The Probability of Weight Collapse in the Weighted Ensemble Kalman Filter

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1. The Weighted Ensemble Kalman Filter

The ensemble Kalman filter (EnKF) has a proven ability to give acceptable results using only a moderate number of ensemble members, even with the high-dimensional systems encountered in applications such as numerical weather prediction.

It does not, however, solve the probabilistic filtering problem in cases where the dynamical model or observation operator are nonlinear, or the model noise or the observation noise are non-Gaussian.

The weighted ensemble Kalman filter (WEnKF) attempts to solve this problem by attaching weights to each ensemble member.

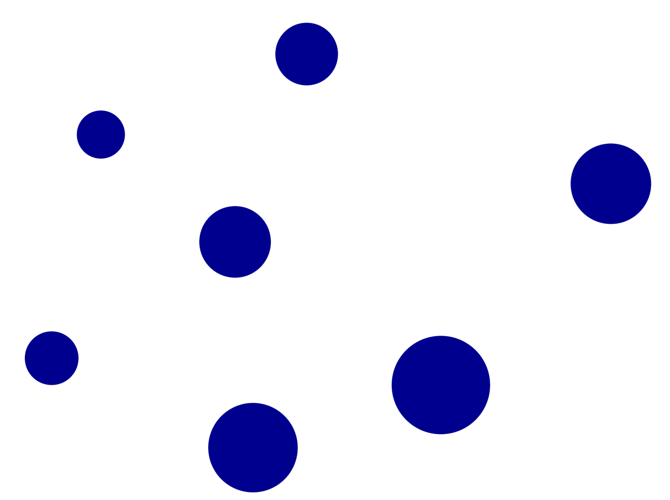

Figure 1: *A Weighted Ensemble*

With weight $w^{(i)}$ attached to ensemble member $\mathbf{x}^{(i)}$, the expected value of the function $f(\mathbf{x})$ can be approximated as

$$E(f(\mathbf{x})) \approx \sum_i f(\mathbf{x}^{(i)}) w^{(i)}$$

WEnKFs can be based on the stochastic EnKF (Papadakis et al., 2010) or on deterministic EnKFs (Beyou et al., 2013).

2. An Experiment that Failed

An attempt was made to apply various formulations of the WEnKF to the stochastic Lorenz-63 system that was tracked with some success by Ades and van Leeuwen (2013) using their equivalent weights particle filter. The attempts with the WEnKF each ended in failure after assimilating the first observation. The filters suffered from catastrophic weight collapse, in which practically all the weight after assimilation went on a single particle.

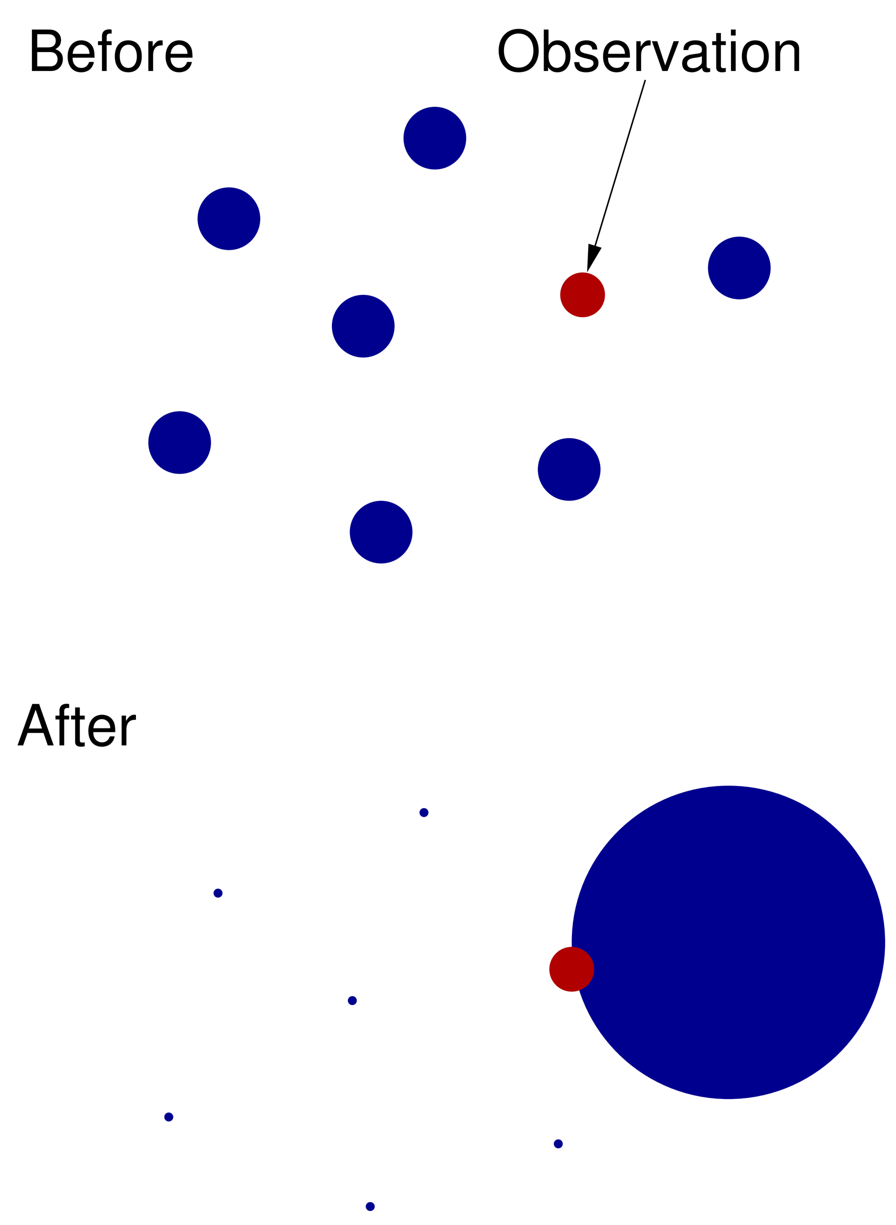

Figure 2: *Catastrophic Weight Collapse*

3. The Probability of Weight Collapse

Making certain approximations, it is possible to estimate the probability density function (PDF) of the weights in a WEnKF (Livings, 2016). In the Lorenz-63 experiment with the weighted stochastic EnKF, $w^{(i)} \propto \exp(-q^{(i)}/2)$ where $q^{(i)}$ is a multiple of a non-central χ^2 random variable.

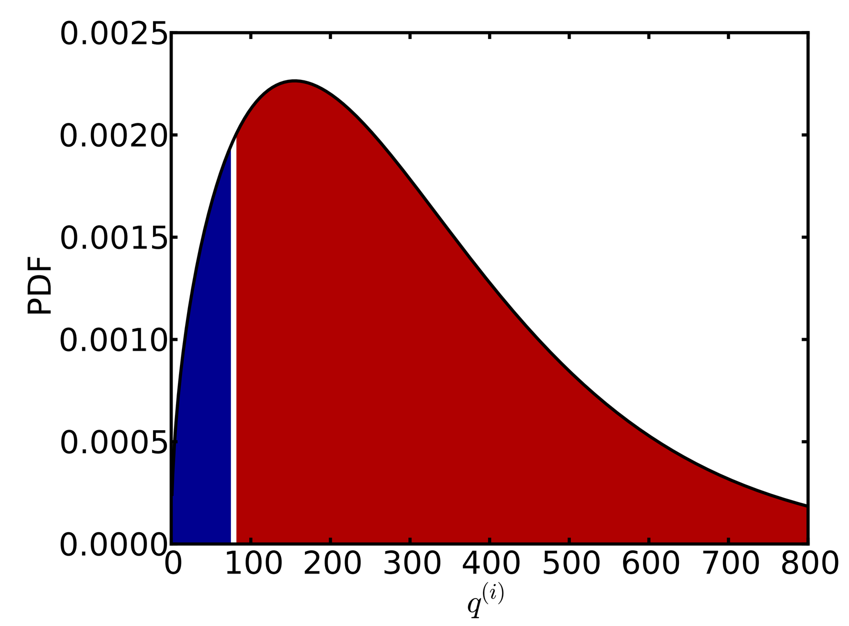

Figure 3: *PDF of $q^{(i)}$*

There are 10 ensemble members. The probability of one member being in the blue region and the other nine members in the red region is 0.33. In this case, more than 9/10 of the weight goes on the one ensemble member and the other nine members get less than 1/10 of the weight between them.

This is one way in which the ensemble can collapse; there are others. Thus 0.33 is an underestimate of the probability of weight collapse. In any case, a filter in which weights collapse one time in three is of limited practical usefulness.

4. Conclusions

The WEnKF is prone to weight collapse, at least in some circumstances. This explains why the WEnKFs in the Lorenz-63 experiments were unable to track the system, whilst the equivalent weights particle filter of Ades and van Leeuwen (2013) could.

The estimation formulae imply that weight collapse should have been a problem in the high-dimensional experiments of Papadakis et al. (2010) and Beyou et al. (2013). Maybe the approximations in the formulae are too crude in these cases; maybe the experiments continued to run because of the additional smoothing procedure applied by those authors.

References

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