Hans-Ertel-Centre for Weather Research
DATA ASSIMILATION BRANCH

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Deutscher Wetterdienst

AN ADAPTIVE ADDITIVE INFLATION SCHEME FOR ENSEMBLE KALMAN FILTERS

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OBJECTIVES AND SCIENTIFIC QUESTIONS

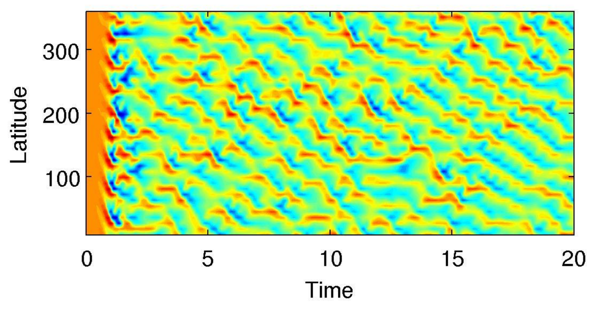
Objectives
In atmospheric data assimilation, the consideration of model error of multiple scales and processes is important in order to associate an optimal weight to background and observations. Besides the question of how to estimate the model error covariance matrix, also its implementation in Ensemble Kalman Filters is an important issue, which shall be addressed here.

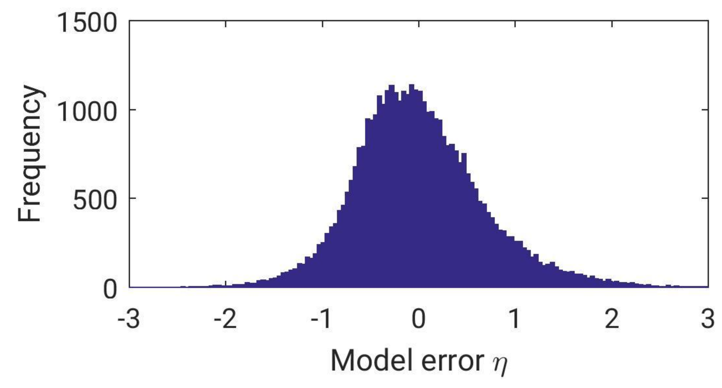
Scientific questions

- How does the skill of an Ensemble Kalman filter depend on the rank of the model error covariance matrix?
- Is it feasible to choose the rank of the model error covariance matrix in a flexible way?

THE LORENZ (1996) MODEL

- Slow time scale ('Resolved processes'): $\partial_t z_i = \underbrace{z_{i-1}(z_{i+1} - z_{i-2})}_{\text{'Advection'}} - \underbrace{z_i}_{\text{'Dissipation'}} + \underbrace{F}_{\text{'Forcing'}} - \frac{hc}{b}\sum_{j=1}^{n} y_{j,i}$
- Fast time scales ('Unresolved processes'): $\partial_t y_{j,i} = cby_{j+1,i}(y_{j-1,i} - y_{j+2,i}) - cy_{j,i} + \frac{hc}{h} z_i$
- Model error given through unresolved processes





MODEL ERROR

$$\underbrace{x_f^{k+1} - x_t^{k+1}}_{\text{Total error}} = M(x_a^k) - M_t(x_t^k)$$

$$= \underbrace{M(x_a^k) - M(x_t^k)}_{=:\zeta_k} + \underbrace{M(x_t^k) - M_t(x_t^k)}_{=:\eta_k}$$

$\zeta^k \sim \mathcal{N}(0, \mathsf{P}_b)$	Error due to initial conditions
$P_b = \text{cov}(M(x_a^k) - \overline{M(x_a^k)})$	Sampling by ensemble
$\eta^k \sim \mathcal{N}(0, \mathsf{Q})$	Model error. *Ad hoc* estimate

Sources of model error:

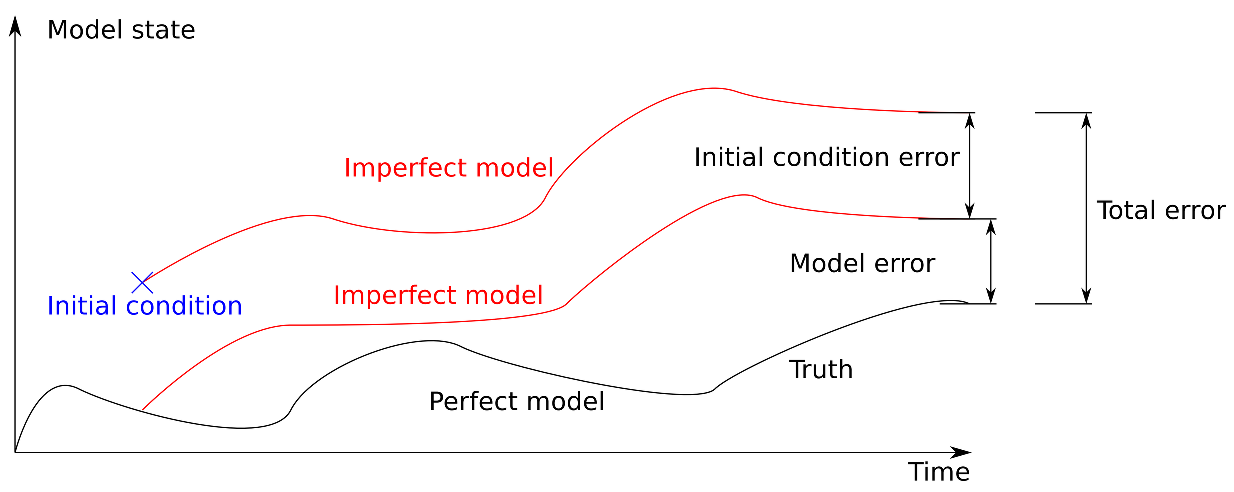
- Unresolved processes
- Imperfect parametrizations
- Inaccurate model parameters

Estimates for model error:

- Differences of forecasts with varying resolution
- Short-term tendencies
- Random states-climate

Properties of and/or assumptions on model error:

- Various scales involved
- May lead to filter divergence if neglected in DA
- Correlated in time?
- Biased?
- Gaussian?
- Correlated to errors due to initial conditions?



ADAPTIVE ADDITIVE INFLATION

- Conventional additive inflation scheme: $\widetilde{\mathsf{X}} = \frac{\mathsf{X}}{\sqrt{n_{ens}-1}} + \frac{\eta}{\sqrt{n_{ens}-1}} \Rightarrow \widetilde{\mathsf{P}} = \widetilde{\mathsf{X}}\widetilde{\mathsf{X}}^T = \widetilde{\mathsf{P}}_b + \mathsf{Q} + \frac{\mathsf{X}\eta^T + \eta\mathsf{X}^T}{n_{ens}-1} \Rightarrow \text{rank}(\widetilde{\mathsf{P}}_b) \leq n_{ens} - 1$
- Alternative: Concatenate error samples: $\widetilde{\mathsf{X}} = (\frac{\mathsf{X}}{\sqrt{n_{ens}-1}}, \frac{\eta}{\sqrt{n_{synth}-1}}) \Rightarrow \widetilde{\mathsf{P}}_b = \widetilde{\mathsf{X}}\widetilde{\mathsf{X}}^T = \mathsf{P}_b + \mathsf{Q} \Rightarrow \text{rank}(\widetilde{\mathsf{P}}_b) \leq n_{ens} + n_{synth} - 2$
(Synthetic ensemble members, phase space augmentation)
- Application to LETKF:

$$\overline{x}_a = \overline{x}_b + \widetilde{\mathsf{P}}_a \widetilde{\mathsf{X}}_b^\mathsf{T} \mathsf{H}^\mathsf{T} \mathsf{R}^{-1}(y_o - \overline{y}_b)$$

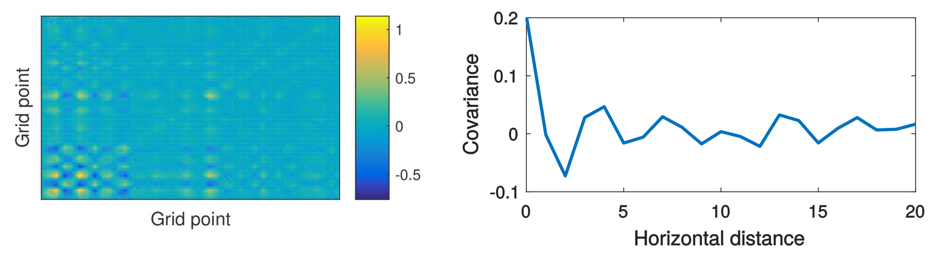
$$\widetilde{\mathsf{P}}^a = \left(\mathbb{I}_{n_{ens}+n_{synth}} + \widetilde{\mathsf{X}}^\mathsf{T}\mathsf{H}^\mathsf{T}\mathsf{R}^{-1}\mathsf{H}\widetilde{\mathsf{X}}\right)^{-1}$$

$$\mathsf{X}_a = \widetilde{\mathsf{X}}\mathsf{T} = \mathsf{X}_b\mathsf{T}_b + \mathsf{X}_{synth}\mathsf{T}_{synth}$$

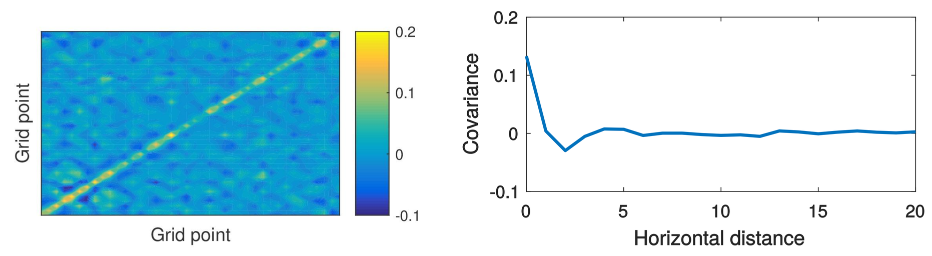
$$\mathsf{T}\mathsf{T}^\mathsf{T} = (\mathbb{I} + \underbrace{\widetilde{\mathsf{X}}^\mathsf{T}\mathsf{H}^\mathsf{T}\mathsf{R}^{-1}\mathsf{H}\widetilde{\mathsf{X}}}_{\stackrel{!}{=}\mathsf{CDC}^\mathsf{T}})^{-1} \widehat{=} \widetilde{\mathsf{P}}_a$$

- Gaussian Resampling: $\mathsf{X}'_a = \mathsf{X}_a\Psi$, $\Psi \sim \mathcal{N}(0,1), \Psi \in \mathbb{R}^{n_{ens}+n_{synth}, n_{ens}}$
- Shrinkage covariance resampling: $\mathsf{X}'_a = \sqrt{\mathsf{P}_{SCE}}\Psi$, $\mathsf{P}_{SCE} = \gamma\mathsf{T} + (1-\gamma)\mathsf{P}_a$
- Remove bias: $\mathsf{X}' \to \mathsf{X}'(\mathbb{I} - \frac{\mathbf{e}\mathbf{e}^T}{n_{ens}})$

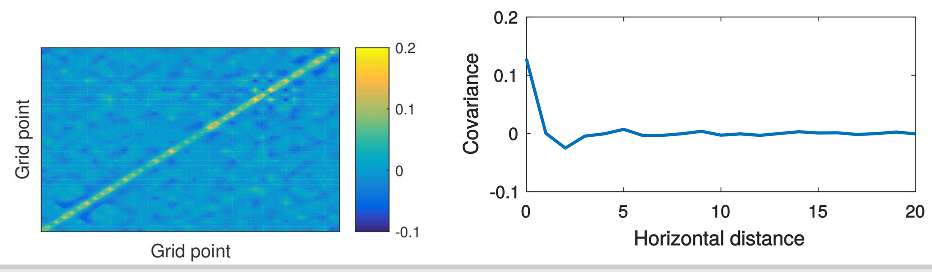
n_synth=10



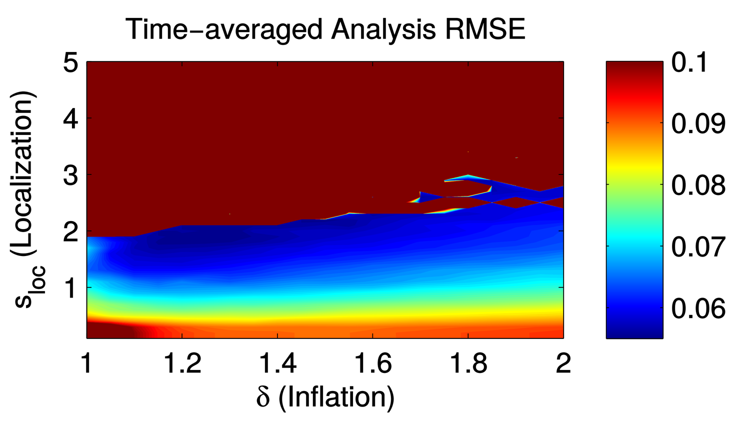
n_synth=100



True covariance



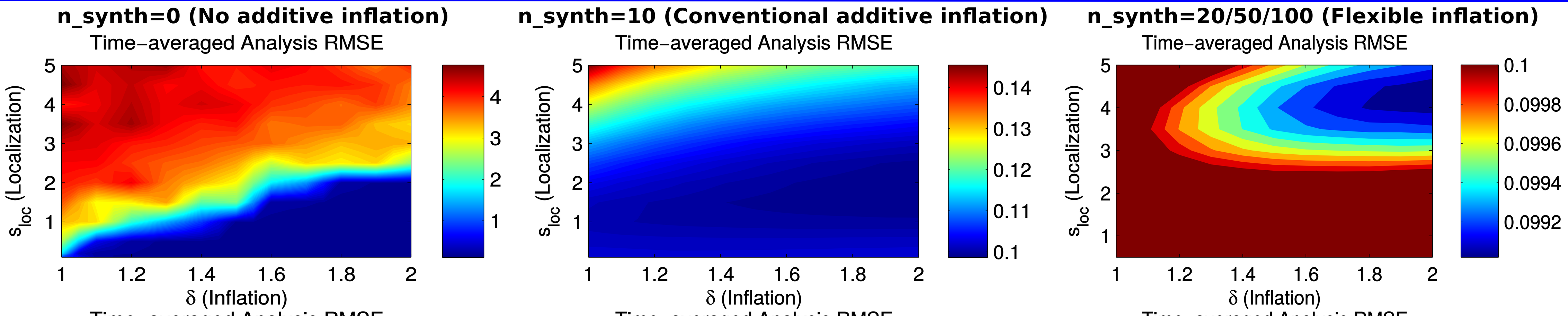
RESULTS (COUPLING≙MODEL ERROR)



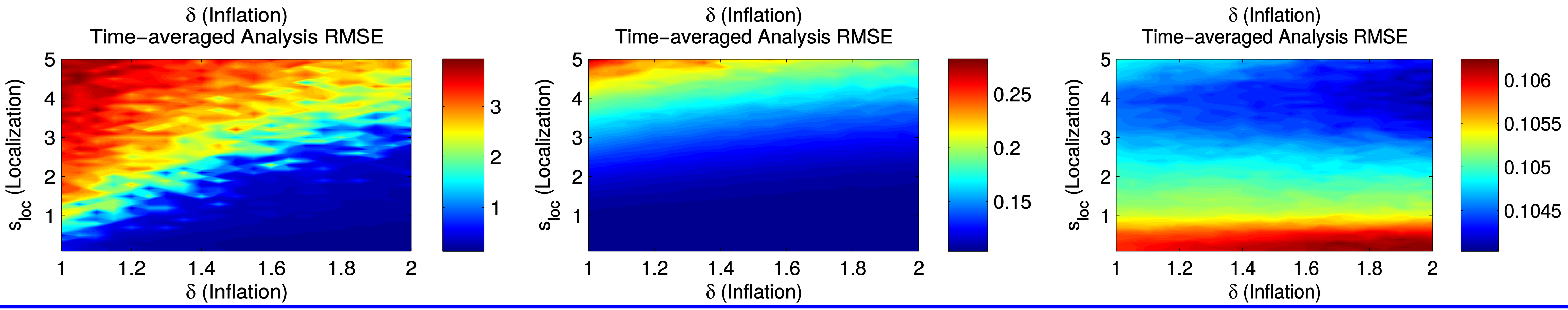
No coupling (h=0)

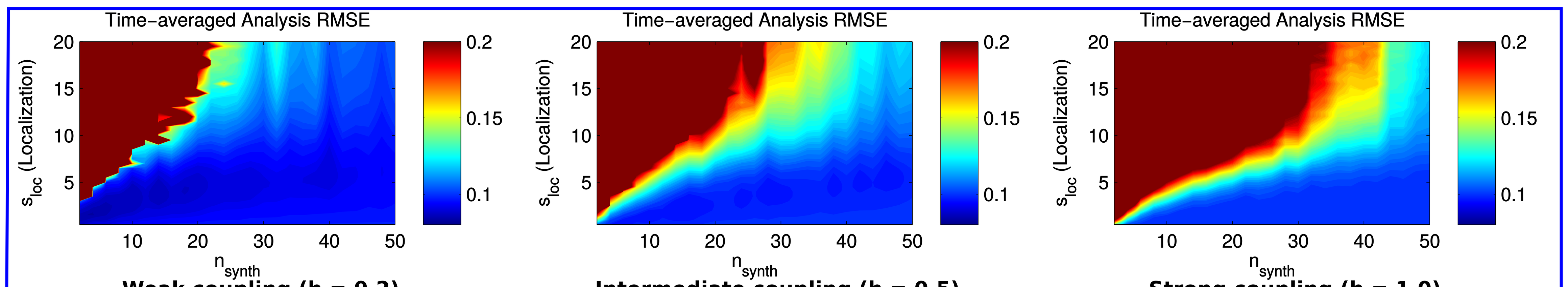
n_synth=0 (No additive inflation) | **n_synth=10 (Conventional additive inflation)** | **n_synth=20/50/100 (Flexible inflation)**

Intermediate coupling (h = 0.5)



Strong coupling (h = 1.0)





Weak coupling (h = 0.2) | **Intermediate coupling (h = 0.5)** | **Strong coupling (h = 1.0)**

- With coupling: Additive inflation needed to avoid filter divergence
- A higher rank of model error covariance matrix...
  - * reduces the minimal analysis error
  - * moves the minimum towards ...
    - larger localization (since less spurious correlations are present)
    - for large enough n_synth, no localization is needed
    - smaller multiplicative inflation
- The computational cost is...
  - * equivalent to a correspondingly larger ensemble in the analysis step
  - * zero in the forecast step

SUMMARY

- In conventional additive inflation only few effects of model error are considered
- By concatenating `synthetic' ensemble members more degrees of freedom can be taken into account
- These `synthetic' ensemble members are resampled onto the `real' members and not forecast causing no additional computational cost in the propagation step
- Experiments show that a higher rank model error covariance matrix leads to reduced analysis RMSE, weaker multiplicative inflation, and higher localization radii

LITERATURE

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