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Using lagged covariances in data assimilation

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Motivation

The motivation for this work is to assimilate observations of the Atlantic Meridional Overturning Circulation (AMOC) that have been made at 26°N in the Atlantic Ocean (Fig. 1) by modifying ocean densities 'upstream' in the Labrador Sea several years earlier. Many previous studies (*e.g.* Fig. 2) have shown that the AMOC is robustly sensitive to anomalies in the Labrador Sea. Using earlier assimilation increments should give better continuity to the circulation and the heat transports, making the model more useful for coupled forecasting.

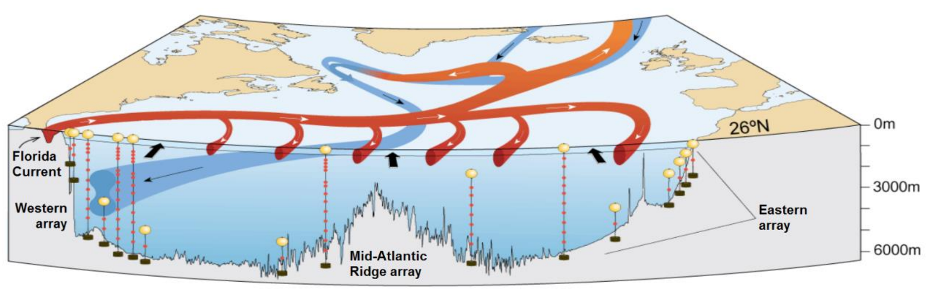

Figure 1: A cartoon of the AMOC and the RAPID observational array at 26°N. Adapted from www.rapid.ac.uk.

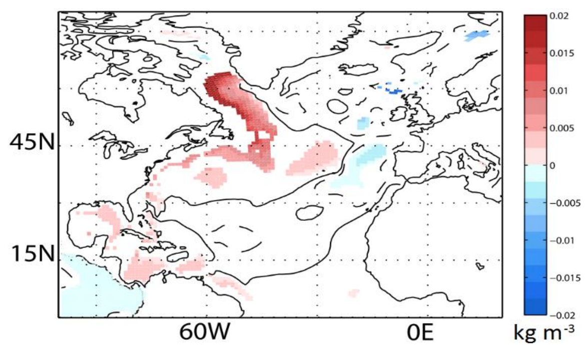

Figure 2: The 26°N AMOC lag-regressed onto density anomalies. Adapted from Polo *et al.* (2014).

This may seem like a classical 4DVar problem that just needs better initial conditions. However given the need for high resolution (essentially operational) models, 4DVar is impractical over a multi-year time window. We aim to use the robustness (state independence) of lagged relationships such as that shown in Fig. 2 to make earlier increments without the use of an adjoint.

Methodology

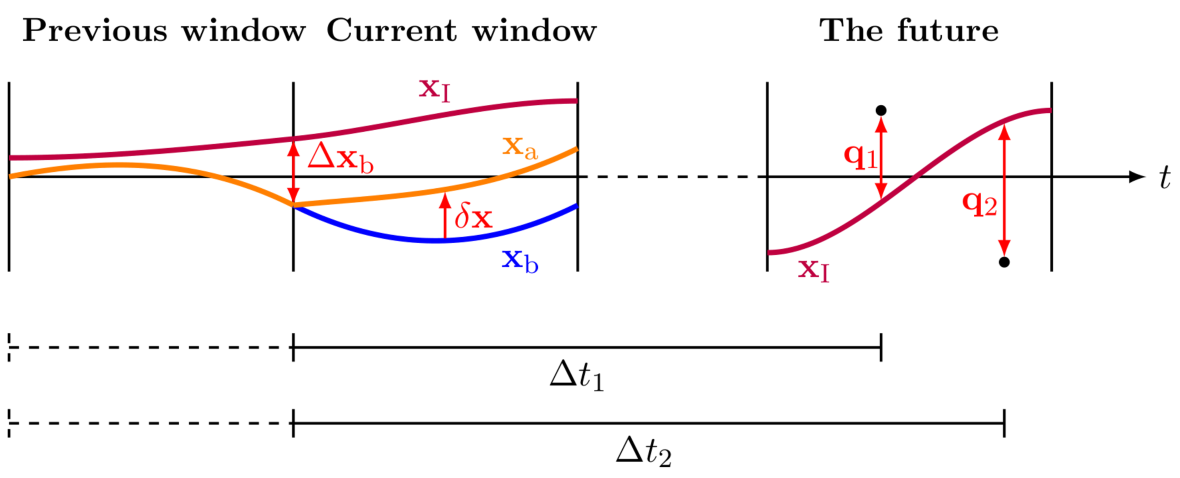

Figure 3: Stage 1 is 3DVar-FGAT producing trajectory $\mathbf{x}_\mathrm{I}$. Outside window (future innovations $\mathbf{q}_1$ and $\mathbf{q}_2$), are not used. Stage 2 repeats Stage 1 but now innovations $\mathbf{q}_1$ and $\mathbf{q}_2$ influence all previous window increments $\delta\mathbf{x}$ using appropriate lagged covariances, *e.g.* at lags Δt_1 and Δt_2. The new background trajectory for the current window is $\mathbf{x}_\mathrm{b}$ and the final analysis trajectory is $\mathbf{x}_\mathrm{a} = \mathbf{x}_\mathrm{b} + \delta\mathbf{x}$.

Consider **Stage 1**, a sequential 3DVar-FGAT assimilation of conventional data in short windows (5-10 days) which is run over a period of several years to produce an initial analysis trajectory $\mathbf{x}_\mathrm{I}$ (Fig 3). Now consider **Stage 2** with additional innovations $\mathbf{q}_i$, from a period towards the end of this initial analysis, which we will assimilate through lagged covariances to influence earlier windows in a repeat run of the 3DVar-FGAT. As the new innovations $\mathbf{q}_i$ have already influenced the previous window (Fig 3) the background trajectory $\mathbf{x}_\mathrm{b}$ is different from $\mathbf{x}_\mathrm{I}$. However a modification of the cost function can account for this, allowing consistent influence from $\mathbf{q}_i$ over several windows.

The new cost function developed to assimilate $\mathbf{q}_i$ has the form:

$$J_\mathrm{c}(\delta\mathbf{x}) = \frac{1}{2}\sum_i\sum_j[\mathbf{q}_i - \mathbf{Z}_i(\delta\mathbf{x} + \Delta\mathbf{x}_\mathrm{b})]^\mathrm{T}(\mathbf{S}^{-1})_{ij}\left[\mathbf{q}_j - \mathbf{Z}_j(\delta\mathbf{x} + \Delta\mathbf{x}_\mathrm{b})\right]$$

where the indices *i* and *j* run over the lags used, $\mathbf{q}_i$ are the innovations between the additional data and $\mathbf{x}_\mathrm{I}$, $\mathbf{Z}_i$ are based on lagged covariances (see below), and $\mathbf{S}$ is the covariance matrix of the innovations. The $\Delta\mathbf{x}_\mathrm{b} = \mathbf{x}_\mathrm{b} - \mathbf{x}_\mathrm{I}$ term always references the new increments back to the initial trajectory.
The lag-covariance matrix $\mathbf{Z}_i$ plays the role of the $\mathbf{H}_i\mathbf{M}_i$ term in classical 4DVar and can be determined by applying linear regression to *e.g.* a long model run. It should be possible to implement the above cost function term within the NEMOVAR ocean assimilation code that is operational at the Met Office and ECMWF.

Error estimation

Multiple sources of error contribute to $\mathbf{S}$, the innovation covariance matrix used in Stage 2. Both the observations and the analysis trajectory from Stage 1 have associated covariances. These covariances, including those between innovations at different lags ($i \neq j$), could be estimated using a method similar to the analysis of innovations described in Desroziers *et al.* (2005). This would occur between Stages 1 and 2.

An additional contribution to $\mathbf{S}$ due to the robustness of the linear regression can be found by determining the robustness of the lagged error covariances that determine $\mathbf{Z}_i$. If the long run is nonstationary, for example, it will have limited predictive power and the J_c term will be heavily downweighted relative to the other terms in the cost function. Error studies for this approach are still underway.

Simulation study

A simple simulation study is used to test the two-stage assimilation. The system has the governing equation $\frac{\partial\mathbf{x}}{\partial t} + u\frac{\partial\mathbf{x}}{\partial z} = 0$, corresponding to the advection of the quantity $\mathbf{x}(z,t)$ around a ring with spatial coordinate z. For Stage 1 data are generated inside each assimilation window and used to produce an initial analysis trajectory, $\mathbf{x}_\mathrm{I}$. For Stage 2, additional data are generated at lags corresponding to multiple window lengths in the future. The innovations between these data and $\mathbf{x}_\mathrm{I}$ are calculated and, together with values of $\mathbf{Z}_i$ determined using a long model run, are used to formulate J_c.

Fig. 4 shows the result of applying this procedure to a system with 64 spatial points and 10 assimilation windows consisting of 1000 time steps each. On average, the second assimilation lies closer to the truth than the first assimilation, indicating the procedure has been successful.

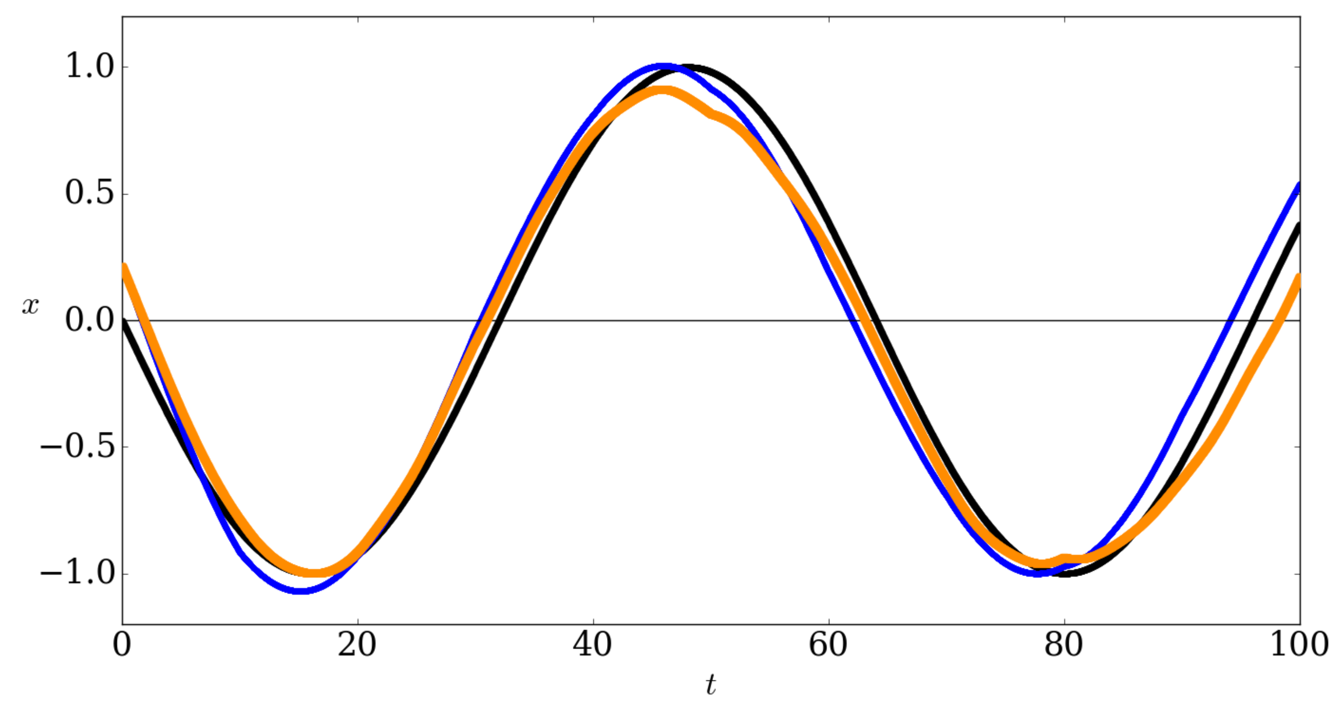

Figure 4: Truth (black), first assimilation (blue) and second assimilation (orange) against time for the simple simulation at one particular value of *z*. There are ten assimilation windows, each of which spans 10 time units.

References

1. McCarthy *et al.* (2015), Progress in Oceanography, 130, 91-111.
2. Polo *et al.* (2014), J. Phys. Oceanogr., 44, 2387-2408.
3. Desroziers *et al.* (2005), Q. J. R. Meteorol. Soc., 131, 3385-3396.

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