# Combining data assimilation and moving meshes for moving boundary processes: application to ice sheet modelling

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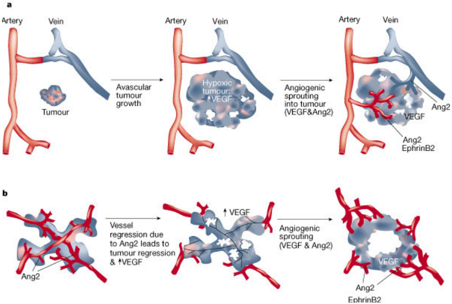
# Outline

1. Motivations
2. Moving meshes
3. Data assimilation

# Outline

# Motivation: estimate the evolution of moving boundary problems

Many applications can be modelled by moving boundary problems.



# Motivation: assimilate new sources of observations

Figure: Retreat of the Rhone glacier, comparison between postcard from 1870 and actual view in Gletsch, Switzerland (Image credit: Dominic Buettner for The New York Times)

# Outline

# A simplified model

Radially-symmetric grounded ice sheet under Shallow Ice Approximation

$$
\begin{cases}
\partial_t h = m - \frac{1}{r}\,\partial_r(r\,h\,U_r) & r \in (0, r_l(t)) \\
U_r = -\,c\,h^{n+1}\,|\partial_r h|^{n-1}\,\partial_r h & r \in (0, r_l(t)) \\
U_r = \partial_r h = 0 & r = 0 \\
h(0, r) = h_0(r) & r \in (0, r_l(0)) \\
h(t, r_l(t)) = 0 & t \geq 0
\end{cases}
$$

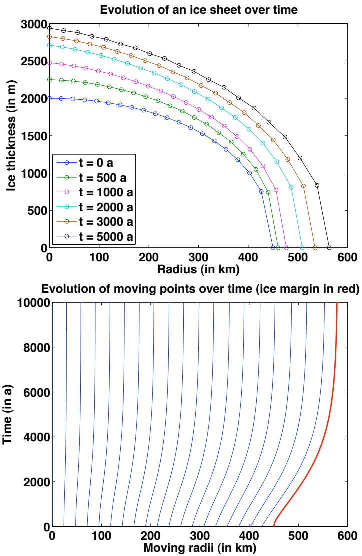
with:

- $r$ radius between 0 (ice divide) and $r_l(t)$ (ice sheet margin)
- $h(t, r)$ ice thickness
- $U_r(t, r)$ vertically averaged ice velocity
- $m(t, r)$ surface mass balance
- $c > 0$ constant, $n$ exponent ($n = 3$)

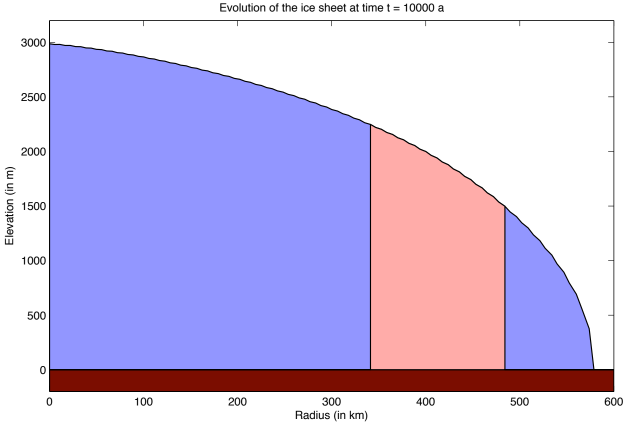
# Moving mesh method

- Physical quantities ($h$, ...) calculated on a moving grid with a fixed number of evolving radii $\widehat{r}_i(t)$.
- One moving radius for
  - ice divide $\widehat{r}_1(t) = 0$
  - ice sheet margin $\widehat{r}_{n_r}(t) = r_l(t)$
- **Strategy**:
  At given time, geometry of ice sheet known
  - calculate velocity of moving radii.

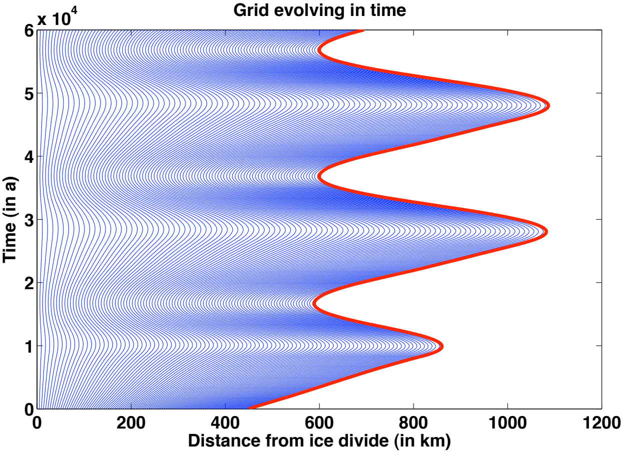
  $\longrightarrow$ next time step
  - update radii
  - update ice sheet geometry



# An example with 100 moving points



# Trajectory of moving points



# Conserved mass fraction

- Trajectories of moving radii are defined such that **relative volume fraction** $\mu_i$ between 0 and $\widehat{r}_i(t)$ kept constant in time.

$$\mu_i = \frac{2\pi}{\theta(t)} \int_0^{\widehat{r}_i(t)} r\, h(t,r)\, dr \quad \text{with} \quad \theta(t) = 2\pi \int_0^{r_I(t)} r\, h(t,r)\, dr$$

- Velocity of moving radii obtained implicitly by differentiating

$$\frac{d}{dt}\left(\int_0^{\widehat{r}_i(t)} r\, h(t,r)\, dr\right) = \mu_i \frac{d}{dt}\left(\int_0^{r_I(t)} r\, h(t,r)\, dr\right)$$

- The ice thickness profile is updated using

$$h(t, \widehat{r}(t)) = \frac{\theta(t)}{\pi} \frac{d\mu(\widehat{r})}{d(\widehat{r}^2)}$$

# Outline

# What we estimate

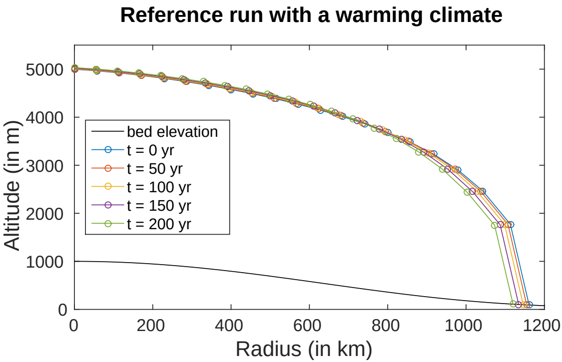
Location of moving points and ice thickness at these locations (number of grid points $n_r = 21$)

$$\mathbf{x} = \begin{pmatrix} \mathbf{x}_h \\ \mathbf{x}_r \end{pmatrix} \quad \text{with} \quad \mathbf{x}_h = \begin{pmatrix} h_1 \\ \vdots \\ h_{n_r-1} \end{pmatrix}, \mathbf{x}_r = \begin{pmatrix} \widehat{r}_2 \\ \vdots \\ \widehat{r}_{n_r} \end{pmatrix}$$

# What we observe

Observations are obtained from a reference run at different times ($t = 1, 2, \ldots, 10$ yr) and perturbed with a Gaussian noise:

- surface ice velocity $u_s$ at 20 different locations, $\sigma_u^o = 30$ m/yr, uncorrelated noise.
- position of ice sheet margin, $\sigma_r^o = 50$ km.



# Initial ensemble

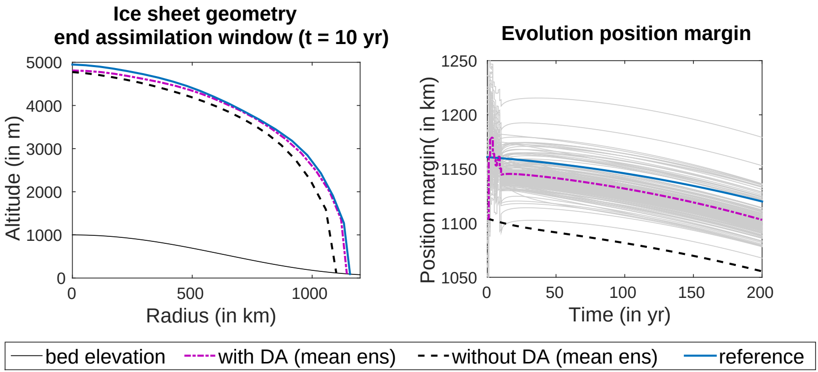
The $N_e = 200$ members of the initial ensemble are generated from a background state with an added Gaussian noise $\mathcal{N}(0, \mathbf{B})$:

- Background state: $\mathbf{x}^b = 0.95\,\mathbf{x}^{ref}(0)$.
- Background covariance matrix:

$$\mathbf{B} = \begin{pmatrix} \mathbf{B}_h & \mathbf{B}_{rh}^T \\ \mathbf{B}_{rh} & \mathbf{B}_r \end{pmatrix}$$

- Cross-covariances $\mathbf{B}_{rh}$ set to zero.
- Covariances for ice thickness $\mathbf{B}_h$: SOAR function with $\sigma_h = 200$ m and length scale $L_h = 240$ km for the spatial correlations.
- Covariances for point locations $\mathbf{B}_r$: SOAR function with $\sigma_r = 60$ km (and reduced variance for points close to $r = 0$) and length scale $L_r = 240$ km for the spatial correlations.

# DA results with inflation 1.10



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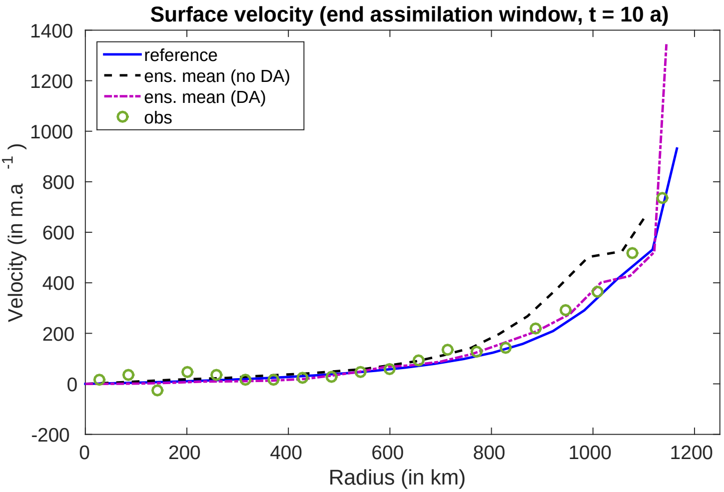

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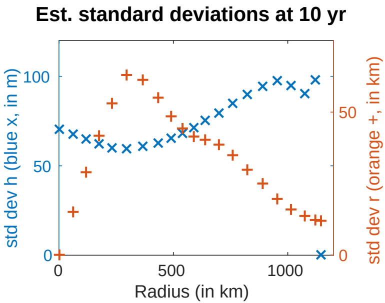


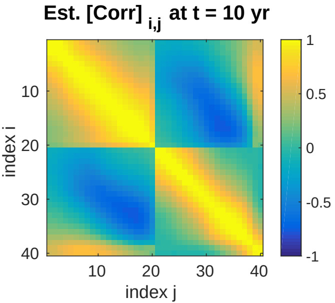

# Conclusion and Prospects

Conclusions:

- Successful combination of DA with moving meshes in 1d is done by including the position of grid nodes in the state vector.
- Ensemble Kalman filter gives information on cross-covariance between the grid and physical variables.
- This approach allows the straightforward assimilation of the position of boundaries.

Prospects:

- Extend the approach to 2d cases.
- Generalize the approach to other moving boundary problems.

# Thank you for your attention!