Ensemble Transform Kalman Filter to Image Deblurring Problems

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Overview

Our goal is to apply the ensemble transform Kalman filter (ETKF) [1] to the image deblurring problem as a prototype of inverse problems which can be modeled as

$$y = Ax^{\dagger} + \epsilon,$$

where, from the blurred and noisy image y, we aim to recover the original image $x^{\dagger}$ contaminated by an ill-conditioned matrix A. Sample applications also include

- Signal processing,
- Tomographic X-ray projection data,
- Reconstruction of initial temperature from a diffusion process, etc.

Since inverse problems are always ill-posed because of the decaying singular values of A, the use of the ETKF is based on

1. a span of initial ensemble that is chosen according to the spatial relationship of y, imposing additional information to regularize the filtered solution,
2. an artificial linear dynamical system [2] that is constructed to adapt the ETKF as an iterative method to the inverse problems,
3. a second use of the ETKF [3] that is performed within the same assimilation cycle with respect to the gradient of the analysis mean for further regularization. It is because members of the analysis ensemble depart from the analysis mean vigorously around edges.

The Artificial Linear Dynamical System

The ETKF is applied to the inverse problem under the artificial linear dynamical system:

$$\begin{aligned} \widehat{x}_n &= x_{n-1} \\ y &= A\widehat{x}_n + \epsilon, \end{aligned}$$

where x_{n-1} denotes the analysis state at the previous step and $\widehat{x}_n$ denotes the background state. The major steps are:

1. Calculate a spatial covariance matrix of y.
2. Generate a span of initial ensemble by adding perturbations to y. The perturbations follow the Gaussian distribution with zero mean and the covariance generated in step 1.
3. Blur each ensemble member with the distortion operator A.
4. Apply the ETKF to the set of blurred ensemble with respect to the blurred and noisy image y.
5. Obtain the analysis mean and a set of analysis ensemble.
6. Repeat steps 3 - 5 iteratively. The analysis mean is the filtered image.

An experiment of 10 iterations is included. The method requires only one iteration to get a sharpened image (Figure 2, Left). However, as the iteration goes on, the noise is magnified and the filtered results become worse and worse (Figure 2, Right; Figure 3).

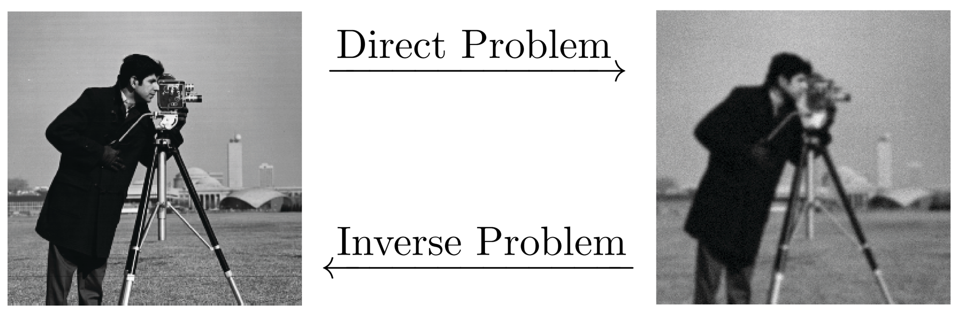

Figure 1: Left: An original image. Right: A blurred noisy image ($\|y - x^{\dagger}\|_2 = 4.190$ with SNR: 27 dB.)

Figure 2: Left: The filtered image after the 1st iteration (rel. err = 3.2988). Right: The filtered image at the 10th iteration (rel. err = 6.4807).

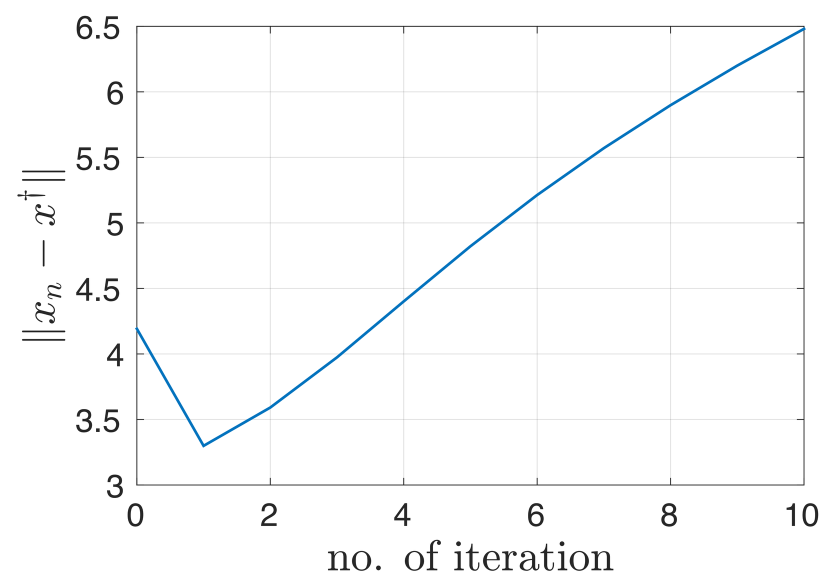

Figure 3: The quality of filtered images deteriorates as number of iterations.

Two-stage Use of the ETKF

By inspection, we found that, after the first iteration, the members of the analysis ensemble depart from the analysis mean in large extent, especially at regions across the edges of the figure.

Thus, within the same assimilation cycle, a second use of the ETKF is performed with respect to the gradient of the analysis mean $D\overline{x}$; the distortion operator A is replaced by a differential operator D. Mathematically, it aims to obtain a new analysis mean by minimizing the cost function

$$J(x) = (x - x_n)^T P^{\dagger}(x - x_n) + (Dx_n - Dx)^T R^{-1}(Dx_n - Dx),$$

where P is the ensemble covariance matrix and R is the error covariance matrix whose entries are chosen in proportional to absolute intensities of pixels of the analysis mean. The corresponding results are:

Figure 4: The 1st (rel. err = 3.1670) and the 10th (rel. err = 3.1669) filtered images are shown.

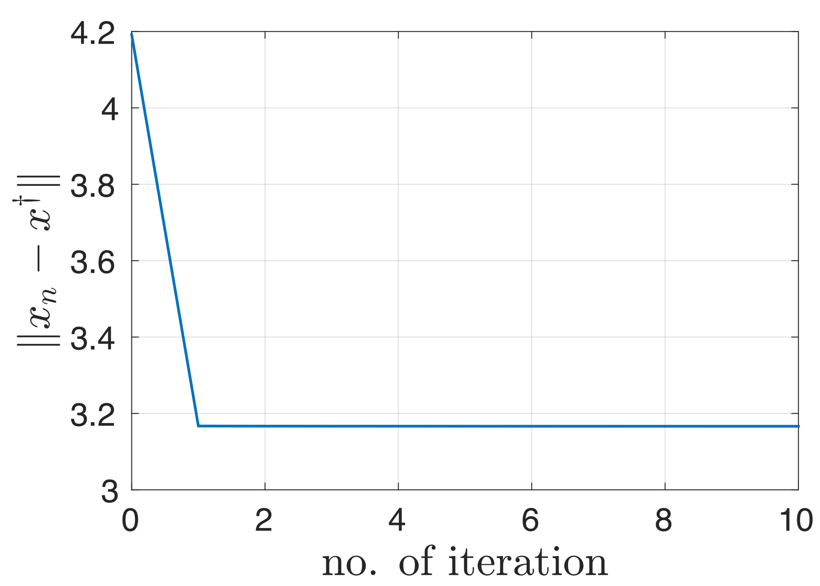

Figure 5: The method converges after the two-stage use of the ETKF and the error is suppressed.

Video Deblurring

To move a further step, we may extend the method to deblur:

- an out-of-focus image due to hand-shaking (image stabilization),
- slices of blurred images (video).

The problems are modeled by modifying the artificial model as a dynamical one

$$\begin{aligned} \widehat{x}_n &= x_{n-1} \\ y_n &= A\widehat{x}_n + \epsilon_n, \end{aligned}$$

The observations y_n form a sequence of blurred video data. The blurred data for $n = 1, 2$ and 3 are shown in the upper row of the Figure 6. The two-stage use of the ETKF is performed conditioned to the observations sequentially.

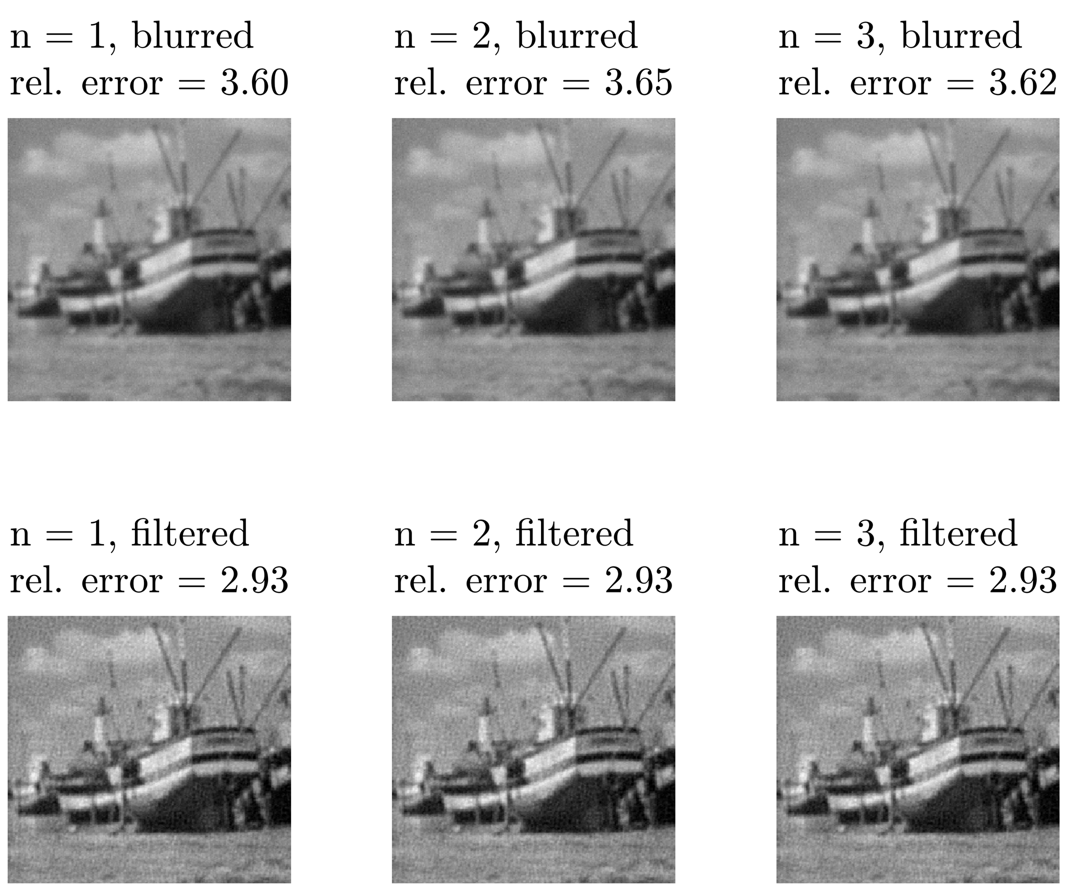

Figure 6: Bottom: The filtered video data are sharpened.

From numerical experiments, it should be remarked that if the changes between the successive blurred observations y_{n-1} and y_n are too rapid, the deblurring effects are not obvious. In other words, the video data should have large extent of overlapping for effective filtering.

Video Deblurring (cont.)

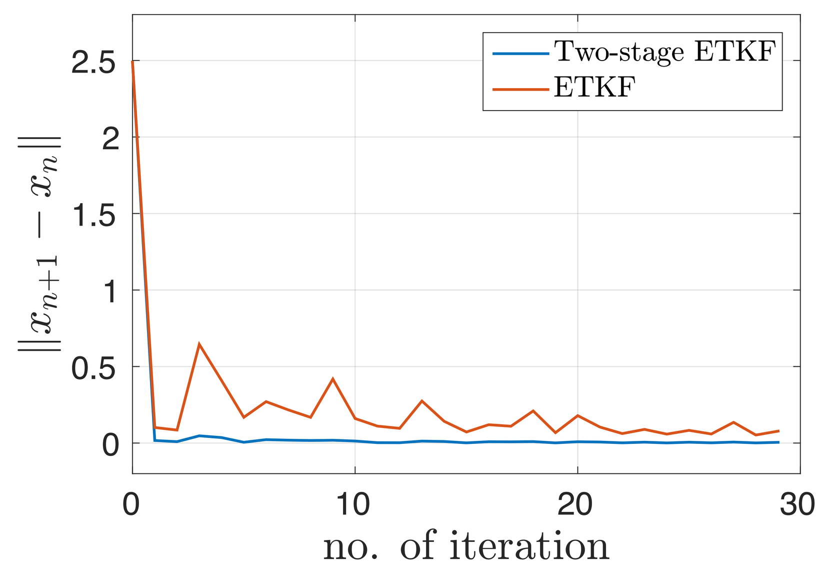

Figure 7: An experiment of 20 video data, $\{y_1, y_2, \ldots y_{20}\}$. Blue: The successive differences of the filtered images are small, illustrating that the two-stage method is effective. Red: The oscillations denote the differences between two successive filtered images which means the deblurring effect is inferior to that of the two-stage method.

Conclusion and Ongoing Work

With demonstrations of applying the two-stage ETKF to both static and dynamical image deblurring problems, we conclude that the method manages to produce sharpened images; the magnification of the additive noise is alleviated by the second use of the ETKF.

As suggested in Figure 5, the error can not be further reduced up to the relative error around 3.1670 which indeed corresponds to the magnified noise. For betterment of the results, ones may consider:

- the method of generation of initial ensembles. Since the span of ensemble imposes additional information to the inverse problem, a priori knowledge such as skeletons of images may be used to generate the initial ensemble.
- further regularization on the additive noise along with the ensemble Kalman filter. An in-looped algorithm may be devised to smooth out the additive noise.

References

[1] Hunt, Brian R and Kostelich, Eric J and Szunyogh, Istvan. Efficient data assimilation for spatiotemporal chaos: A local ensemble transform Kalman filter. In *Physica D: Nonlinear Phenomena 230* (2007)
[2] Iglesias, Marco A and Law, Kody JH and Stuart, Andrew M Ensemble Kalman methods for inverse problems. In *Inverse Problems 29* (2013)
[3] Johns, Craig J and Mandel, Jan. A two-stage ensemble Kalman filter for smooth data assimilation. In *Environmental and Ecological Statistics 15* (2008)