

Constraining a C-cycle model data fusion problem

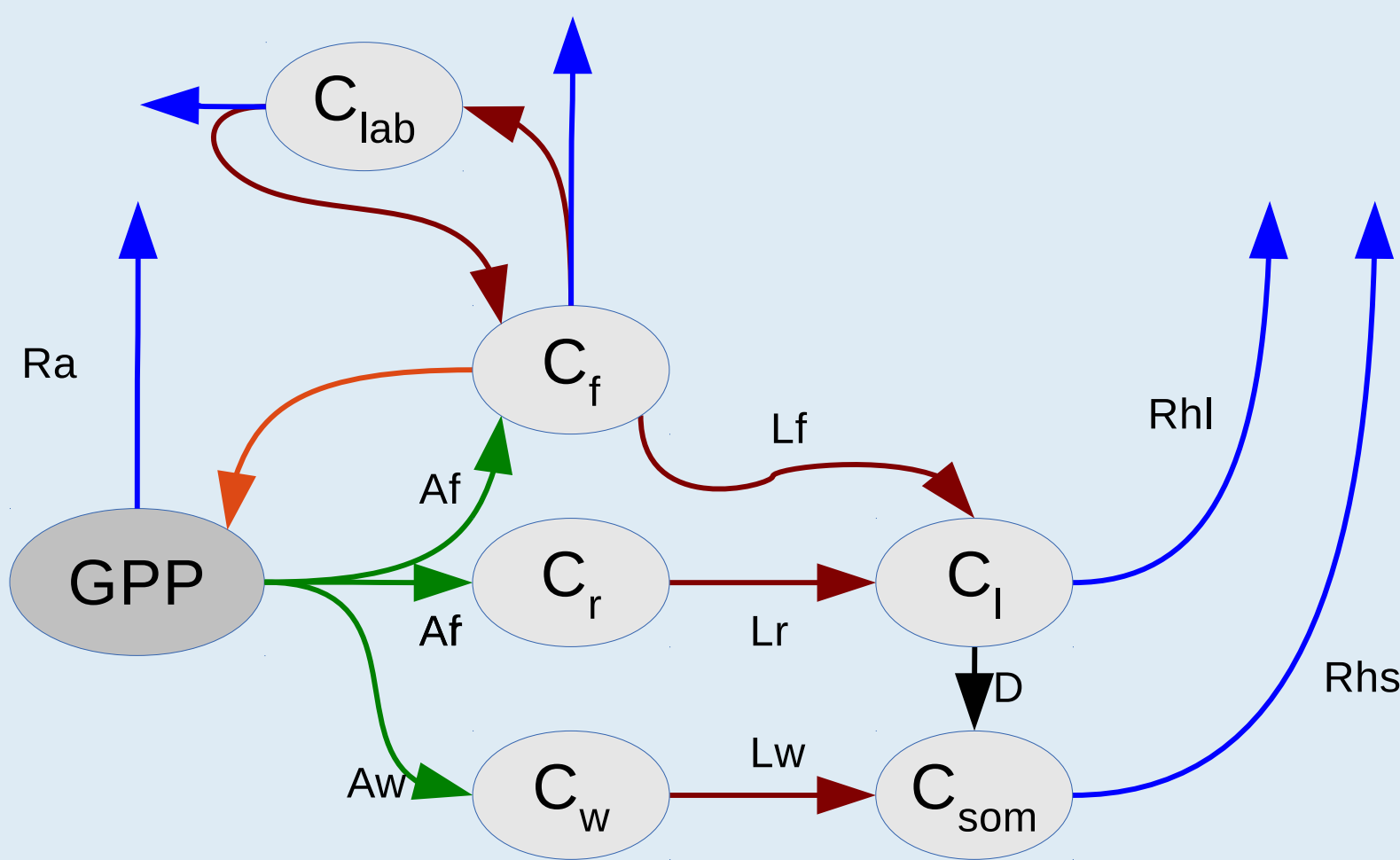
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Context

For over a decade the Data Assimilation-Linked Ecosystem model (DALEC) has been extensively used to confront our knowledge of the C-cycle for terrestrial ecosystem with Earth Observation data using different inverse modelling techniques. Many studies have demonstrated the validity of this approach in improving the parametrization of the model and in refining Carbon fluxes predictions. Various strategies based on empirical knowledge have been used. Here we use MODIS LAI observations together with ecological *common sense* within a variational framework to regularize this otherwise ill-posed problem.

C-cycle model

DALEC depicts a terrestrial ecosystem as a set of six carbon pools (labile C_{lab} , foliar C_f , wood C_w , root C_r , litterfall C_l and soil organic matter C_s) linked via allocation fluxes. At a monthly time step gross primary productivity (GPP) is calculated as a function of meteorological drivers and following a mass conservation principle it is then allocated to the different carbon pools or release in the atmosphere via respiration.



Here we consider 19 control variables for the model: p_1 to p_{17} parametrize allocation, decomposition and phenology processes, p_{18} and p_{19} are the initial values for C_w and C_s , the values of the remaining initial carbon pools are given by long term equilibrium states calculated as functions of the control variables (p_1, \dots, p_{19}).

Observations and Constraints

To constrain the 19 control variables we use monthly MODIS LAI observations. LAI only depends on 10 of the 19 variables of our problem. To bring additional information Bloom and Williams (2014) introduced **ecological and dynamical constraints** (EDC), $g(x) < 0$, including

- turnover rates ($p_1 < p_9, \dots$),
- C pools growth ($C^{(year)} < \alpha C^{(year+1)}$),
- root foliar allocation constraints,
- limited exponential decay,
- steady state proximity.

Other conditions such as

$$\text{LAI}(\text{summer}) < a,$$

$$\underline{\text{NEE}} < \mathbf{E}[\text{NEE}] < \overline{\text{NEE}},$$

can be introduced to further constrain the parameter space.

Variational data assimilation

Variational data assimilation aims at best combining observations, model and prior knowledge by minimizing a cost function

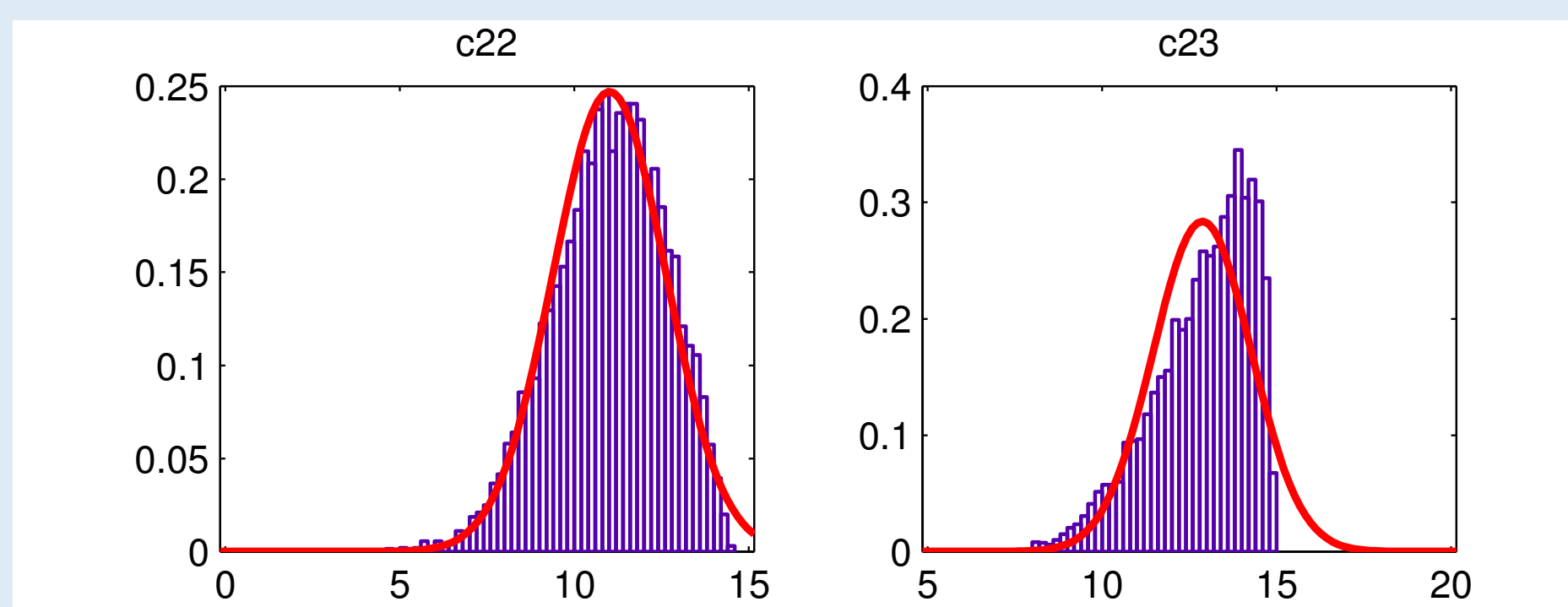
$$J(x) = \frac{1}{2} \|h(x) - y\|_R^2 + \lambda \frac{1}{2} \|g(x) - c\|_\Sigma^2,$$

using a gradient based method where

$$\nabla J(x) = H^T R^{-1} (h(x) - y) + \lambda G^T \Sigma^{-1} (g(x) - c).$$

The covariance matrix of the solution is given by the inverse of the Hessian of J which is approximated by

$$\mathcal{H} \approx H^T R^{-1} H + \lambda G^T \Sigma^{-1} G.$$



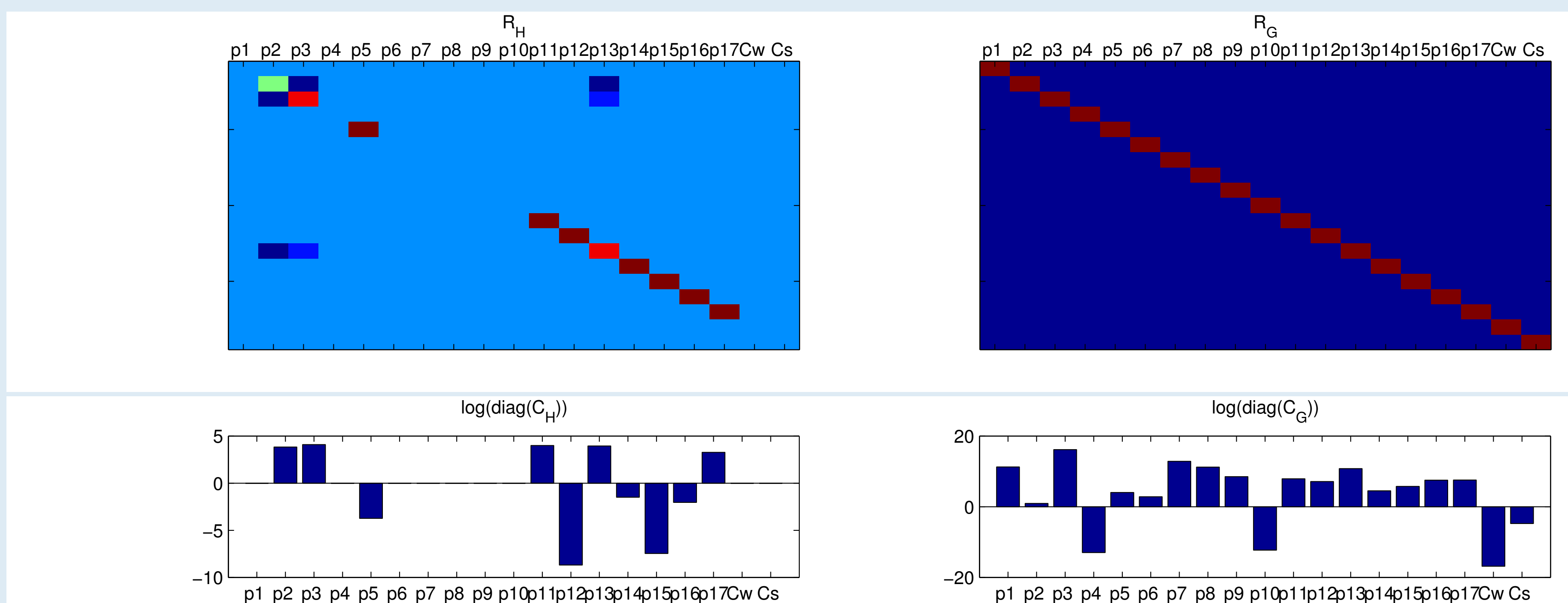
The second term of the cost function encodes the EDC. At the given site 1000 parameter sets satisfying the EDC are generated, fitting a multivariate Gaussian distribution with mean c and covariance Σ provides a reasonable approximation for the EDC values as shown for c_{22} and c_{23} .

Linear analysis

Studying the linear inverse problems $H z = d$ and $G z = e$ using resolution matrices and unit covariance matrices

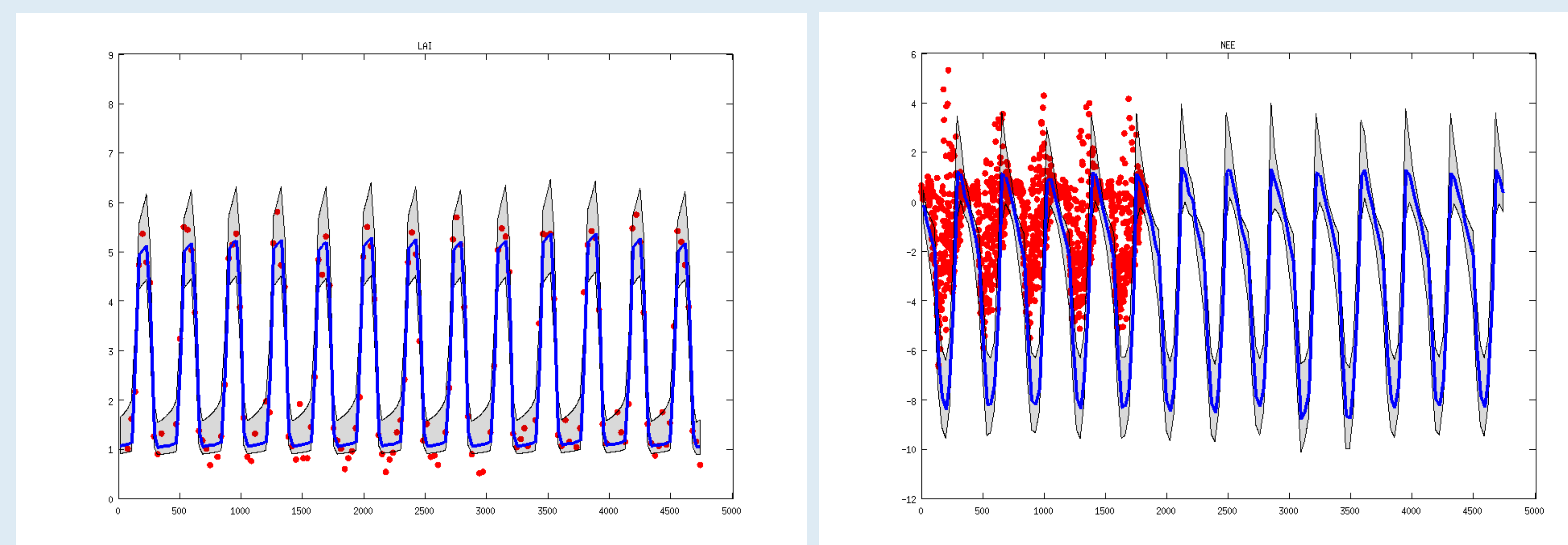
$$R_H = H H^{-g}, \quad R_G = G G^{-g}, \quad C_H = H^{-gT} H^{-g}, \quad C_G = G^{-gT} G^{-g},$$

allows us to assess the information content of each component of the data assimilation problem.



Experiment

We study an example at Howland forest (evergreen forest, USA) for which we consider 12 years of MODIS LAI monthly observations. The figure below on the left shows the results of the variational assimilation for LAI: observations are represented in red, the optimal trajectory is represented in blue, the gray area represents the 90% confidence area obtained by propagating the uncertainty using an ensemble method. To validate our results we show on the figure below on the right how the solution performs against NEE observations at the site.



Ref

- [1] S. Delahaies, I. Roulstone, and N. Nichols (2013). Regularisation of a carbon cycle model-data fusion problem. University of Reading preprint series.
- [2] Bloom, A.A. M. Williams (2014). Constraining ecosystem carbon dynamics in a data-limited world: integrating ecological common sense in a model-data-fusion framework. Biogeosciences Discussions 11, 12733-12772.