dependent covariance localization

Impact on dynamical balance – Rotational Part

Vertical profile of average normalized departure from (n-l) balance



Summary and Conclusions

- S-D localization is feasible and straightforward to implement in EnVar, but more expensive than using single-scale localization.
  - In the S-D localization experiments reported here: 3x more expensive.

- Results using a horizontal-scale-dependent horizontal localization indicate small forecast improvements in our global NWP system.

- In terms of dynamical balance, S-D localization does not seem to present any issue for the rotational part of the analysis increments.

- Finding the optimal S-D localization setup is **not** straightforward.
  - For localization radii, taking only sampling errors into account might not be appropriated.

Future Work

- Test S-D localization in higher resolution limited-area applications.
- Examine the impact of adding vertical-scale-dependent vertical localization.

Vertical scale ↑

hLoc-small vLoc-large	hLoc-medium vLoc-large	hLoc-large vLoc-large
hLoc-small vLoc-medium	hLoc-medium vLoc-medium	hLoc-large vLoc-medium
hLoc-small vLoc-small	hLoc-medium vLoc-small	hLoc-large vLoc-small

Horizontal scale →

Questions?

(before we brexit the symposium)

4DEnVar Formulation

- **In 4D-Var…** the 3D analysis increment is evolved in time using the TL/AD forecast model (here included in $\mathbf{H}_{4D}$):

$$J(\Delta\mathbf{x}) = \frac{1}{2}(H_{4D}[\mathbf{x}_b] + \boxed{\mathbf{H}_{4D}\Delta\mathbf{x}} - \mathbf{y})^T \mathbf{R}^{-1}(H_{4D}[\mathbf{x}_b] + \boxed{\mathbf{H}_{4D}\Delta\mathbf{x}} - \mathbf{y}) + \frac{1}{2}\boxed{\Delta\mathbf{x}^T\mathbf{B}^{-1}\Delta\mathbf{x}}$$

- **In EnVar…** the background-error covariances and analysed state are explicitly 4-dimensional, resulting in cost function:

$$J(\Delta\mathbf{x}_{4D}) = \frac{1}{2}(H_{4D}[\mathbf{x}_b] + \boxed{\mathbf{H}\Delta\mathbf{x}_{4D}} - \mathbf{y})^T \mathbf{R}^{-1}(H_{4D}[\mathbf{x}_b] + \boxed{\mathbf{H}\Delta\mathbf{x}_{4D}} - \mathbf{y}) + \frac{1}{2}\boxed{\Delta\mathbf{x}_{4D}^T\mathbf{B}_{4D}^{-1}\Delta\mathbf{x}_{4D}}$$

EnVar is ~10x computationally cheaper than 4DVar

Hybrid **B** formulation

$$\mathbf{B}_{4D} = \beta_{nmc}\mathbf{B}_{nmc} + \beta_{ens}\sum_{k=1}^{N_{ens}}\left(\mathbf{e}_k\mathbf{e}_k^T\right)\circ\mathbf{L}$$

($\mathbf{e}_k$ is k^{th} ensemble perturbation divided by sqrt(N_{ens}-1))

EnVar Formulation - Preconditioning

- Preconditioned cost function formulation at EC:

$$J(\xi) = \frac{1}{2}\xi^T\xi + \frac{1}{2}\left(H_{4D}(\mathbf{x}_b) + \mathbf{H}\Delta\mathbf{x}(\xi) - \mathbf{y}\right)^T \mathbf{R}^{-1}\left(H_{4D}(\mathbf{x}_b) + \mathbf{H}\Delta\mathbf{x}(\xi) - \mathbf{y}\right)$$

- In EnVar with hybrid covariances, the control vector (ξ) is composed of 2 vectors:

$$[\xi] = \begin{bmatrix} \xi_{nmc} \\ \xi_{ens} \end{bmatrix} \rightarrow [\xi_{ens}] = \begin{bmatrix} \xi_{ens}^{1} \\ \vdots \\ \xi_{ens}^{N_{ens}} \end{bmatrix}$$

- The analysis increment is computed as (e_k is *k*'th ensemble perturbation divided by sqrt(N_{ens}-1)):

$$\Delta\mathbf{x}(\xi) = \beta_{nmc}^{1/2}\mathbf{B}_{nmc}^{1/2}\xi_{nmc} + \beta_{ens}^{1/2}\sum_{k=1}^{N_{ens}}\mathbf{e}_k \circ \left(\mathbf{L}^{1/2}\xi_{ens}^{k}\right) \rightarrow \mathbf{B} = \beta_{nmc}\mathbf{B}_{nmc} + \beta_{ens}\sum_{k=1}^{N_{ens}}\left(\mathbf{e}_k\mathbf{e}_k^T\right)\circ\mathbf{L}$$