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The Error of Representation: Treatment and Estimation

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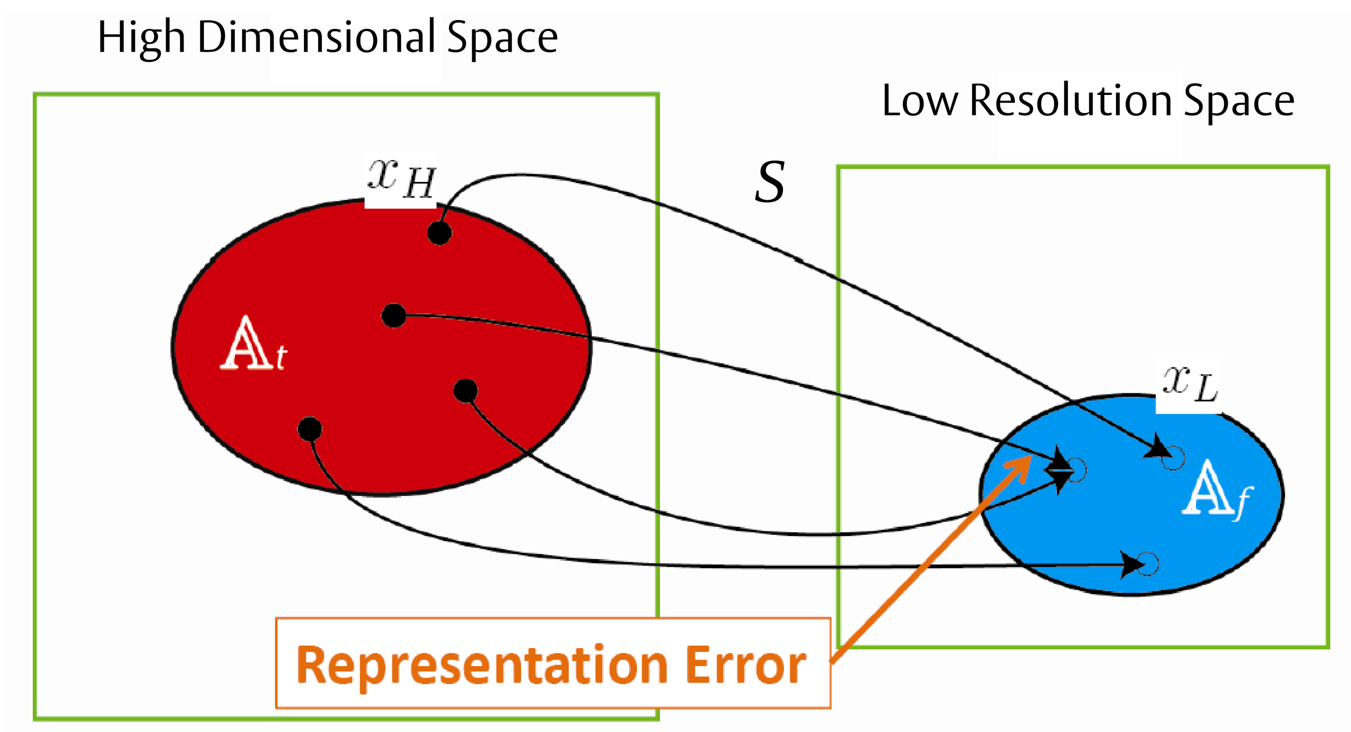
Introduction

Data assimilation (DA) aims to provide statistical estimates of the states of a dynamical system given a model of the system, a prior model forecast, and observations of the states over time. To reduce computational cost, DA is often performed on a low resolution model using observations of the full high dimensional system. Classical DA techniques used with the low resolution model do not take into account the representation error arising from scale mismatches between the low and high resolution systems. Here we present a modified DA scheme for use with a low resolution model that includes the representation error and provides more accurate estimates of the high resolution states[1,2,3].

Background

Let S map high dimensional state space into low resolution state space:



Not all high dimensional states can be represented by low resolution states and S is not invertible. The pseudo-inverse of S is denoted $S^{\dagger}$. We write $x_L = Sx_H$, $v_L = y - H_L\bar{x}_L$, $< v_L >= H_H\bar{x}_H - H_L\bar{x}_L$, where y is an observation in high dimensional space, and let P_L denote the low resolution prior error covariance matrix.

Classical Approach

The analysis in high dimensional space is classically given by:

$$\bar{x}^c_H = \bar{x}_H + S^{\dagger}G_c[\, v_L - < v_L >]$$

where

$$G_c = P_L H_L^T [H_L P_L H_L^T + R]^{-1}$$

and R is a *diagonal* matrix of (possibly inflated) observation error variances.

Bayesian Reduced Resolution (BRR) Approach

The analysis in high dimensional space accounting for representation error is given by:

$$x^a_H = \bar{x}_H + S^{\dagger}G_h[\, v_L - < v_L >]$$

where

$$G_h = [P_L H_L^T + P_{LH}][H_L P_L H_L^T + R^*]^{-1}$$

$$P_{LH} = SP_H[H_H - H_L S]^T$$

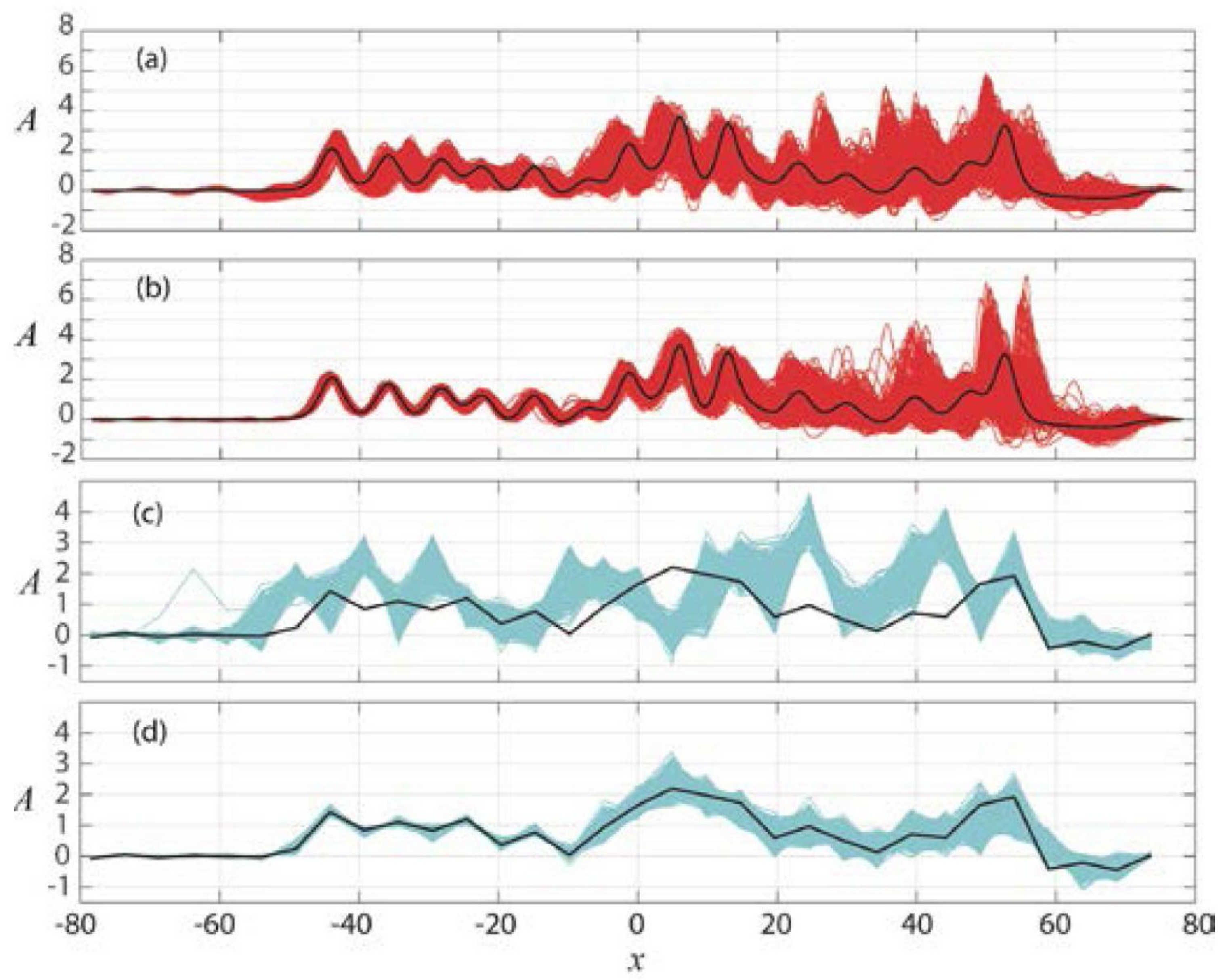
$$R^* = R_I + H_H P_H H_H^T - H_L P_L H_L^T$$

which is the best linear unbiased estimate for the problem[1].

Experiments

We run twin experiments with a numerical model of the nonlinear, variable-coefficient Korteweg-de Vries equation on a fine and a coarse grid, such that the map S between the state spaces is a spectral cut. The solution on the high resolution grid contains large amplitude, high speed solitary waves that are not fully resolved on the low resolution grid. Observations are taken from the high resolution model with a known error variance. An ensemble DA method is used on each grid with a large ensemble size and localization.

The **results** show that the posterior ensembles on the high resolution grid obtained by each approach are reasonable, but the ensemble variance produced by the BRR approach is much smaller (Figures **(a)** and **(b)**). The posterior ensembles on the low dimensional grid produced by the BRR approach are, however, much more accurate estimates of the projected high dimensional state than those of the classical approach (Figures **(c)** and **(d)**).



Caption: Example posterior ensemble members for the 400th cycle. The solid black line in all plots is the high dimensional state. In (a) and (b) the high resolution posterior ensemble for the classical and Bayesian reduced resolution methods are shown, respectively. In (c) and (d) the low resolution posterior ensemble for the classical and Bayesian reduced resolution methods are shown, respectively.

Conclusion

A new DA approach is presented that accounts for representation error between high and low dimensional systems. The approach is shown in twin experiments to be successful. Further research is in progress.

References

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2. Hodyss, D and Satterfield, E. , The Treatment, Estimation, and Issues with Representation Error Modelling, Data Assimilation for Atmospheric, Oceanic and Hydrologic Applications, Volume 3, Springer (*to appear*).
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