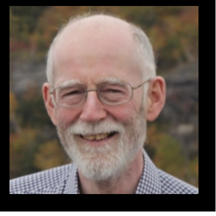
Generalised Localisation

Improving ensemble-based covariance estimates for use in hybrid variational assimilation

Andrew Lorenc

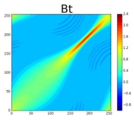
Met Office



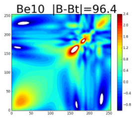
There are several methods, as well as the well known localisation, which can improve the covariance estimates from a small ensemble. I demonstrate and tune them in a toy system and trial some in a full NWP system.

Toy Problem

256 points in a circle, with true error covariance $\mathbf{B}_t$ a Gaussian-shaped function of distance, with large-scale variation in both the error variance and the correlation scale.

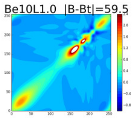


Sample from $\mathbf{B}_t$ of (N_e=10) perturbations with covariance $\mathbf{B}_e$. Errors in estimated $\mathbf{B}$ are measured using the RMS of elements of $|\mathbf{B}\text{-}\mathbf{B}_t|$ (Frobenius norm).

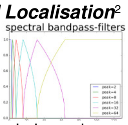


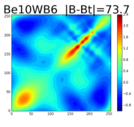
Improving the noisy $\mathbf{B}_e$ by:

- ***Horizontal localisation***[1] applied using a Schur product $\mathbf{B}=\mathbf{L}\circ\mathbf{B}_e$, where $\mathbf{L}$ is a localisation matrix of correlations with a specified scale.

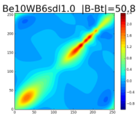


- ***Wavebands Spectral Localisation***[2] A crude spectral localisation is achieved, projecting the ensemble onto wavebands and assuming independence. This has the effect of smoothing $\mathbf{B}_e$.

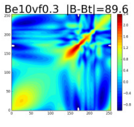




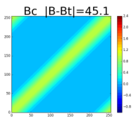
- ***Scale-dependent localisation***. Use a different spatial localisation matrix for each waveband, with localisation scale increasing with wavelength.



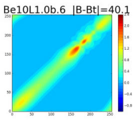
- ***Variance filtering***[3]. Smoothing with a specified scale is applied to the variance field – the correlations are unaltered.



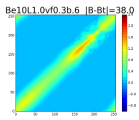
- ***Hybridisation***. A "climatological" $\mathbf{B}_c$ used the same function as $\mathbf{B}_t$, without the spatial variation of scale and variance. The hybrid combines $\mathbf{B}_c$ (shown) with the localised $\mathbf{B}_e$.



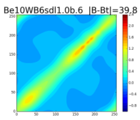
In the Ensemble Kalman Filter, only localisation is used. A potential advantage of ensemble-variational (EnVar) methods is that they can easily be combined, for example as in the plots below – the settings have been chosen to minimise $|\mathbf{B}\text{-}\mathbf{B}_t|$.



localisation & hybrid



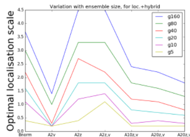
localisation & variance filter & hybrid

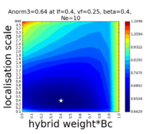


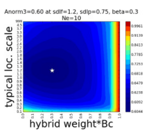
wavebands & scale-dep. loc. & hybrid

Choosing the coefficients

Optimal settings depend on N_e and the methods used. I search parameter-space for the minimum (averaged over 512 samples) of $|\mathbf{B}\text{-}\mathbf{B}_t|$ or $|\mathbf{A}|$-norms equal to the mean analysis error for one of 5 observation distributions.







Wavebands + scale-dep. loc., ≈ variance filter + loc.

BUT Localising with the optimal scale is less important with scale-dep. loc.

Conclusions

Toy Experiments

1. All the methods tested could make significant improvements to the sampled covariances.
2. The need for all the methods decreased with ensemble size and the optimal coefficients varied accordingly.
3. [Waveband localisation + scale-dependent localisation] was better than [horizontal localisation + variance filtering] (both run without hybridisation).
4. With hybridisation the difference was less. However the scale-dependent localisation was more robust to the use of sub-optimal localisation scales.
5. The benefit of waveband localisation and scale-dependent localisation was not very sensitive to the algorithms used to define the bands and the scale dependence. The bands shown, with localisation scale increasing as wavelength^p with p≈0.75, gave as good results as any tried.
6. If not using wavebands, the variance filter was simple and beneficial. (It could also provide benefit in improved background variance fields for observation quality control.)
7. The simple $|\mathbf{B}\text{-}\mathbf{B}_t|$ norm did not always give the same "optimal" settings as the analysis error norms. The latter halved the optimal localisation scales for dense observations.

Real NWP Experiments

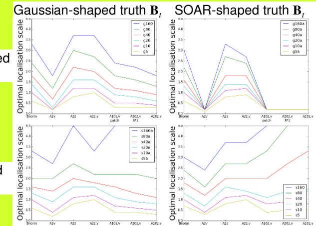
1. The best localisation scale increased with effective ensemble size, and varied with region (N.Hem., Tropics, S.Hem.).
2. The use of 4 wavebands, with no change in localisation, consistently gave a small improvement (as predicted by the toy).Tuning scale-dependent localisation proved surprisingly difficult. My third try improved the Tropics and N.Hem., but degraded the S.Hem. Overall it was beneficial but more tuning is needed. *(Probably the regional dependence of optimal scale is relevant.)*
3. The use of time-lagged ensemble perturbations consistently gave a small improvement.
4. The use of time-shifted perturbations gave additional benefit. But changing the shifts used from [3,6] hours to [1,2,3,4,5,6] hours did not.
5. Combining the best methods, it was possible to nearly match results from an N_e=200 ensemble, using only N_e=44.

Further Work

With all the methods (including the N_e=200 ensemble) it proved difficult to improve all S.Hem. scores. ***Why?***

The methods should be equally applicable to hybrid-4DVar (our current operational system) as to hybrid-4DEnVar tested here. ***Trials are needed – if successful, the methods can be implemented quickly.***

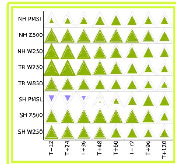
We need better ways of choosing the correct settings. One hope is Ménétrier's method[4], which calculates parameters to minimise $|\mathbf{B}\text{-}\mathbf{B}_\infty|$ ($\mathbf{B}_\infty$ from a hypothetical infinite ensemble). This has two potential problems: it assumes that $\mathbf{B}_\infty$=$\mathbf{B}_t$, and that minimising $|\mathbf{B}\text{-}\mathbf{B}_t|$ gives the best analysis. This can be illustrated by specifying $\mathbf{B}_t$ to be different from $\mathbf{B}_\infty$. Below $\mathbf{B}_t$ & $\mathbf{B}_\infty$ used different shaped covariances with the same variance and scale. In the top-right plot, a short localisation is best, for the dense obs networks, for all N_e, correcting the Gaussian-shaped ensemble, which is too broad at short scales. Ménétrier's method should give the top-left **Bnorm** scales.



Real NWP System

Experiments[5] showed that hybrid-4DVar or 4DEnVar, using simple spatial localisation, were significantly improved using N_e=200 instead of N_e=44. This improvement is my target:

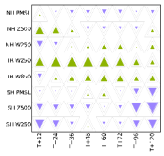
Can a similar improvement be obtained by better estimation of covariances from a smaller ensemble?



Methods for improving covariances

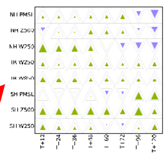
I ran hybrid-4DEnVar with a Ne=44, N320 ensemble and verified 84 N512 forecasts against independent ECMWF analyses. Results are all plotted like the diagram above; ▲ / ▼ indicate that the trial was better / worse than its control.

- ***Horizontal localisation***



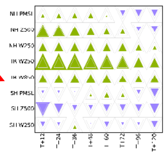
The target experiments increased localisation scale for N_e=200 from L=600km to L=800km. At N_e=44 that change had mixed impact. (For the lagged & shifted ensembles below it was positive.)

- ***Hybridisation*** The same weights were used throughout, so this was not investigated.



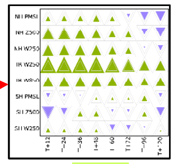
- ***Wavebands Spectral Localisation*** I used 4 wavebands and kept L=600km for each.

- ***Scale-dependent localisation***.



This was unexpectedly difficult to get right, my first attempt (not shown) was uniformly worse. My third attempt with L=[8115,665,230,120] km improved the Tropics and most of the N.Hem., but degraded the S.Hem., compared to the bands all using L=600km.

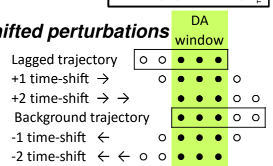
The boxed figure shows the impact of Wavebands + scale-dependent localisation.



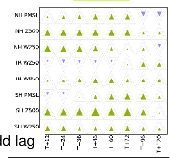
- ***Variance filtering*** This requires modifications to our covariance software, so was not tested.

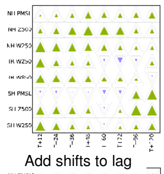
- ***Use of time-lagged & shifted perturbations***

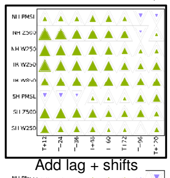
Lagging uses perturbations valid at the correct time, from longer forecasts from the previous cycle. Shifting uses perturbations valid at slightly different times.



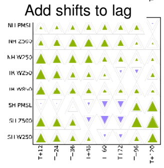
Adding a lagged ensemble with 3hr trajectory steps gave a small improvement everywhere. Adding 2 time-shifts to these as in the diagram above gave the left figure; comparing to the original experiment (with shorter scales) gave the boxed figure.

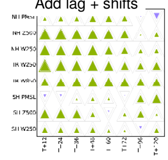






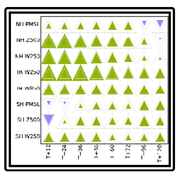
The original 4DEnVar suite used 1hr steps; this allow us to add 6 time-shifts to each trajectory. This more than doubles the DA cost and gives no benefit, as seen in the corresponding lower figures.





Combined system

Combining the time-lag & shift and wavebands & scale-dependent methods with N_e=44 (see boxed results above) nearly matches the target performance of an N_e=200 ensemble.



Met Office FitzRoy Road, Exeter, Devon, EX1 3PB United Kingdom
Tel: +44 1392 886227 Fax: +44 1392 885681
Email: andrew.lorenc@metoffice.gov.uk

References

[1] Hamill, T. M., J. S. Whitaker and C. Snyder, 2001: "Distance-dependent filtering of background error covariance estimates in an ensemble Kalman filter." *Mon. Weather Rev.*, **129**, 2776-2790.
[2] Buehner, M. and Charron, M. (2007), Spectral and spatial localization of background-error correlations for data assimilation. *Q.J.R. Meteorol. Soc.*, **133**: 615–630.
[3] Raynaud, L., Berre, L. and Desroziers, G. (2009), Objective filtering of ensemble-based background-error variances. *Q.J.R. Meteorol. Soc.*, **135**: 1177–1199.
[4] Ménétrier, Benjamin and Thomas Auligné, 2015: Optimized Localization and Hybridization to Filter Ensemble-Based Covariances. *Mon. Wea. Rev.*, **143**, 3931–3947.
[5] Bowler N. E., A. M. Clayton, M. Jardak, P. Jermey, A. C. Lorenc, M. A. Wlasak, D. M. Barker, G. W. Inverarity, R. Swinbank. "The effect of improved ensemble covariances on hybrid variational data assimilation" *Under revision.*