Simulation of error cycling

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with inputs from

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Motivations and questions

EDA and innovations for diagnosing contributions (background errors, observation errors, model errors) in error cycling over one week.

Revisit formalism & some previous EDA experiments in the litterature :

$\Rightarrow$ Respective global amplitudes of these 3 error sources ?

$\Rightarrow$ Evolution of error contributions during the cycling ?

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- EDA context at Météo-France
- Quasi-linear expansion of forecast errors
- Old background errors / Recent observation errors
- Observation errors / Model errors

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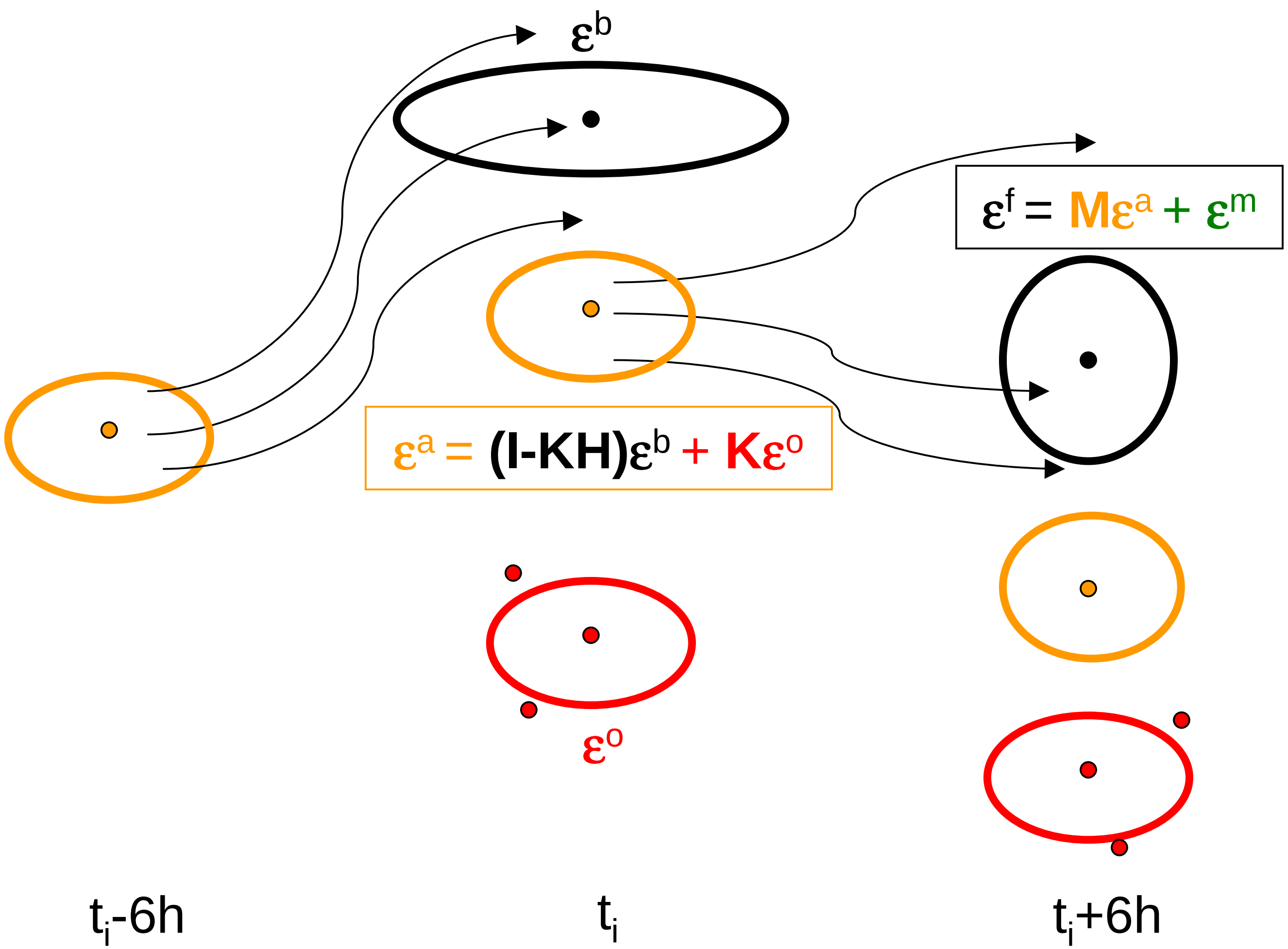
Ensemble of perturbed Data Assimilations (EDA) : simulation of error cycling



(e.g. Houtekamer et al 1996, Fisher 2003, Berre et al 2006)

Global operational EDA at Météo-France

- 25 members, T479 (40 km) L105, Arpege 4D-Var (minim T149), 6h cycle.

- 4D-Var analysis perturbations :

  observation perturbations (drawn from **R**, incl. spatial corr. for AMVs),

  background perturbations (evolved analysis pertbs and model pertbs).

- Multiplicative inflation of forecast perturbations,

  using innovation-based diagnostics.

- Spatially filtered variances for observation QC and minimisation,

  wavelet-filtered 3D correlations.

An EDA is also being developed at mesoscale (AROME, oper 2018).

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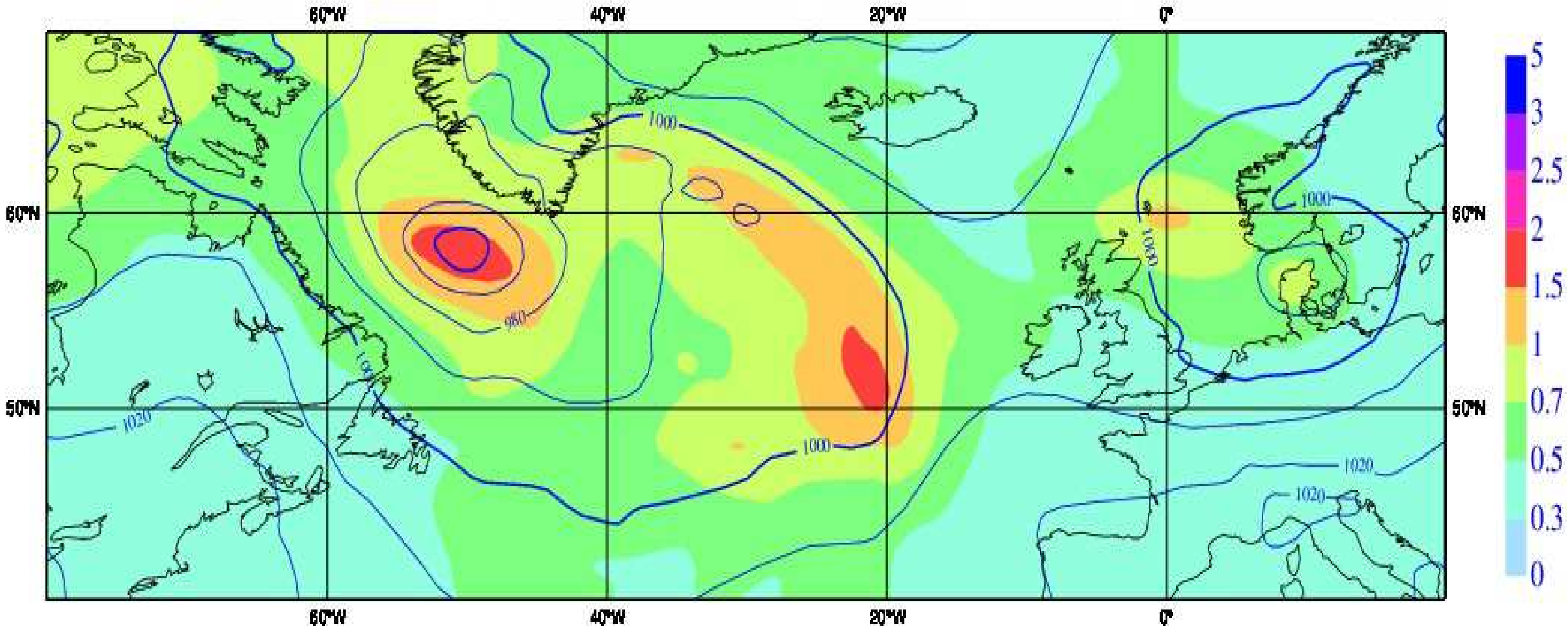
Dynamics of background error variances with EDA



Standard deviations of surface pressure (hPa) (2/2/2010)
(superimposed with MSLP analysis, in hPa).

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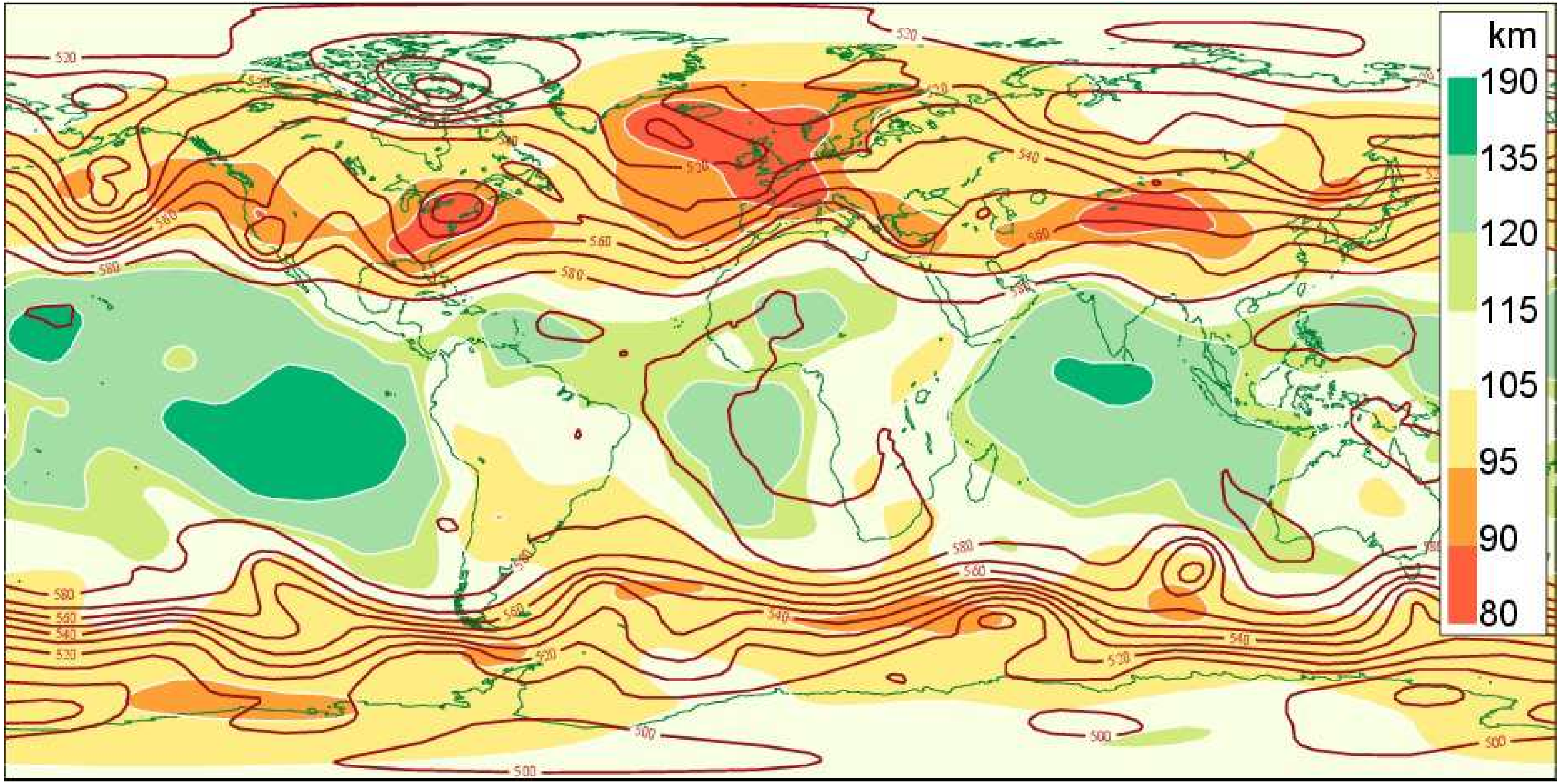
(e.g. Berre et al 2007, Raynaud et al 2012)

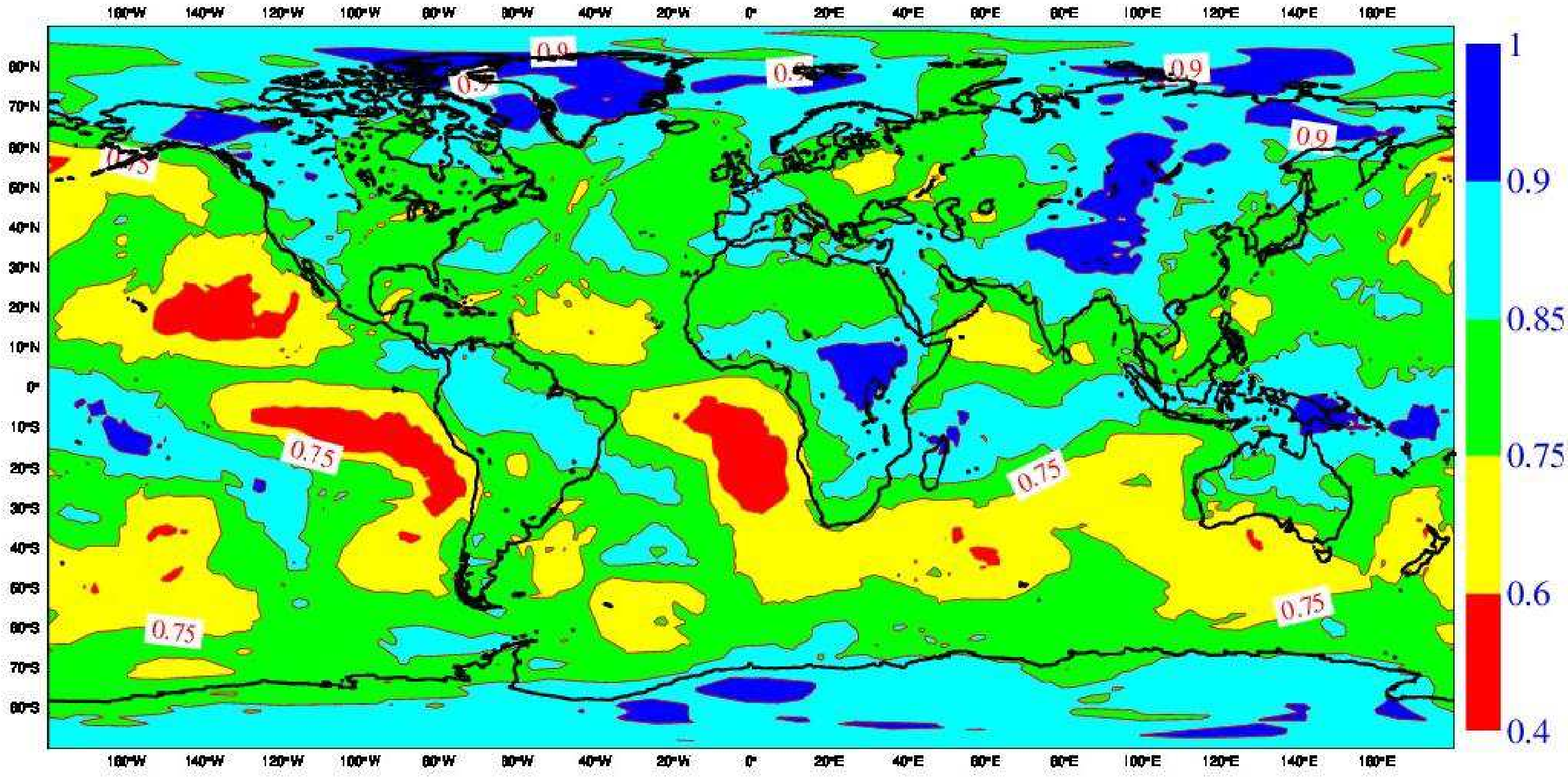
Dynamics of horizontal correlations from ensemble and wavelet filtering



Length scales (km) for wind near 500 hPa (28/2/2010),
superimposed with geopotential

(Berre, Varella et Desroziers 2015)

Dynamics of vertical correlations from ensemble and wavelet filtering



Vertical correlations of temperature between 850 & 870 hPa (28/2/2010)

(Berre, Varella and Desroziers 2015)

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Expansion of forecast error contributions (quasi-linear framework)

At a given step t_0 of the cycling :

$$\varepsilon^a_0 = (\mathbf{I}-\mathbf{K}_0\mathbf{H}_0)\,\varepsilon^b_0 + \mathbf{K}_0\,\varepsilon^o_0$$

$$\varepsilon^f_0 = \mathbf{M}_0\,\varepsilon^a_0 + \varepsilon^m_0$$

$$= \mathbf{M}_0(\mathbf{I}-\mathbf{K}_0\mathbf{H}_0)\,\varepsilon^b_0 + \mathbf{M}_0\mathbf{K}_0\,\varepsilon^o_0 + \varepsilon^m_0$$

At a later step t_i (e.g. one week later) :

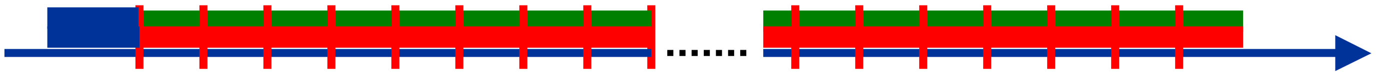
$$\varepsilon^f_i = \mathbf{T}_{i+1}\,\varepsilon^b_0 + \sum_{j\geq 0}\mathbf{T}_{i-j}\,\mathbf{M}_j\mathbf{K}_j\,\varepsilon^o_j + \sum_{j\geq 0}\mathbf{T}_{i-j}\,\varepsilon^m_j$$

where $\mathbf{T}_{i-j} = \prod_k \mathbf{M}_k\,(\mathbf{I}-\mathbf{K}_k\mathbf{H}_k)$ = cycling operator
(over k successive analysis/forecast steps from t_j to t_i).

(El Ouaraini and Berre 2011)

Age of error contributions

Error contributions over one week, from t_0 to t_i :



Full expansion of forecast errors at t_i :

$$\varepsilon^f_i = \mathbf{T}_{i+1}\, \varepsilon^b_0 + \sum_{j \geq 0} \mathbf{T}_{i-j}\, \mathbf{M}_j\, \mathbf{K}_j\, \varepsilon^o_j + \sum_{j \geq 0} \mathbf{T}_{i-j}\, \varepsilon^m_j$$

where $\mathbf{T}_{i-j} = \prod_k \mathbf{M}_k(\mathbf{I}-\mathbf{K}_k\mathbf{H}_k)$ = cycling operator

(over k successive analysis/forecast steps from t_j to t_i).

(El Ouaraini and Berre 2011)

Several processes in error cycling

Expansion of forecast errors :

$$\varepsilon^f_i = \mathbf{T}_{i+1}\,\varepsilon^b_0 + \sum_{j\geq 0} \mathbf{T}_{i-j}\,\mathbf{M}_j\,\mathbf{K}_j\,\varepsilon^o_j + \sum_{j\geq 0} \mathbf{T}_{i-j}\,\varepsilon^m_j$$

where $\mathbf{T}_{i-j} = \prod_k \mathbf{M}_k\,(\mathbf{I}-\mathbf{K}_k\mathbf{H}_k)$ = cycling operator.

- **Old background errors** ε^b_0 :
  repeted analysis damping & model propagation.

- **Recent observation errors** ε^o_j :
  filtering (**K**) & propagation (**M**) ; damping & propagation (**T**) ; accumulation (Σ).

- **Recent model errors** ε^m_j :
  damping & propagation ; accumulation.

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Links with EDA and innovations

Same equation for errors and perturbations :

$$\varepsilon^f_i = \mathbf{T}_{i+1}\, \varepsilon^b_0 + \sum_{j \geq 0} \mathbf{T}_{i-j}\, \mathbf{M}_j\, \mathbf{K}_j\, \varepsilon^o_j + \sum_{j \geq 0} \mathbf{T}_{i-j}\, \varepsilon^m_j$$

Simulation of error cycling : cycle observation and model perturbations added to deterministic system (e.g. with 4D-Var and non linear forecasts included).

Observation and model perturbations are often (+/- indirectly) derived from innovation-based estimates of covariances.

Amplitudes of some estimated (accumulated) error contributions can be diagnosed and compared using EDA and innovations.

Experimental framework

- Revisit some sensitivity experiments with a previous EDA configuration :
  no model perturbations (offline tuning of horizontally averaged variances) ;
  same **K** in all compared configurations (static spectral correlations,
  flow-dependent variances).

- Comparison « cold start » / « warm start » ensembles :
  diagnosis / evolution of old background perturbations.

- EDA configurations without (recent) model perturbations :
  ° to diagnose the 2 other contributions
  (old background errors, recent observation errors).
  ° to estimate model error variance
  by comparison with innovation-based estimates of forecast errors.

- Offline diagnosis of global amplitudes (variances) of perturbations.

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Contribution of recent observation errors

$$\varepsilon^f_i = \mathbf{T}_{i+1}\,\varepsilon^b_0 + \sum_{j\geq 0} \mathbf{T}_{i-j}\,\mathbf{M}_j\mathbf{K}_j\varepsilon^o_j + \sum_{j\geq 0} \mathbf{T}_{i-j}\,\varepsilon^m_j$$

Contribution of recent observation errors to forecast errors :

$$\varepsilon^{fo}_i = \sum_{j\geq 0} \mathbf{T}_{i-j}\,\mathbf{M}_j\mathbf{K}_j\varepsilon^o_j$$

Simulation and evolution using EDA :

* use $\varepsilon^b_0=0$ (cold start = start from unperturbed background),

then cycle with ε^o_j ($j\geq 0$) (and with $\varepsilon^m_j = 0$).

* compute evolution of EDA spread : $\sigma(\varepsilon^{fo}_t = \sum \mathbf{T}_{t-j}\,\mathbf{M}_t\mathbf{K}_t\varepsilon^o_t)$.

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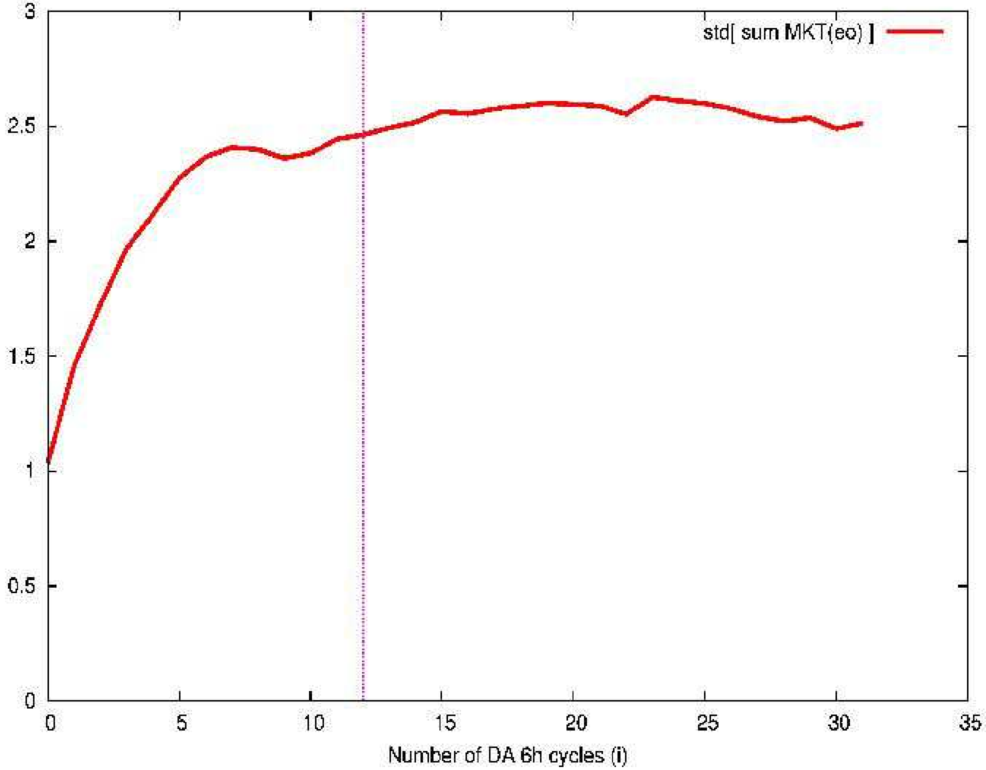
Accumulation of recent observation errors

Evolution of $\sigma(\varepsilon^{fo}_t = \sum_{j \geq 0} \mathbf{T}_{t-j} \mathbf{M}_j \mathbf{K}_j \varepsilon^o_j)$

at successive analysis steps (t_0, t_1, … t_i)

$\sigma(\varepsilon^{fo}_t)$

(vorticity (10^{-5} s^{-1}), 500 hPa)



Observation error contribution increases (accumulation), then it stabilizes after 3-5 days of cycling, due to analysis damping : $\mathbf{T}_{t-j} = \prod_k \mathbf{M}_k (\mathbf{I}-\mathbf{K}_k\mathbf{H}_k)$.

(El Ouaraini and Berre 2011, see also Fisher et al 2005)

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Accumulation of recent observation errors

Evolution of $\varepsilon^{fo}_t = \sum_{j \geq 0} \mathbf{T}_{t-j}\, \mathbf{M}_j \mathbf{K}_j \varepsilon^{o}_j$

at successive analysis steps (t_0, t_1, t_2, … t_i) :

$$\varepsilon^{fo}_0 = \mathbf{M}_0 \mathbf{K}_0\, \varepsilon^{o}_0$$

$$\varepsilon^{fo}_1 = \mathbf{M}_1 \mathbf{K}_1\, \varepsilon^{o}_1 + \mathbf{T}_1\, \varepsilon^{fo}_0$$

$$\varepsilon^{fo}_2 = \mathbf{M}_2 \mathbf{K}_2\, \varepsilon^{o}_2 + \mathbf{T}_1\, \varepsilon^{fo}_1 + \mathbf{T}_2\, \varepsilon^{fo}_0$$

….

$$\varepsilon^{fo}_t = \mathbf{M}_t \mathbf{K}_t\, \varepsilon^{o}_t + \mathbf{T}_1\, \varepsilon^{fo}_{t-1} + \mathbf{T}_2\, \varepsilon^{fo}_{t-2} + \ldots + \mathbf{T}_{12}\, \varepsilon^{fo}_{t-12} + \sum_{k>12} \mathbf{T}_k\, \varepsilon^{fo}_{t-k}$$

…

T is mainly damping, so old obs. error contrib. (with time-lag k>12) become negligible, and $\sigma(\varepsilon^{fo}_t)$ stabilizes ~ at step t = 12 (3 days).

Contribution of old background errors

$$\varepsilon^f_i = \mathbf{T}_{i+1}\,\varepsilon^b_0 + \sum_{j\geq 0} \mathbf{T}_{i-j}\,\mathbf{M}_j\,\mathbf{K}_j\,\varepsilon^o_j + \sum_{j\geq 0} \mathbf{T}_{i-j}\,\varepsilon^m_j$$

Contribution of « old » background errors :

$$\varepsilon^{fb}_i = \mathbf{T}_{i+1}\,\varepsilon^b_0 \qquad \text{with } \mathbf{T}_{i+1} = \prod_{k=0}^{i} \mathbf{M}_k\,(\mathbf{I}-\mathbf{K}_k\mathbf{H}_k)$$

Simulation and evolution using EDA :

use warm start = ensemble started 6 days before t_0 :

* use $\varepsilon^b_0 = \sum_{j<0} \mathbf{T}_{0-j}\,\mathbf{M}_j\,\mathbf{K}_j\,\varepsilon^o_j$ (= contribution of old observation errors),

  then cycle with ε^o_j ($j \geq 0$) (and with $\varepsilon^m_j = 0$).

* compute evolution of sqrt of $\sigma^2(\varepsilon^{fb}_t) = \sigma^2(\varepsilon^{fb}_t + \varepsilon^{fo}_t) - \sigma^2(\varepsilon^{fo}_t)$

  (~sqrt of EDA variance difference between warm start and cold start)

Evolution of old background error contribution

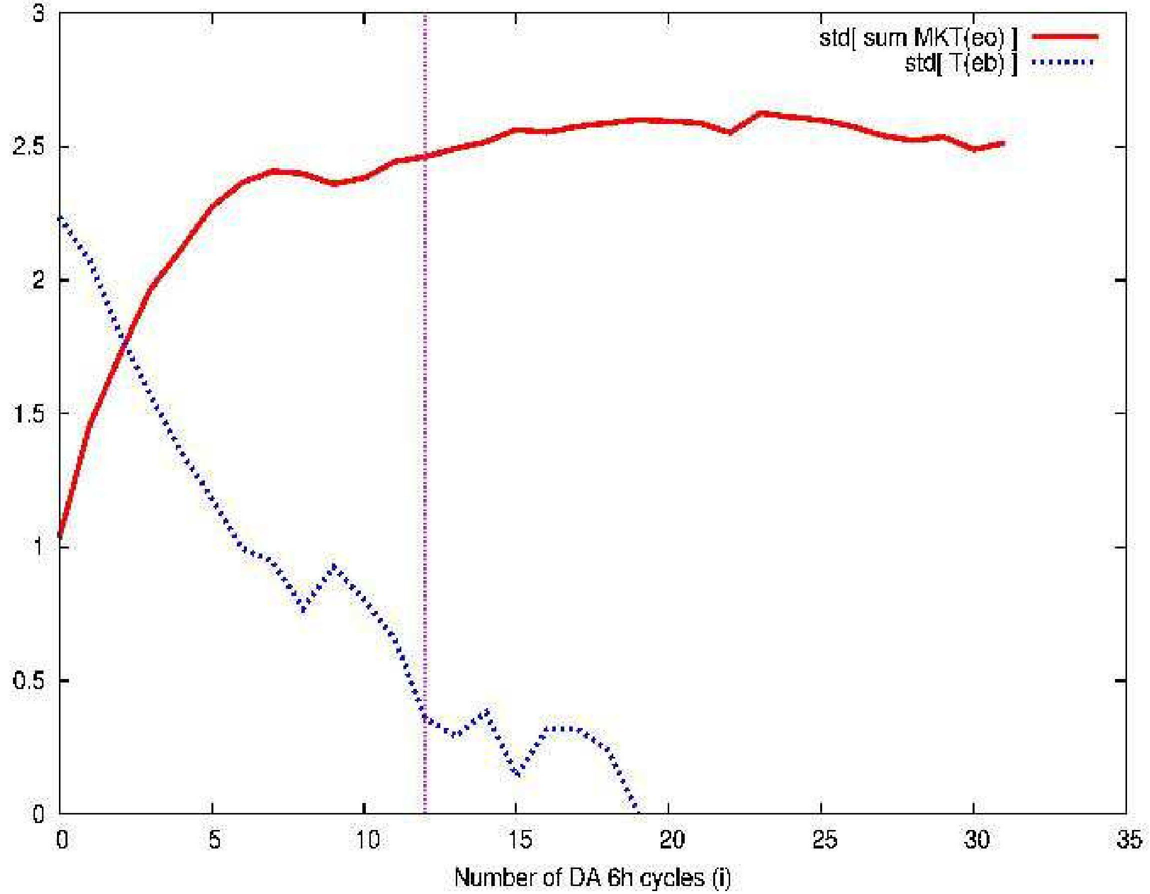
$\sigma(\varepsilon^{fb}_t = \mathbf{T}_{t+1}\, \varepsilon^b_0)$

with

$\mathbf{T}_{t+1} = \prod_{k=0}^{t} \mathbf{M}_k\, (\mathbf{I}-\mathbf{K}_k\mathbf{H}_k)$

$\sigma(\varepsilon^{fo}_t)$

(vorticity ($10^{-5}\ s^{-1}$), 500 hPa)



$\varepsilon^b_0 = \sum_{j<0} \mathbf{T}_{0-j}\, \mathbf{M}_j\, \mathbf{K}_j\, \varepsilon^o_j$ (contribution of old observation errors)

$\sigma(\varepsilon^{fb}_0) > \sigma(\varepsilon^{fo}_0)$ (for vorticity).

$\sigma(\varepsilon^{fb}_i = \mathbf{T}_{i+1}\, \varepsilon^b_0)$ decreases ~ linearly / analysis damping.

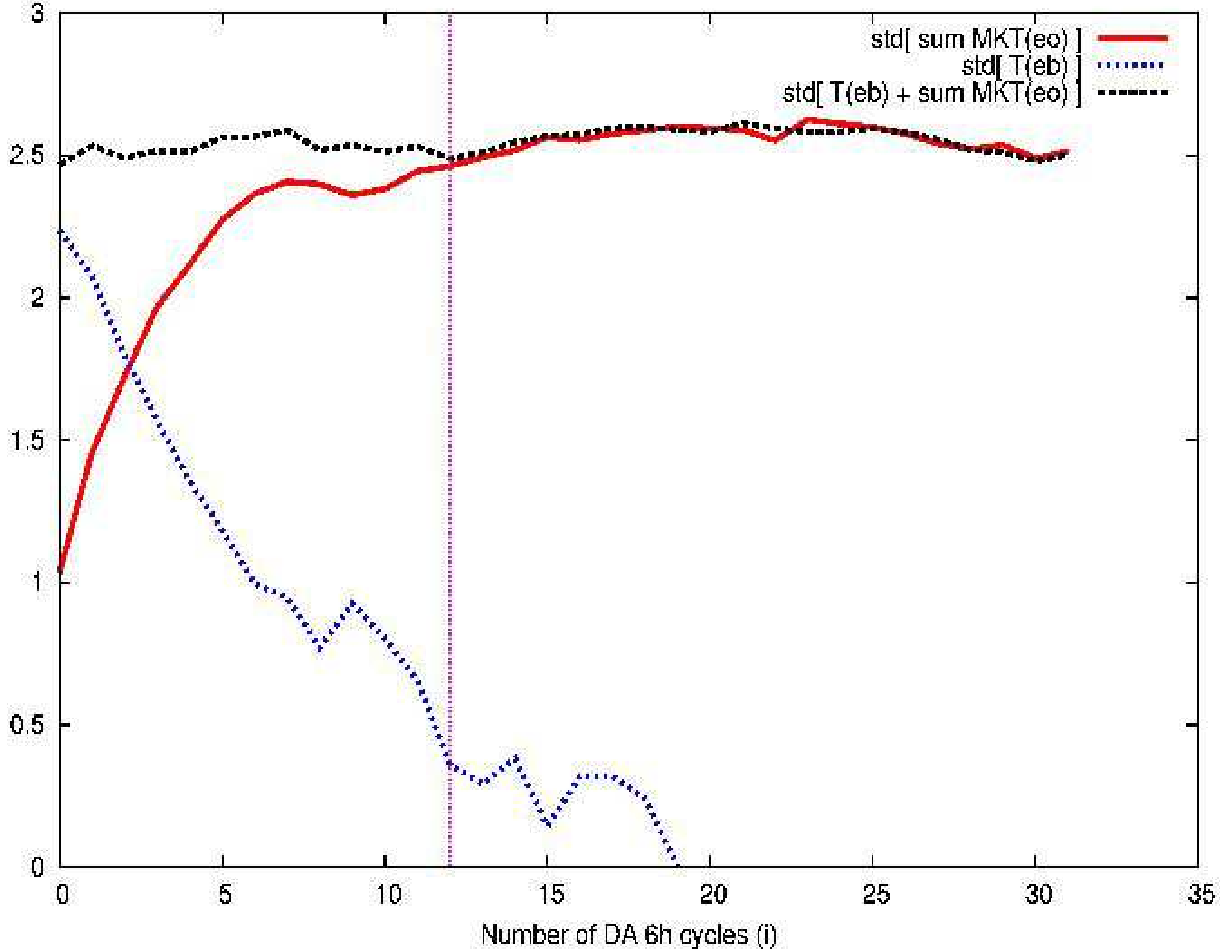
Contribution of old+recent observation errors

$\sigma(\varepsilon^{fb}_t + \varepsilon^{fo}_t)$

$\sigma(\varepsilon^{fb}_t = \mathbf{T}_{t+1}\, \varepsilon^{b}_0)$

$\sigma(\varepsilon^{fo}_t)$

(vorticity (10^{-5} s^{-1}), 500 hPa)

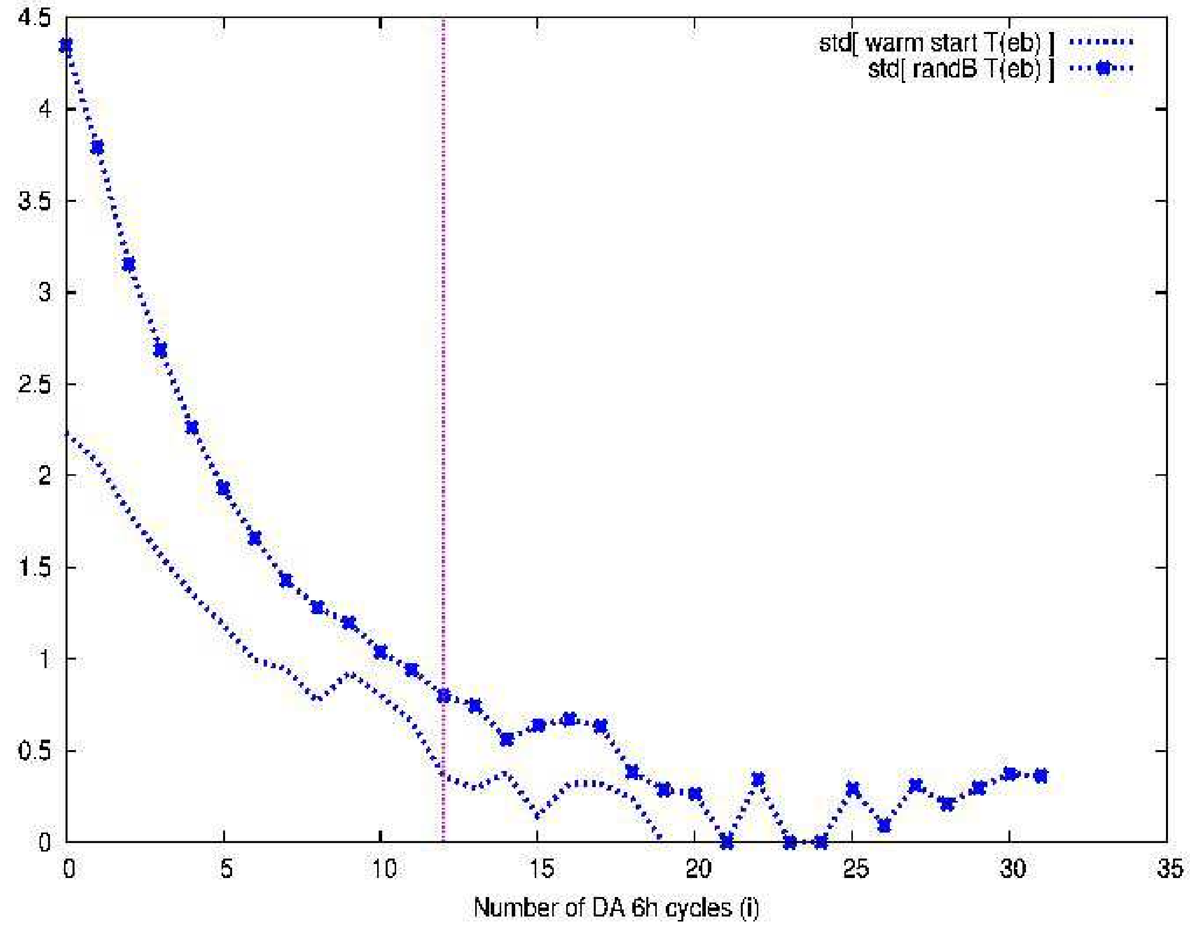


Total contribution of (old+recent) obs errors is stable :
damping of old observation errors is compensated by
accumulation of recent obs errors.

Dependence on amplitude of old background errors

$\sigma(\varepsilon^{fb}_t = \mathbf{T}_{t+1}\, \varepsilon^b_0)$

(vorticity ($10^{-5}\ s^{-1}$), 500 hPa)



$\varepsilon^b_0 = \sum_{j<0} \mathbf{T}_{0-j}\, \mathbf{M}_j\, \mathbf{K}_j\, \varepsilon^o_j$ (old observation errors)

$\varepsilon^b_0 = \mathbf{B}^{1/2}\, \eta$ (old model errors + old observation errors)

$\sigma(\varepsilon^{fb}_t = \mathbf{T}_{t+1}\, \varepsilon^b_0)$ decreases ~ linearly / analysis damping.

First conclusions / formalism + experiments

- Quasi-linear expansion of forecast errors / contributions of old background errors, recent observation errors, recent model errors.
- Compare cold/warm start EDA (without recent model perturbations) to diagnose error contrib. of old background and recent observations.
- Old background errors vanish (~linearly) after 3-5 days of cycling.
- Compensated by accumulation of recent observation errors.
- This may help for interpretation/diagnosis of total forecast errors (~ recent observation & model errors).

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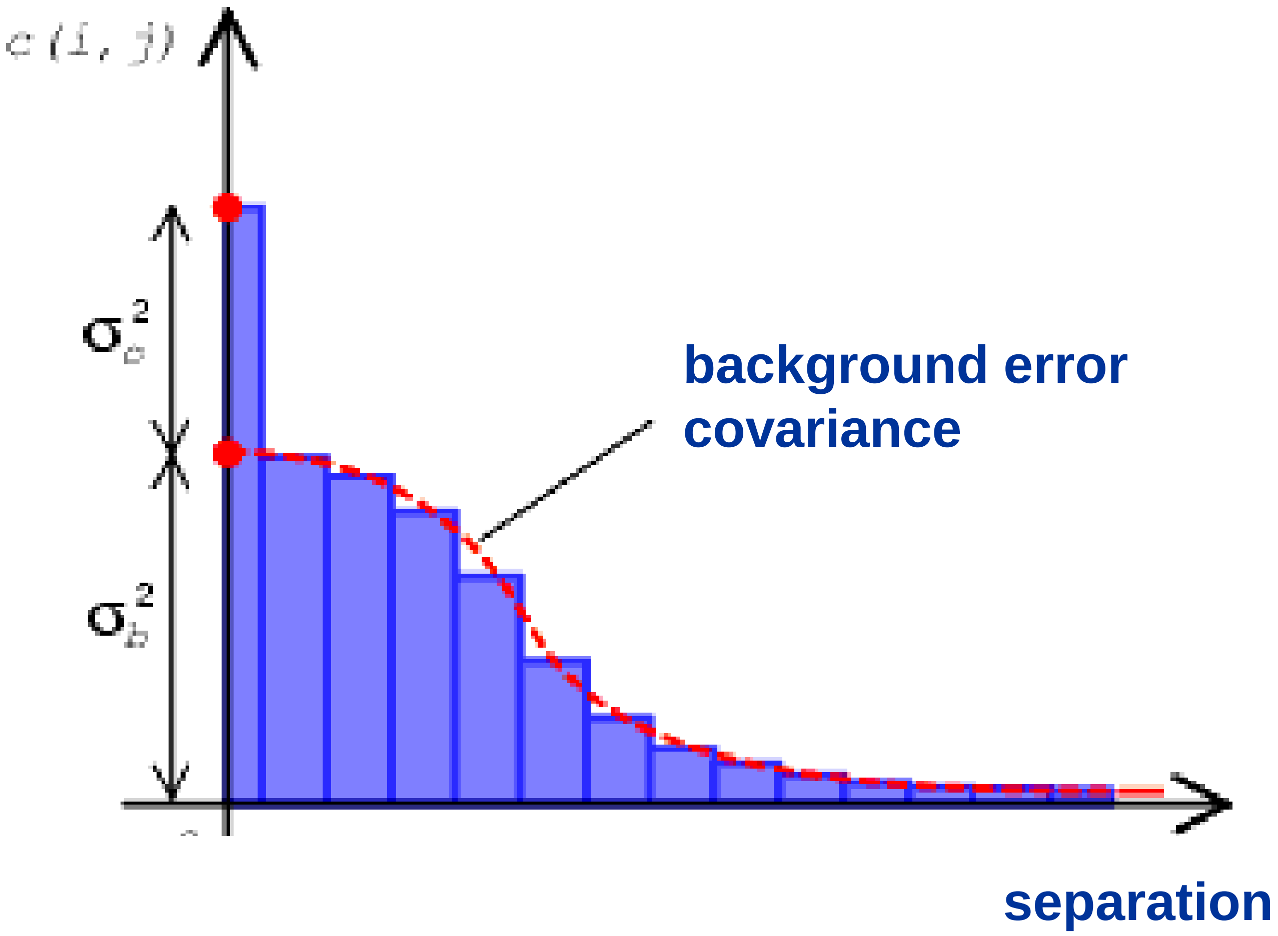
Innovation covariances



$$E[\,(\mathbf{y}-\mathbf{Hx}^b)(\mathbf{y}-\mathbf{Hx}^b)^T\,] = \mathbf{R} + \mathbf{HBH}^T$$

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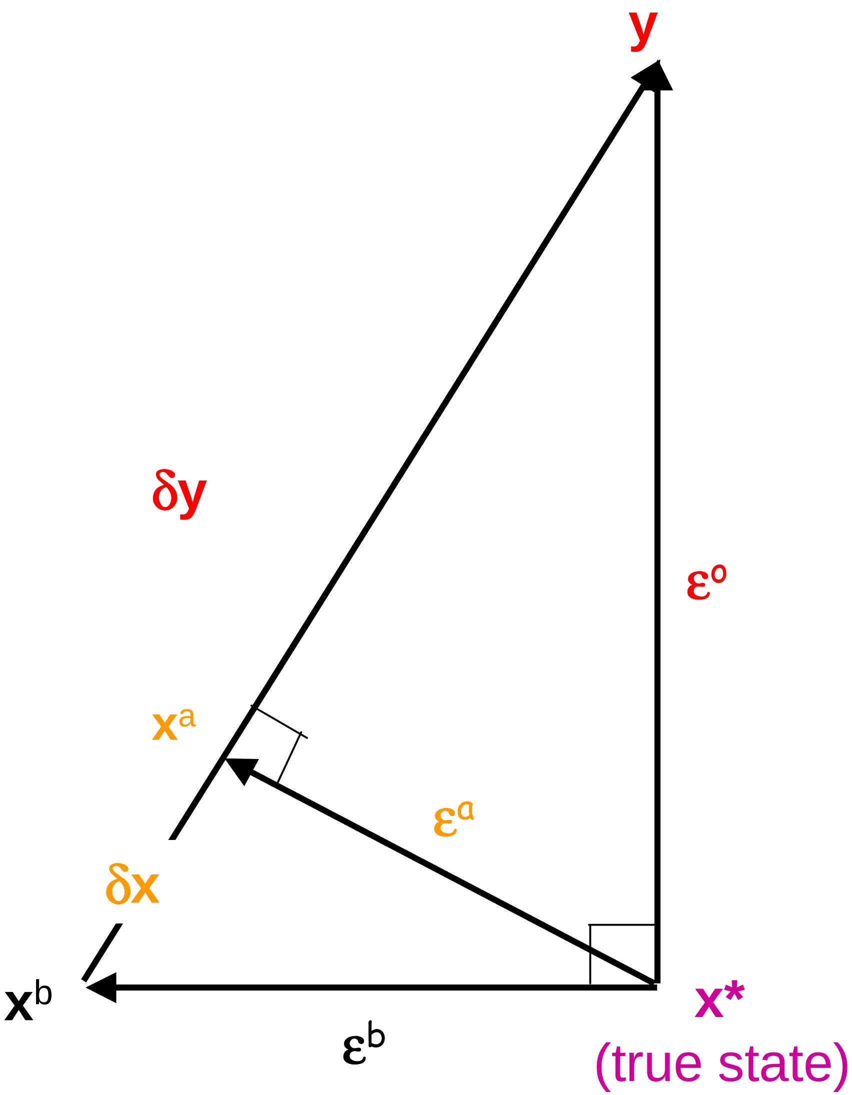
Covariances of analysis residuals



$\delta \mathbf{y} = \mathbf{y} - H(\mathbf{x}^b)$ (innovation)

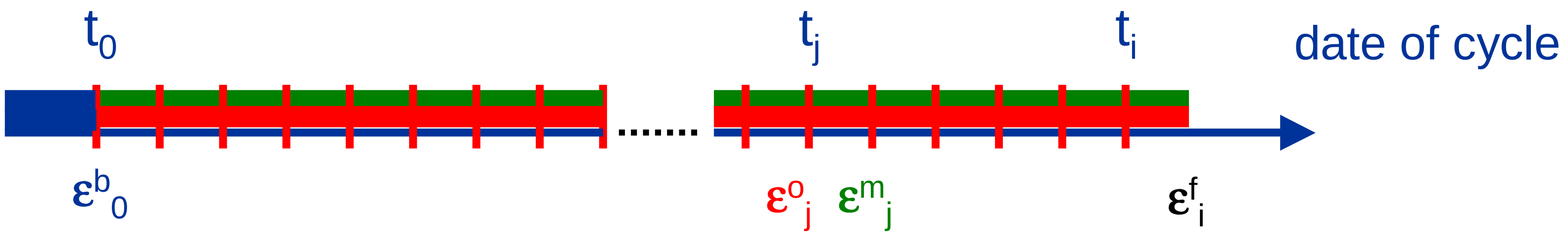
$\mathbf{H}\, \delta \mathbf{x} = H(\mathbf{x}^b) - H(\mathbf{x}^a)$ (increment)

$E[\mathbf{H}\, \delta \mathbf{x}\, \delta \mathbf{y}^T] = \mathbf{HB^*H}^T$

$E[(\mathbf{y} - H(\mathbf{x}^a))\, \delta \mathbf{y}^T] = \mathbf{R^*}$

(Desroziers et al 2005)

Estimation of model error contributions (to forecast errors)



Error ε^f_i of forecast issued from cycle t_i :

$$\varepsilon^f_i = \mathbf{T}_{i+1}\,\varepsilon^b_0 + \underbrace{\sum_{j\geq 0} \mathbf{T}_{i\text{-}j}\,\mathbf{M}_j\,\mathbf{K}_j\,\varepsilon^o_j}_{\text{obs. errors}} + \underbrace{\sum_{j\geq 0} \mathbf{T}_{i\text{-}j}\,\varepsilon^m_j}_{\text{model errors}} \qquad [\,\mathbf{T}_{i\text{-}j} = \prod_k \mathbf{M}_k\,(\mathbf{I}\text{-}\mathbf{K}_k\mathbf{H}_k)\,]$$

obs. errors model errors

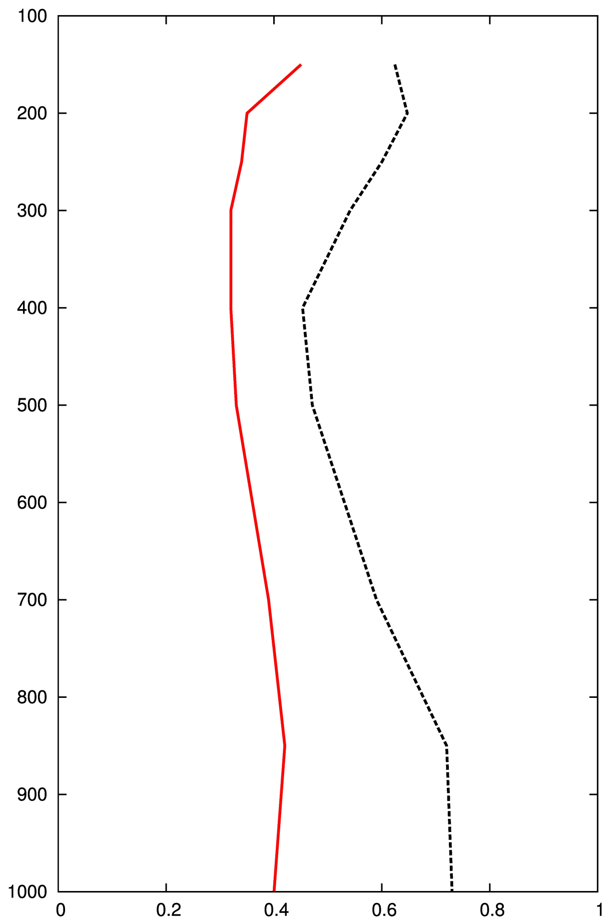
accumulated from t_0 to t_i

$\Rightarrow$ Compare $\sigma_{\delta y}($ $\varepsilon^f_i = \varepsilon^{fb0}_i + \Sigma\,\mathbf{T}_{i\text{-}j}\,\mathbf{M}_j\,\mathbf{K}_j\,\varepsilon^o_j + \Sigma\,\mathbf{T}_{i\text{-}j}\,\varepsilon^m_j\,)$ (innovation-based)

with $\sigma_{EDA}($ $\varepsilon^{fb0}_i + \Sigma\,\mathbf{T}_{i\text{-}j}\,\mathbf{M}_j\,\mathbf{K}_j\,\varepsilon^o_j\,)$ (using unperturbed model from t_0 to t_i).

Total forecast error versus observation error contribution

Pressure (hPa)



$\sigma_{\delta y}(\varepsilon^{f}_{i} = \varepsilon^{fb0}_{i} + \sum \mathbf{T}_{i-j}\mathbf{M}_{j}\mathbf{K}_{j}\varepsilon^{o}_{j} + \sum \mathbf{T}_{i-j}\, \varepsilon^{m}_{j})$

$\sigma_{EDA}(\varepsilon^{fb0}_{i} + \sum \mathbf{T}_{i-j}\mathbf{M}_{j}\mathbf{K}_{j}\varepsilon^{o}_{j})$

Standard deviation of forecast errors
(aircraft observations of temperature (K))

(Raynaud et al 2012)

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Total forecast error versus observation error contribution

$$\sigma^2_{\delta y}(\varepsilon^{fb0}_i + \Sigma\, T_{i-j} M_j K_j \varepsilon^o_j + \Sigma\, T_{i-j}\, \varepsilon^m_j) \sim \sigma^2(\Sigma\, T_{i-j} M_j K_j \varepsilon^o_j) + \sigma^2(\Sigma\, T_{i-j}\, \varepsilon^m_j)$$

If $\mathbf{cov}(\varepsilon^{fb0}_i , \varepsilon^{fm}_i) \sim 0$, i.e.

if the one-week period is large enough to neglect time correlations

between old background errors & recently accumulated model errors,

in addition to :

$$\sigma^2_{EDA}(\varepsilon^{fb0}_i + \Sigma\, T_{i-j} M_j K_j \varepsilon^o_j) = \sigma^2_{EDA}(\Sigma\, T_{i-j} M_j K_j \varepsilon^o_j) \qquad \text{if } \sigma^2(\varepsilon^{fb0}_i) \sim 0$$

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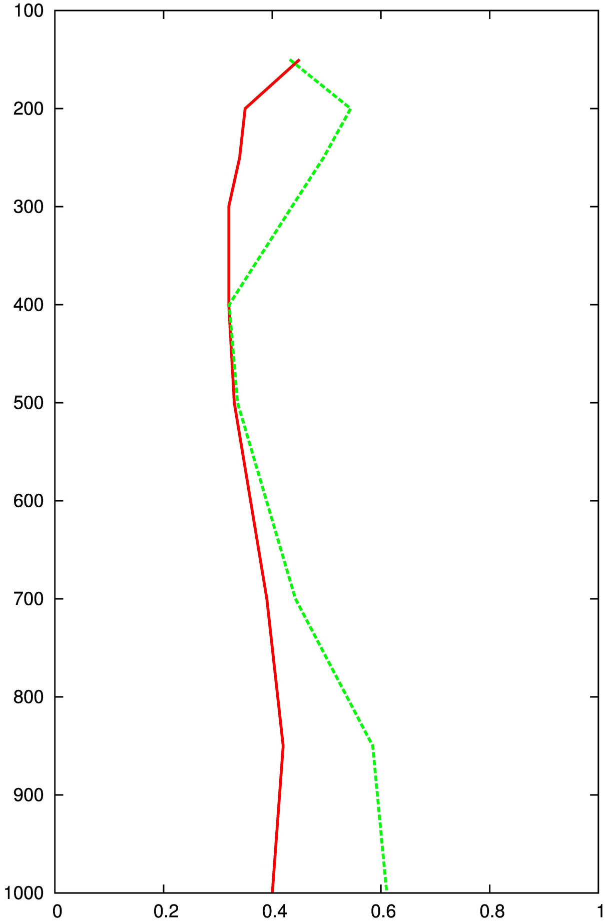
Quantification of model error accumulated during cycling

Pressure (hPa)



$\sigma_{\delta y/EDA}(\varepsilon^{fm}_i = \sum T_{i-j} \varepsilon^{m}_j)$

$\sigma_{EDA}(\varepsilon^{fo}_i = \sum T_{i-j} M_j K_j \varepsilon^{o}_j)$

Standard deviation of error contributions
aircraft observations of temperature (K))

Possible guidance for model error representation ?

- Variance of model error accumulated over 3-5 days can be diagnosed :
  model error contrib. ~ one half of 6h forecast error variance ;
  obs. error contrib. ~ other half.

- Variance of model error accumulated over 6h: similar formalism,
  but assumptions/diagnostics on temporal correlations required.

- Importance of innovation-based estimates (**R** and **B**).

- Possible model error representations ($\mathbf{Q}^{1/2}\eta$, SPPT, SKEB, etc)
  may be compared/adjusted with such diagnostics.

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Conclusions

- EDA for estimation of flow-dependent covariances,
  combined with spatial filtering methods.

- EDA (and innovations) for diagnosing contributions to error cycling :
  - obs. errors contribute ~ within the last 3-5 days of DA cycling.
  - old background errors vanish ~ after 3-5 days of DA cycling.
  - model error contrib. are similar in amplitude to obs. error contrib.

- LAM : contribution of LBC errors over 1/3 of ALADIN-France domain
  due to advection in the cycling (El Ouaraini et al 2015).

- Extension to diagnostics of spatial structures,
  pursue comparison EDA vs innovations, etc.

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Thank you for your attention