Unexpected Background Error Statistics of Convective-Scale Atmospheric Models

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Abstract

Data assimilation is often used to help make sense of observations made from the traditional meteorological observing systems, from new radar networks, and from space. Effective data assimilation relies on accurate uncertainty statistics of all the data that are used, including those of observations and from the numerical model's forecasts used as the prior ('background') state.

The background error covariance scheme used in the Met Office's current variational data assimilation system is designed to represent errors in large-scale processes, even though many of the models that it serves allow small-scale (convective-scale) process. The work reported here looks at how well the Met Office's current scheme copes with fine-scale features of numerical models and reveals some unexpected characteristics.

Keywords: Data assimilation, Balance, Convective-scale, Numerical Weather Prediction

General idea of data assimilation (DA)

- The forecast ($\mathbf{x}_\mathrm{B}$, blue) has errors that are correlated between model variables x_1 and x_2.
- The observation ($\mathbf{y}$, yellow) is an imperfect observation of only x_1.
- The analysis ($\mathbf{x}_\mathrm{A}$, red) is a combination of $\mathbf{x}_\mathrm{B}$ and $\mathbf{y}$ found from Bayes Theorem.
- The way that $\mathbf{x}_\mathrm{B}$ and $\mathbf{y}$ are combined depend strongly on their error properties.

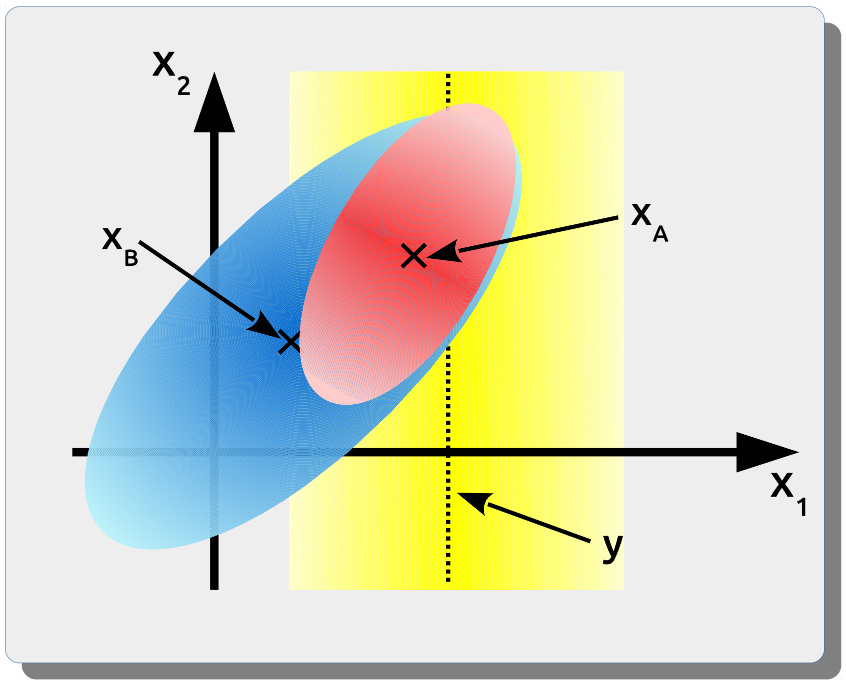


Fig. 1: Combining a model forecast (background state, $\mathbf{x}_\mathrm{B}$) with an observation ($\mathbf{y}$) to give an analysis ($\mathbf{x}_\mathrm{A}$). The coloured shapes indicate the error co-variances of the data. In the case of the background state, errors in components x_1 and x_2 are correlated. Here there is only one observation (of x_1). The analysis and its uncertainty depend upon $\mathbf{x}_\mathrm{B}$, $\mathbf{y}$ and how their errors are characterised.

Data assimilation for large-scale systems

- The $\mathbf{B}$-matrix (describing the blue error bubble in Fig. 1) is modelled by allowing forecast errors to respect geostrophic (Fig. 2) and hydrostatic (Fig. 3) balances.
- This is good for large-scale motion where the flow is balanced.

pressure perturbations	$\delta p = \delta p_\mathrm{b}(\delta \mathbf{v}_\mathrm{h}) + \delta p_\mathrm{u}$
temperature perturbations	$\delta\theta = \delta\theta_\mathrm{b}(\delta p_\mathrm{b}) + 0$

$$\underbrace{\nabla_\mathrm{h}^2 \delta p_\mathrm{b} \approx \rho_0 \mathbf{k} \cdot [\nabla \times (f \delta \mathbf{v}_\mathrm{rot})]}_{\text{geostrophic relationship between forecast errors}} \qquad \underbrace{\frac{R\theta_0^2}{c_p g}\frac{\partial}{\partial z}\left[\frac{1}{p_0}\left(\frac{p_0}{p_{1000}}\right)^{R/cp} \delta p_\mathrm{b}\right] = \delta\theta_\mathrm{b}}_{\text{hydrostatic relationship between forecast errors}}$$

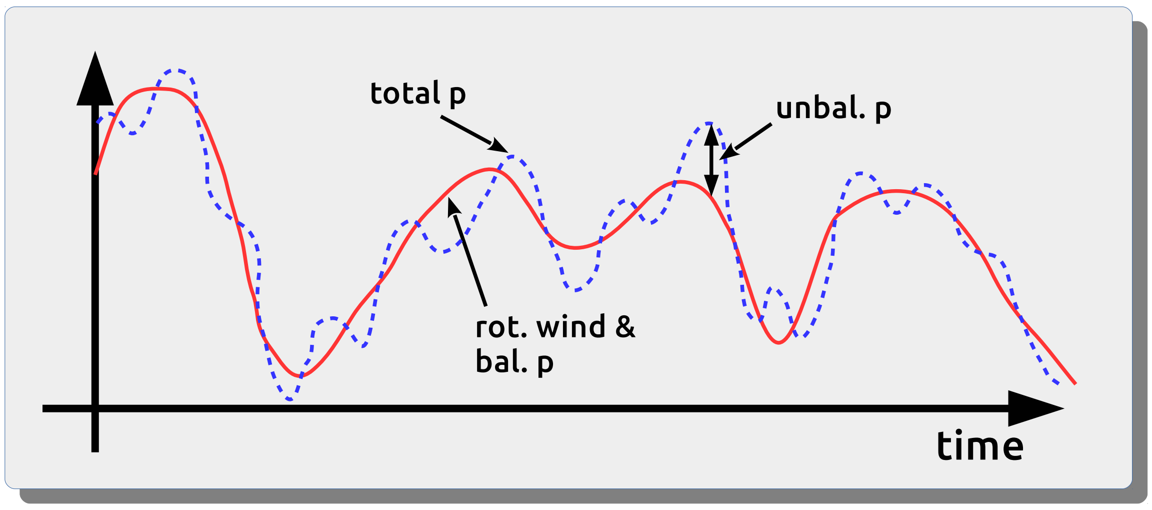


Fig. 2: Schematic evolution of δp in a large-scale system (where geostrophic balance is appropriate). The rotational wind, $\delta \mathbf{v}_\mathrm{r}$, correctly predicts δp_b by geostrophic balance. On top of that is geostrophically unbalanced pressure, δp_u.

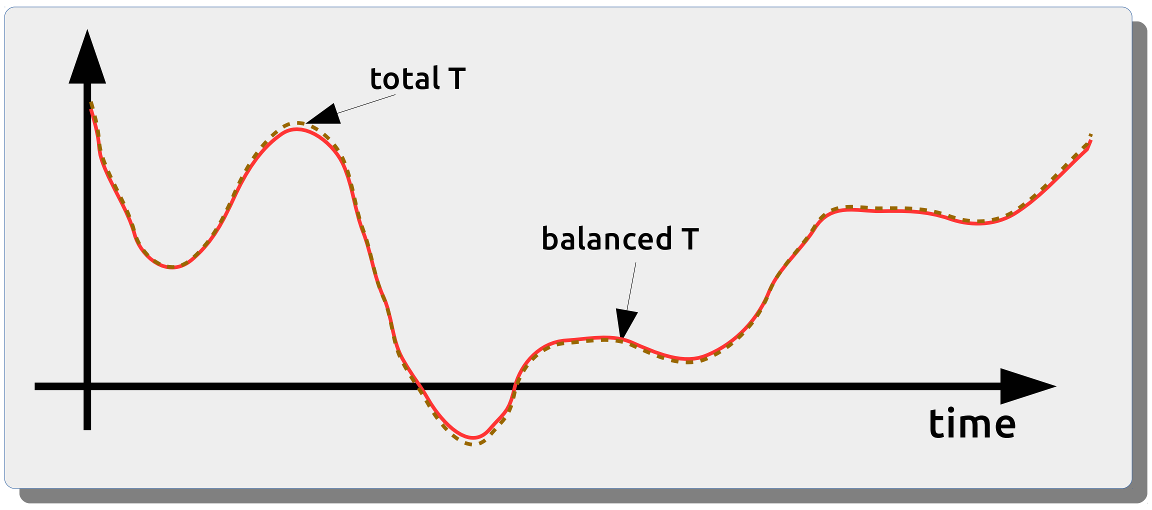
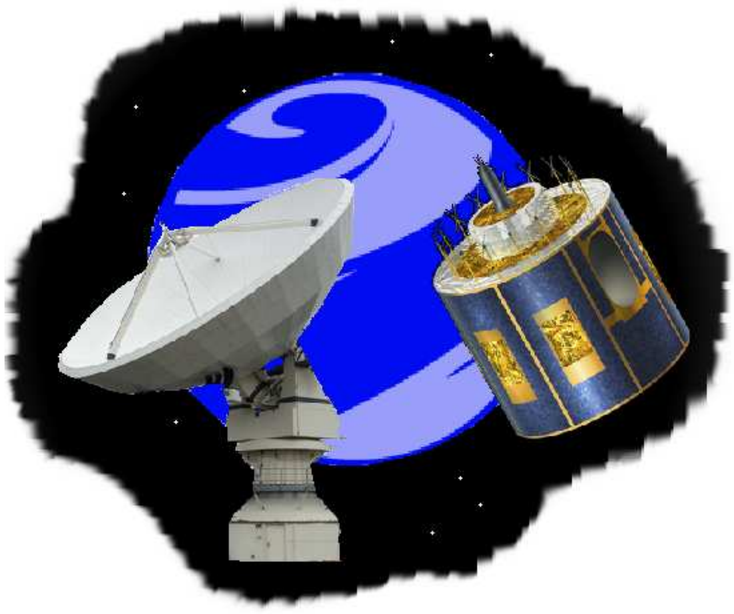


Fig. 3: Schematic evolution of δT in a large-scale system (where hydrostatic balance is appropriate). δp correctly predicts δT_b by hydrostatic balance. The hydrostatically unbalanced temperature, δT_u, is negligible.

Data assimilation for convective-scale systems (currently 1.5km model grid box size)

- Interest is expanding into convective-scale DA.
  - Observation hungry.
  - Shorter predictability timescales.
  - Many scales of motion.
    - Balance appropriate for the larger scales of motion.
    - Balance diminishing for the smaller scales.



- DA systems still model the $\mathbf{B}$-matrix with balance relations. We hypothesize that in such DA:
  - This will make the DA go awry (Fig. 4).
  - Analysis increments of δp_{b} will be anomalously large.
  - Analysis increments of δp_{u} will also be anomalously large (and opposite to δp_{b}), trying to undo the false δp_{b}.
    - $\therefore$ δp_{b} and δp_{u} will be anomalously correlated.
  - We can diagnose these properties from an ensemble of convective-scale forecasts.
  - We can artificially 'damp-down' the balance as a function of scale.

$$\delta p = \Lambda \delta p_{\mathrm{b}}(\delta \mathbf{v}_{\mathrm{h}}) + \delta p_{\mathrm{u}}$$

where $\Lambda \leq 1$ is a damping factor which has the expected properties:

large-scale motion	$\Lambda = 1$
small-scale motion	$\Lambda \to 0$

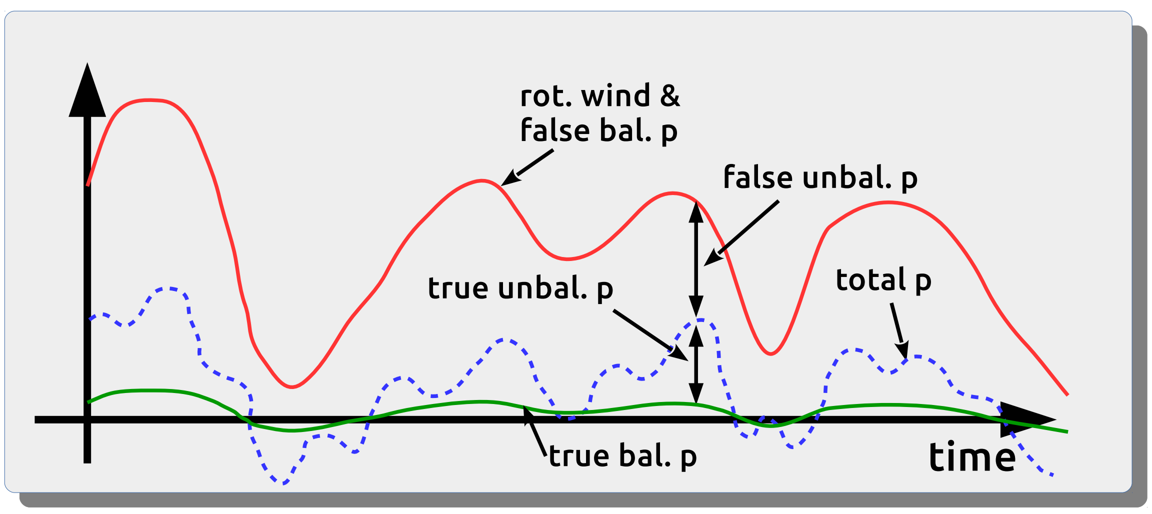


Fig. 4: Hypothesised evolution of temperature in a convective-scale system (where geostrophic balance is inappropriate). The rotational component of the wind incorrectly predicts the balanced pressure.

Diagnostics from a Met Office 24-member convective-scale ensemble (Southern UK domain)

- Standard variational DA uses the background error covariances implied by the $\mathbf{B}$-matrix model (different from the true covariances):

$$\begin{aligned} \mathrm{var}_{\mathrm{true}}(\delta p) &= \mathrm{var}_{\mathrm{implied}}(\delta p) + \mathrm{anomaly}, \\ \text{where anomaly} &= 2\mathrm{cov}(\delta p_{\mathrm{b}} \delta p_{\mathrm{u}}) = 2\mathrm{cov}(\delta p_{\mathrm{b}}[\delta p - \delta p_{\mathrm{b}}]), \\ \delta p_{\mathrm{b}} &: \text{ pressure perturbation found from the geo. bal. equation}, \\ \delta p_{\mathrm{u}} &: \text{ residual}, \ \delta p_u = \delta p - \delta p_{\mathrm{b}}. \end{aligned}$$

- Stats for a mid-tropospheric level (level 35) are in Fig. 5.

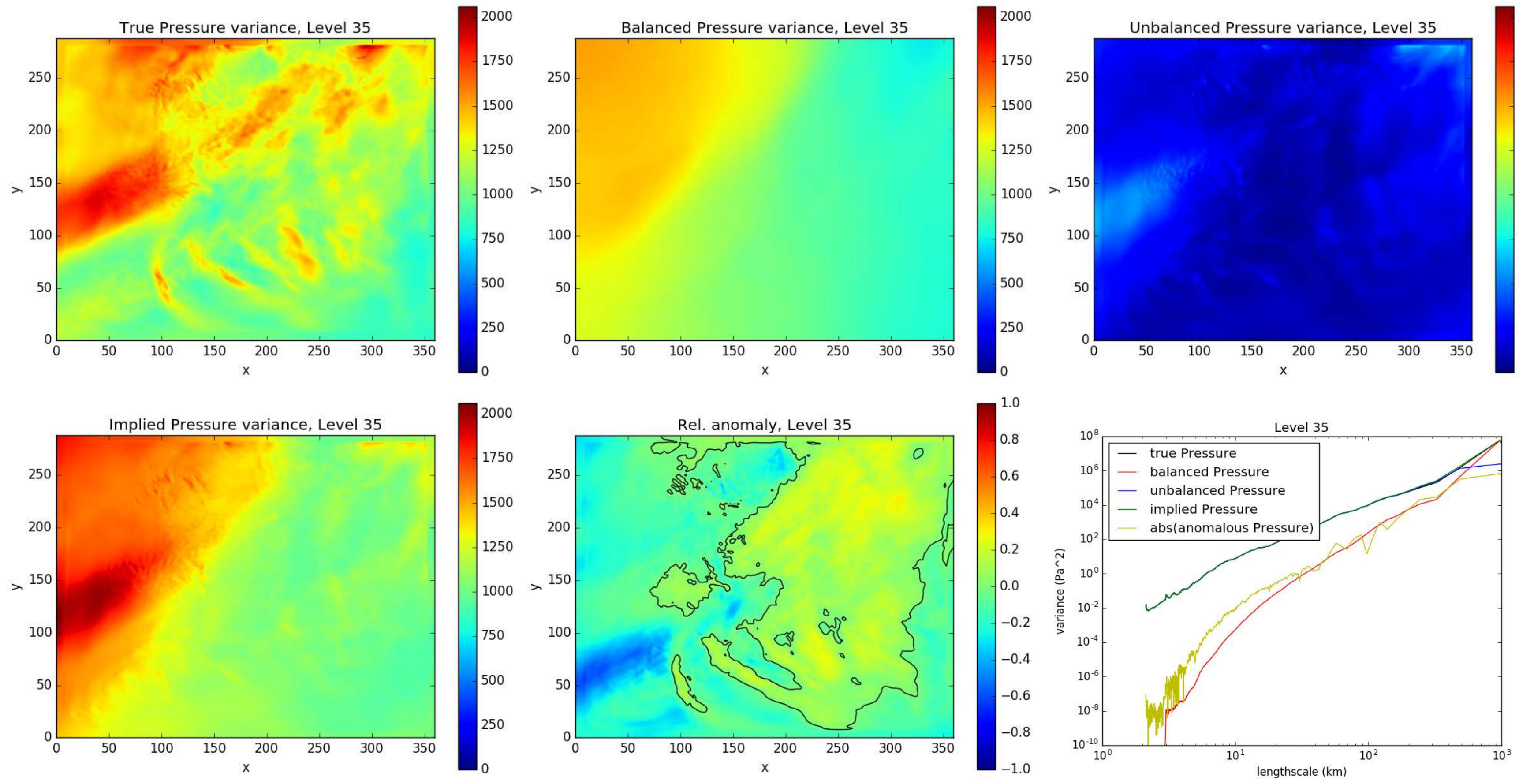


Fig 5: First row: true pressure variance (lev. 35) ($\mathrm{var}(\delta p)$) from the Met Office ensemble when a cold front is traveling eastwards; the balanced variance ($\mathrm{var}(\delta p_{\mathrm{b}})$); the unbalanced variance ($\mathrm{var}(\delta p_{\mathrm{u}})$). Second row: implied pressure variance ($\mathrm{var}(\delta p_{\mathrm{b}}) + \mathrm{var}(\delta p_{\mathrm{u}})$); the relative anomaly ($2\mathrm{cov}(\delta p_{\mathrm{b}} \delta p_{\mathrm{u}})/\mathrm{var}(\delta p)$); statistics as a function of horizontal scale. The implied pressure variance is that 'seen' by the DA if it were calibrated by the ensemble. For a perfect model of $\mathbf{B}$ this would be the same as the true pressure variance.

- Modifying the balance:

$$\begin{aligned} \mathrm{var}_{\mathrm{true}}(\delta p) &= \mathrm{var}_{\mathrm{implied}}(\delta p) + \mathrm{anomaly}, \\ \text{where now anomaly} &= 2\mathrm{cov}(\Lambda \delta p_{\mathrm{b}}[\delta p - \Lambda \delta p_{\mathrm{b}}]). \end{aligned}$$

- Looking for zero anomaly, and Λ can be estimated from data:

$$\Lambda = 0, \qquad \Lambda = \frac{\mathrm{cov}(\delta p_{\mathrm{b}} \delta p)}{\mathrm{var}(\delta p_{\mathrm{b}})}.$$

- Values for Λ are in Fig. 6.

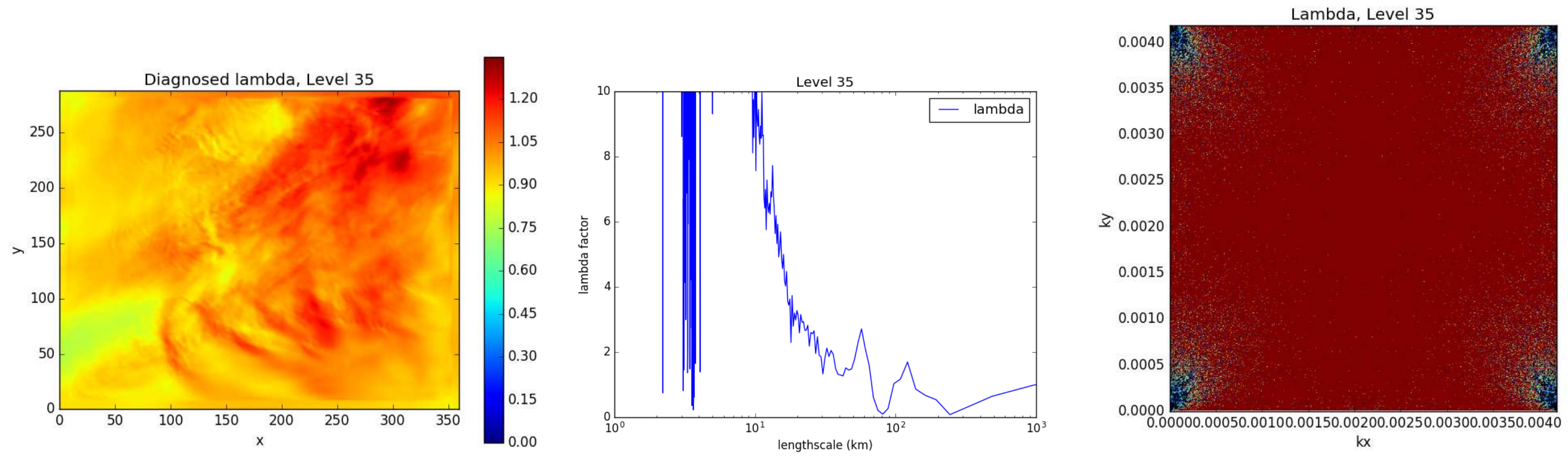


Fig 6: Value of Λ which is needed to multiply the diagnosed δp_{b} to give zero pressure variance anomaly in the data assimilation. Left: Λ as a function of position. Middle: Λ as a function of horizontal scale. Right: Λ as a function of horizontal wave-vector.

Comments

- The 'balanced pressure scaling factor' (Λ) diagnosed here does not behave as expected.
  - It is not found that $\Lambda \to 0$ for smaller scales.
  - It is not found that $\Lambda \leq 1$ always.
- These experiments need to be repeated for other data sources:
  - Other ensembles.
  - Sets of forecast differences as a proxy for forecast errors.
- Hydrostatic balance remains a good approximation, even at the smallest scales in this model.
- Even though Meteorological centres around the world are starting to use ensemble-based DA, it is normally in a hybrid form where the $\mathbf{B}$-matrix is still needed.