Diagnosis and normalisation of wavelet-induced background error variances

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Introduction

- Needs to approximate and model matrix **B**.
- A wavelet block-diagonal approach (Fisher, 2003) is used operationally at Météo-France (Berre et al., 2015), in the global model ARPEGE, and at ECMWF (Bonavita et al., 2016).
- Wavelet **B** computed from an ensemble to get flow-dependent correlations.
- Local spatial averages of correlations (through the wavelet block-diagonal approach), which reduces sampling noise effects.
- Block-diagonal approach has an effect on specified grid point variance.
- Diagnosis and renormalisation of wavelet induced variances.

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Wavelet B

B modelled as a sequence of operators:

$$\mathbf{B} = \mathbf{P}\,\mathbf{\Lambda}^{1/2}\,\mathbf{C}\,\mathbf{\Lambda}^{T/2}\,\mathbf{P}^{T}$$

P balance operator, $\mathbf{\Lambda}^{1/2}$ contains grid-point σ^b, **C** contains correlations.

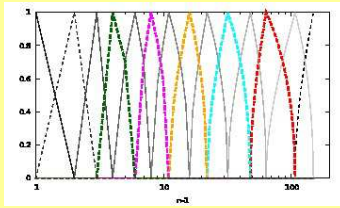
Wavelet block-diagonal representation of **C**:

$$\mathbf{C} = \mathbf{W}^{-1}\,\mathbf{D}\,\mathbf{W}^{-T},$$

where

$$\mathbf{W}^{-1} = (\mathbf{S}^{-1}\hat{\mathbf{R}}_0\,\mathbf{S} \ldots \mathbf{S}^{-1}\hat{\mathbf{R}}_J\,\mathbf{S}),$$

and $\hat{\mathbf{R}}_j$ are diagonal matrices containing filters $\hat{\mathbf{r}}_j$.



Filters $\hat{\mathbf{r}}_0$ (left) to $\hat{\mathbf{r}}_{13}$ (right) used in wavelet transform at T149

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Wavelet induced variance fields

Wavelet induced variance fields can be obtained by formally computing the variance of random draws of the form

$$\boldsymbol{\sigma}^2 = \mathbf{var}\,(\,\mathbf{W}^{-1}\,\mathbf{D}^{1/2}\,\boldsymbol{\eta}),$$

where $\boldsymbol{\eta}$ are uncorrelated random draws.

Using the fact that **D** contains no inter-scale correlations but also no spatial correlations for a given scale j, it is possible to show that

$$\boldsymbol{\sigma}^2 = \tilde{\mathbf{W}}^{-1}\,\mathbf{d},$$

where **d** concatenates wavelet grid point variance fields for all scales j, and

$$\tilde{\mathbf{W}}^{-1} = (\mathbf{S}^{-1}\hat{\tilde{\mathbf{R}}}_0\,\mathbf{S} \ldots \mathbf{S}^{-1}\hat{\tilde{\mathbf{R}}}_J\,\mathbf{S})$$

is a wavelet-like inverse transform with diagonal matrices $\hat{\tilde{\mathbf{R}}}_j$ containing the spectral coefficients

$$\hat{\tilde{r}}_j[n] = \hat{\tilde{h}}_j[n] / \sqrt{2n+1}$$

of $\tilde{\mathbf{h}}_j = \mathbf{h}_j^2$, with n total spectral wave number, and

$$\hat{h}_j[n] = \hat{r}_j[n]\,\sqrt{2n+1}.$$

$\tilde{\mathbf{W}}^{-1}$ can be built easily from the existing code $\mathbf{W}^{-1}$.

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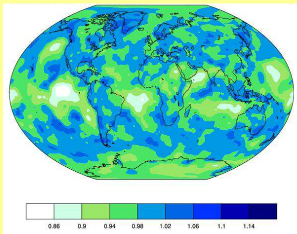
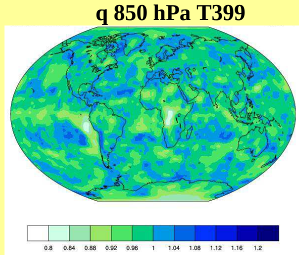
Diagnosis of wavelet-induced standard deviations

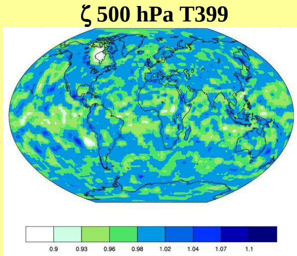
unbal t 500 hPa T399



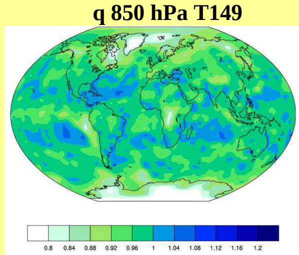
q 850 hPa T399



ζ 500 hPa T399



q 850 hPa T149



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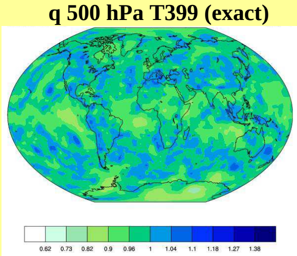
Diagnosis of wavelet-induced standard deviations

q 500 hPa T399 (exact)

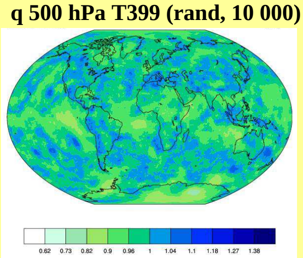


Diagnosed length scale L

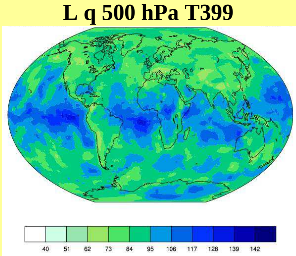
$$L_x = \sigma(\varepsilon) / \sigma(\partial\varepsilon / \partial x)$$

$$L = \sqrt{(L_x^2 + L_y^2)/2}$$

q 500 hPa T399 (rand, 10 000)



L q 500 hPa T399



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Conclusion

- Wavelet block-diagonal representation of **C** at Météo-France and ECMWF.
- Mathematical expression of induced variance fields.
- Possible computation by re-using and modifying wavelet code.
- Cheaper and more precise computation than with randomisation.

- Deviation from 1 depends on variable, level and spectral resolution, but remains mainly small.
- Relationship with correlation length scale:
  - $\sigma < 1$: large L,
  - $\sigma > 1$: small L, with respect to surrounding areas.

- A normalisation of variance fields can be applied and brings slightly positive results.

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